Yr11 Yearly Examination Ext 1 2005

QUESTION 1 (20 marks)

(a) Find the coordinates of point C(x,y) which divides the interval joining A (-1,3) and B(5,7) **externally** in the ratio 2:3. 3

(b) Solve and graph on number line $\frac{x+1}{x-3} \geq 2$ 4

(c) Find the acute angle between the line y=2x-1 and 2y+3x=4 4

(d) Show that $\tan 75^\circ = 2+\sqrt{3}$ 3

(e) Differentiate $x^2\sqrt{x^2+1}$ 3

(f) (i)Sketch the graph of $y=|x-2|$ and y=x on same set of axes 2

(ii) For what values of x is $|x-2| \leq x$ 1

QUESTION 2 (22 marks)

(a) A polynomial $P(x) = 9x^3 - 25x^2 + 10bx - b^2$

(i) Find the remainder when P(x) is divided by x-1 2

(ii) Find the value(s) of b if P(x) is divisible by x-1 3

(b) If α, β and γ are the roots of $2x^3 + 3x^2 - 5x + 1 = 0$
Find

(i) $\alpha+\beta+\gamma$ (ii) $\alpha\beta+\beta\gamma+\alpha\gamma$ (iii) $\alpha\beta\gamma$ 3

(iv) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ 2

(v) $\alpha^2+\beta^2+\gamma^2$ 2

(vi) $(\alpha\beta)^2\gamma + \alpha(\beta\gamma)^2 + (\alpha\gamma)^2\beta$ 2

(c) (i)Factorise fully the polynomial $P(x) = x^3 - 6x^2 - 9x + 14$ 3

(ii) Solve P(x)=0 1

(iii) Graph y = P(x) 2

(d) Simplify $\frac{5^n + 5^{n+2}}{5^{n+1}}$ 2

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QUESTION 3 (29 marks)

(a) Solve for $0 \le \theta \le 360°$ to nearest degree where necessary

(i) $2\cos 2\theta + 1 = 0$ 4

(ii) $3\tan^2\theta = \sec^2\theta + 2$ 4

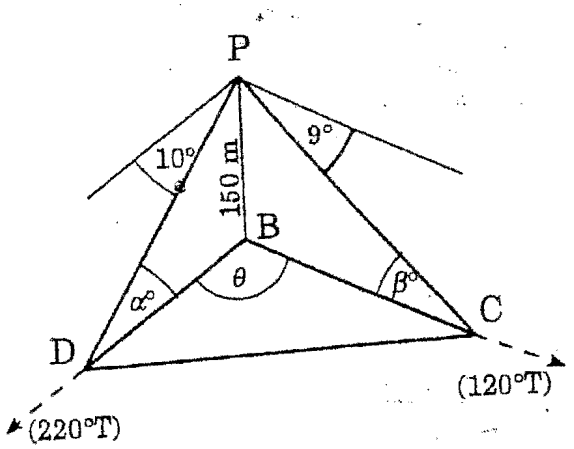
(iii) $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$ 4

(b) If $t=\tan\frac{x}{2}$, prove that

$$\sin x = \frac{2t}{1+t^2}$$ 2

(c) Solve $5\sin x + 12\cos x = 2$ for $0 \le \theta \le 360°$ 4

(d) A ship is observed at the point C from the top of a cliff BP, 150m high in the direction 120°T from B, at an angle of depression 9°, 10 minutes later, it is seen in the direction 220°T from B at an angle of depression of 10°.



Let $\angle PDB = \alpha°$, $\angle PCB = \beta°$ and $\angle CBD = \theta°$

(i) Write down the values of α, β and θ in degrees 3

(ii) Show that $BD = 150\tan 80°$, and $BC = 150\tan 81°$ 3

(iii) Hence show that $CD^2 = 150^2(\tan^2 80° + \tan^2 81° - 2\tan 80°\tan 81°\cos 100°)$ 3

(iv) Find CD 2

QUESTION 4 (19 marks)

(a) A combination lock has 4 dials each with 10 digits. How many possible combinations are there? 1

(b) How many 3 letter words can be made from NORWAY? 1

(c) How many ways can the letters of BACHARACH be arranged using all the letters? 3

(d) A team of 10 boys and 10 girls prefects is to be a chosen from a group of 21 boys and 19 girls. Leave answers in unsimplified form.

(i) How many different ways can the prefects be chosen? 3

(e) If Katrina and Jonathan are hoping to be chosen ,find the probability that

(α) Both will be chosen? 2

(β) Only one will be chosen? 3

(e) To win Lotto you choose 6 numbers out of the numbers 1 to 45 different numbers

(i) In how many ways can this be done 2

(ii) Tas does not select the numbers 11,12,13,14,15 and 16 for Lotto as he says this combination would never win . Is he correct ? Give brief reasons. 2

(f) Find the number of ways in which 3 boys and 3 girls can be seated around a circular table so that no two people of the same sex sit next to each other. 2

QUESTION 5 (21 marks)

(a) (i)Write down the coordinates of the focus, S , and the equation of the directrix of the parabola $x^2=4ay$ 2

(ii) Find the equation of the tangent to $x^2=4ay$ at the point $P(2ap,ap^2)$ 3

(iii) The tangent at P meets the directrix at Q. Find the coordinates of Q. 2

(iv) Find the gradients of the lines PS and QS 2

(v) Prove that PQ subtends a right angle at the focus. 2

(b) (i) Show that the point A $(6p,3p^2)$ lies on the parabola $x^2=12y$ 2

(ii)If B is the point $(6q,3q^2)$. Find the equation of AB 3

(iii)If AB passes through the point C(8,0) show that $4(p+q)=3pq$ 2

(iv) Write down the coordinates of the midpoint M of AB 2

(v)Find the locus of M 3

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QUESTION 1

(a) $(-1, 3)$ $(5, 7)$

$2:-3$ 1 for -ve

$\frac{3+10}{-1}, \frac{-9+14}{-1}$ 1 for each point.

$(-13, -5)$

If answer $\left(\frac{7}{5}, \frac{23}{5}\right)$ 2 marks

(b) $x \neq 3$ ①

$x+1 = 2x-6$

$7 = x$ ①

①

$x=0 \quad \frac{-1}{3} \geq 2$ ✗

$x=4 \quad 5 \geq 2$ ✓

$x=10 \quad \frac{11}{7} \geq 2$ ✗.

$3 < x \leq 7$ ①

(c) $m_1 = 2 \quad 2y = 4-3x$

① ② $y = 2 - \frac{3}{2}x$

$m_2 = -\frac{3}{2}$ ①

$\tan\theta = \left|\frac{-\frac{3}{2}-2}{1+\frac{-3}{2}\times 2}\right|$ ①

$\tan\theta = 1.75$

$\theta = 60^\circ 15'$ ①

(d) $\tan(45^\circ+30^\circ) = \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$ ①

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$ ①

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{3+2\sqrt{3}+1}{2}$

$= \frac{4+2\sqrt{3}}{2}$ ①

$= 2+\sqrt{3}$

(e)(i) $\left|\frac{m-\frac{1}{\sqrt{3}}}{1+\frac{m}{\sqrt{3}}}\right| = \tan 75^\circ$

$\sqrt{3}m - 1 = (2+\sqrt{3})(\sqrt{3}+m)$

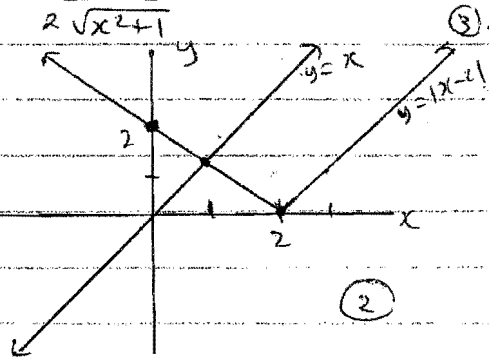
$\sqrt{3}m - 1 = 2\sqrt{3} + 2m + 3 + \sqrt{3}m$

$\therefore 2m =$

(e) $\frac{dy}{dx} = x^2 \times \frac{1}{2}(x^2+1)^{-1/2} \times 2x + 2x\sqrt{x^2+1}$

$= \frac{2x^3}{2\sqrt{x^2+1}} + 2x\sqrt{x^2+1}$ ③.

(f)



②

(i) $x \geq 1$ ①

(ii) $x \geq 1$ ①

Q2d) $\frac{5^n + 5^{n+2}}{5^{n+1}} = \frac{5^n(1+5^2)}{5^{n+1}}$

$= \frac{26}{5}$

QUESTION 3

(a) (i) $\cos 2\theta = -\frac{1}{2}$ (1)

$2\theta = 120, 240, 480, 600$ (1)

$\theta = 60, 120, 240, 300°$ (1).

2 marks if $\theta = 60, 120$ given.

(ii) $3\tan^2\theta = 1 + \tan^2\theta + 2$

$2\tan^2\theta = 3$

$\tan^2\theta = \frac{3}{2}$

$\tan\theta = \pm\sqrt{\frac{3}{2}}$

$\theta = 50°46', 230°46', 129°14', 309°14'$.

(iii) $\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin\theta}$ (1)

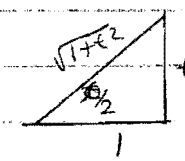
$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$ (1)

$1 = 2\cos\theta$

$\cos\theta = \frac{1}{2}$ (1)

$\theta = 60°, 300°$ (1).

(b)



$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= 2 \times \frac{t}{\sqrt{1+t^2}} \times \frac{1}{\sqrt{1+t^2}}$

$= 2 \times \frac{t}{\sqrt{1+t^2}} \times \frac{1}{\sqrt{1+t^2}}$

$\sin x = \frac{2t}{1+t^2}$ (2)

(c) $5 \times \frac{2t}{1+t^2} + 12 \times \frac{1-t^2}{1+t^2} = 2$

$10t + 12 - 12t^2 = 2 + 2t^2$

$14t^2 - 10t - 10 = 0$

$7t^2 - 5t - 5 = 0$

$t = \frac{5 \pm \sqrt{25+140}}{14}$

$\therefore \tan\frac{\theta}{2} = 1.275$ or -0.560

$\frac{\theta}{2} = 51°53', 150°44'$

$\theta = 103°46', 301°28'$ (4)

(d)(i) $\alpha = 10°$ $\beta = 9°$ $\theta = 100°$ (3)

(ii) $\frac{BD}{PB} = \tan(90-\alpha)$

$BD = 150\tan 80$ similarly

$BC = 150\tan(90-9)$

$= 150\tan 81$ (3)

(iii) using cosine rule

$CD^2 = (150\tan 80)^2 + (150\tan 81)^2 - 150^2\tan 80\tan 81\cos 10°$

$CD^2 = 150^2(\tan^2 80 + \tan^2 81 - \tan 80\tan 81\cos 10°)$ (3)

(iv) $CD^2 = 1379$

(2).

Question 4

(a) $10 \times 10 \times 10 \times 10$

$= 10{,}000$ (1)

(b) ∴ ∴ (1)

$6 \times 5 \times 4 = 120$

(c) $\frac{9!}{3!2!2!}$

1 for $9!$
2 if miss out 1 factorial on bottom.

(d) (i) ${}^{21}C_{10} \times 1 \quad {}^{19}C_{10}$ = (2)
(1) (1)

(ii) ${}^{20}C_{9} \times {}^{18}C_{9} = \frac{21C_{10} \times 19C_{10}}{}$ (2)

(iii) ${}^{20}C_{9} \times {}^{18}C_{10} + {}^{20}C_{10} \times {}^{18}C_{9}$ (3)
$\overline{\qquad\qquad 21C_{10} \times 19C_{10} \qquad\qquad}$

(e) (i) ${}^{45}C_{6} = 8{,}145{,}060$ (2)

(ii) No since this selection has as good a chance as any. Since selection of numbers is independent. (2)

(f) B' G G B B G

$\therefore 3! \times 2! = 12$

Question 5

(a) (i) $(0, a)$ $y = -a$ (2)

(ii) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$= \frac{x}{2a}$

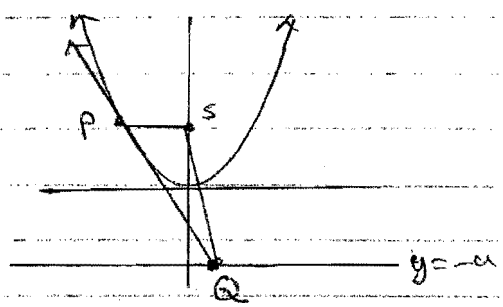
When $x = 2ap$

$\frac{dy}{dx} = p.$

$y - ap^2 = p(x - 2ap)$ (3)

$y = px - ap^2$

(iii)



when $y = -a$

$-a = px - ap^2$

$px = -a + ap^2$

$x = \frac{a(-1+p^2)}{p}$

Q $\left(\frac{a(p^2-1)}{p}, -a\right)$ (2)

(iv) $S(0, a)$ $P(2ap, ap^2)$

$M_{PS} = \frac{ap^2 - a}{2ap}$

$= \frac{p^2-1}{2p}$

$= \frac{p^2-1}{2p}$

QUESTION 5.

$M_{QS} = \dfrac{-a-a}{a(p^2-1)}$

$= \dfrac{-2a}{a(p^2-1)}$

$= \dfrac{-2}{p^2-1}$ (2)

(v) $\therefore M_{PS} \times M_{QS}$

$= \dfrac{p^2-1}{2p} \times \dfrac{-2p}{p^2-1}$

$= -1$

$\therefore PS \perp QS$ (2)

a right angled is subtended at focus.

(b) $x = 6p \quad y = 3p^2$

$LHS = 36p^2$

$RHS = 12 \times 3p^2$

$= 36p^2$

(2)

$\therefore (6p, 3p^2)$ a point on $x^2 = 12y$

(ii) $M_{AB} = \dfrac{3q^2 - 3p^2}{6q - 6p}$

$= \dfrac{3(q-p)(q+p)}{6(q-p)}$

$= \dfrac{p+q}{2}$

$= \dfrac{x}{2}$

$y - 3q^2 = \left(\dfrac{p+q}{2}\right)(x - 6q)$

$2y - 6q^2 = px + qx - 6pq - 6q^2$

$\therefore (p+q)x - 2y - 6pq = 0$ (3)

(iii) when $x = 8 \quad y = 0$

$8(p+q) = 6pq$ (2)

$\therefore 4(p+q) = 3pq$

(iv) $M\left(\dfrac{6p+6q}{2}, \dfrac{3p^2+3q^2}{2}\right)$

$M\left(\dfrac{3p+3q}{2}, \dfrac{3p^2+3q^2}{2}\right)$

(2)

(v) $x = 3(p+q)$

$y = \dfrac{3}{2}(p^2+q^2)$

$(p+q)^2 = p^2 + q^2 + 2pq$

but $pq = \dfrac{4}{3}(p+q)$

$\therefore p^2 + q^2 = (p+q)^2 - \dfrac{8}{3}(p+q)$

$\therefore \dfrac{2}{3}y = p^2 + q^2$

$\dfrac{x}{3} = p + q$

$\therefore \dfrac{2}{3}y = \dfrac{x^2}{9} - \dfrac{8}{9}x$

$\therefore 6y = x^2 - 8x$ (3)

$\therefore 6y = x^2 - 8x$ (3)