GIRRAWEEN HIGH SCHOOL

YEARLY EXAMINATION

YEAR 11

2011

MATHEMATICS

EXTENSION 1

Time allowed: Two hours

(Plus 5 minutes reading time)

Directions to candidates:

- Attempt all questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board approved calculators may be used.
- Start each question on a new page.

Question 1 (10 marks)

Circle the letter corresponding to the correct answer.

(a) If $P(x) = ax^4 - x^3 + 3x^2 - 5$ and $P(1) = -1$, then a is equal to :

(A) 1 (B) 0 (C) 2 (D) -3

(b) Which of the following is a factor of $f(x) = x^4 - 4x^3 - x^2 + 16x - 12$?

(A) $x+1$ (B) $x+2$ (C) x (D) $x+3$

(c) The derivative of $x - \frac{1}{2x}$ is equal to

(A) $\frac{1}{2}$ (B) $1 + \frac{1}{2x^2}$ (C) $1 + \frac{1}{4x^2}$ (D) $1 - \frac{1}{4x^2}$

(d) If $f(x) = x^2 - 6x$, then $f'(4)$ is equal to:

(A) 8 (B) -12 (C) 12 (D) 2

(e) $y = \sqrt{x^2 - 3x + 2}$ expressed as $y = u^n$, $\frac{dy}{du}$ is equal to

(A) $\frac{1}{2}u$ (B) $u^{-\frac{1}{2}}$ (C) $\frac{1}{2\sqrt{u}}$ (D) $\frac{1}{2}u^{\frac{3}{2}}$

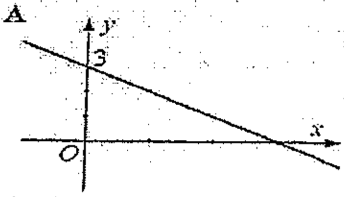
(f) A function is known to have the following properties:

(i) $f(0) = 3$ (ii) $f(x) \geq 0$ for all x.

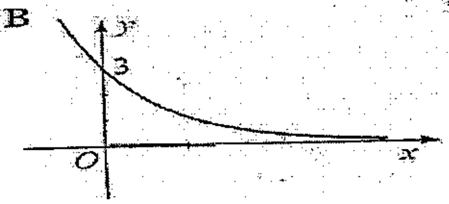
(ii) $f'(x) < 0$ for all x. (iv) $f(x) \to 0$ as $x \to \infty$.

Which of the following graphs could represent this function?

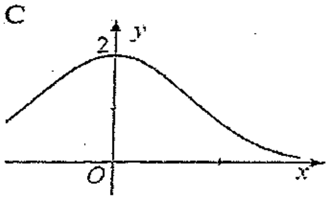
A



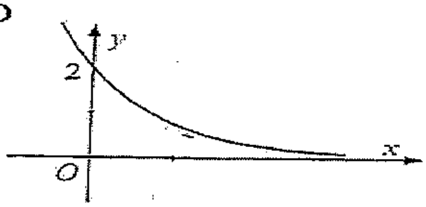
B



C



D



(g) If $y+\frac{1}{y}=3$, then $y^2+\frac{1}{y^2}$ is equal to

(A) 5 (B) 7 (C) 9 (D) 11

(h) If the graph of $f(x)$ crosses the $x-$axis exactly three times, which one of the following rules could not be the rule for f?

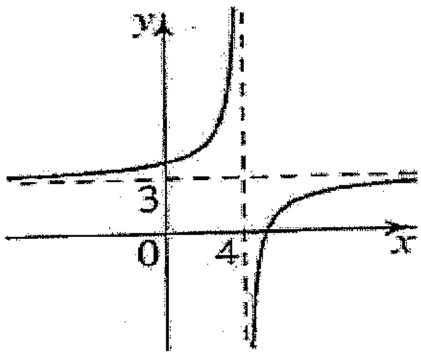
(A) $(x^2-x-6)(x^2-x-12)$ (B) $x(x^2-4)$

(C) $(3-x)(x^4-16)$ (D) $x(x-2)(x+4)(x^2+1)$

(i) $f(x)=\frac{x+1}{x}$, then $f\left(\frac{1}{a}\right)$, in simplified form, is equal to

(A) $1+\frac{1}{1+a}$ (B) $1+a$ (C) $\frac{a}{a+1}$ (D) $\frac{a+1}{a}$

(j) The equation of the graph shown is likely to be:



(A) $y=3+\frac{1}{x-4}$ (B) $y=\frac{1}{x-3}+4$ (C) $3-\frac{1}{x-4}$ (D) $y=\frac{1}{4-x}-3$

Question 2 (19 marks)

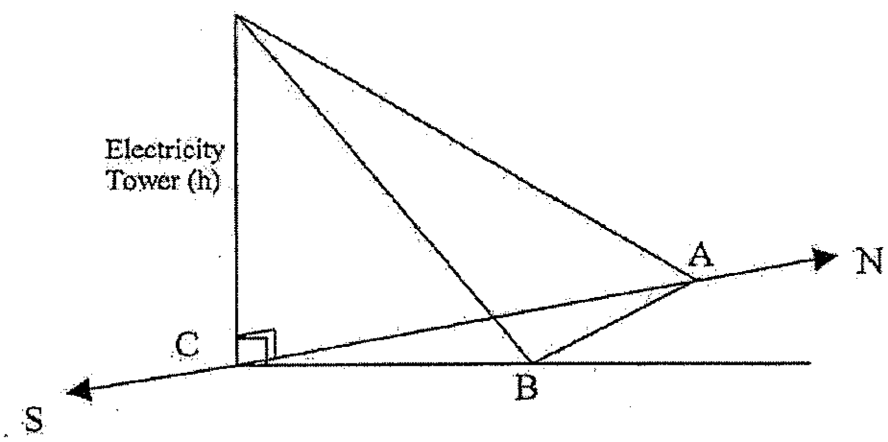
(a) Solve: $\dfrac{4x-1}{x-2} \le 3$ 3

(b) Find the acute angle between the lines $2x+3y-5=0$ and $5x+2y+1=0$ correct to the nearest minute. 3

(c) Find the coordinates of the point P which divides the interval joining $A(4,8)$ and $B(-3,1)$ externally in the ratio 2:1. 3

(d) Differentiate: $y=\dfrac{3x+1}{\sqrt{x+1}}$ 3

(e) Leon walks on level ground, in a northerly direction, away from an electricity tower. When he arrive at a point A, the angle of elevation to the top of the tower is $23°$. Luke walks on level ground on a bearing of $032°T$ from the same tower, until he reaches point B, and notices that the angle of elevation is $17°$. The distance between A and B is 55m. Let h be the height of the tower and assume that the tower base C, is perpendicular to the ground.



NOT TO SCALE

(i) Copy the diagram above onto your booklet and clearly mark on it all the information given. 2

(ii) Find expressions for AC and BC in terms of h. 2

(iii) Hence, or otherwise, find the height h of the tower to the nearest metre. 3

Question 3 (19 marks)

(a) Find the value of k if the remainder is 20 when x^3+3x^2+7x-k is divided by $x-2$. 2

(b) Given that $(x-3)$ and $(x+2)$ are factors of $P(x)=x^3-6x^2+px+q$. Find the values of p and q. 4

(c) The cubic equation $2x^3-x^2+x-3=0$ has roots α,β and γ. Evaluate:

(i) $\alpha+\beta+\gamma$ **(ii)** $\alpha\beta+\alpha\gamma+\beta\gamma$ **(iii)** $\alpha\beta\gamma$ 3

(iv) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ **(v)** $\alpha^2\beta\gamma+\alpha\beta^2\gamma+\alpha\beta\gamma^2$ **(vi)** $\alpha^2+\beta^2+\gamma^2$ 6

(d) Solve the equation $4x^3+32x^2+79x+60=0$ given that one root is the sum of the other two, using sum and product of roots. 4

Question 4 (21 marks)

(a) Find the exact value of:

(i) $\cos 15°$ **(ii)** $2\cos^2 22\frac{1}{2}°-1$ 4

(b) Prove that $\frac{2\cos A}{\cos ecA-2\sin A}=\tan 2A$ 4

(c) Solve: $5\sin 2\theta=2\sin\theta$ 4

(d) Solve using t method.

$2\sin\theta-3\cos\theta=3, 0°\leq\theta\leq 360°$ 4

(e) (i) Express $3\cos x+2\sin x$ in the form $A\cos(x-\alpha)$, clearly stating the value of A and α (to the nearest degree) 2

(ii) Hence, solve $3\cos x+2\sin x=\sqrt{13}$, $0°\leq x\leq 360°$ 3

Question 5 (24 marks)

(a) How many different words can be formed with the letters of the word

EQUATION so that

(i) the words begin with E.

(ii) the words begin with E and end with N.

(iii) the words begin and end with a consonant. 4

(b) An SRC meeting is arranged for 20 students along two sides of a long table with 10 chairs on each side. Four students wish to sit on one particular side and two on the other side. In how many ways they can be seated? 3

(c) **(i)** Find how many arrangements can be made by taking all the letters of the word MATHEMATICS.

(ii) In how many of them do the vowels occur together? 4

(d) From a class of 17 students, including Vivian and Manraj, four students are selected for a committee. How many committees are possible if

(i) there are no restrictions.

(ii) both Vivian and Manraj are selected.

(iii) neither Vivian nor Manraj is selected.

(iv) either Vivian or Manraj, not both is selected. 5

(e) In how many ways can 5 boys and 5 girls be arranged in a circle if

(i) there are no restrictions.

(ii) boys and girls alternate.

(iii) two particular boys want to be together.

(iv) a particular boy want to be opposite to a particular girl.

(v) two particular girls do not want to sit next to each other. 8

Question 6 (15 marks)

(a) Find the Cartesian equation of the parabola $x = 6t, y = 3t^2$. 2

(b) Find the parametric equations for the parabola $x^2 = 2y$ 3

(c) **(i)** Find the equation of the chord joining $P(-8,8)$ and $Q(2,\frac{1}{2})$ where P and Q are points on the parabola $x^2 = 8y$. 2

(ii) Show that PQ is a focal chord. 2

(d) **(i)** Show that the equation of the tangent to the parabola $x^2 = 4ay$ at the point t is $y - tx + at^2 = 0$. 2

(ii) Show that the point of intersection of the tangents at $t = p$ and $t = q$ on the parabola $x = 4t, y = 2t^2$ is $(2(p+q), 2pq)$. 4

END OF QUESTION PAPER

Year 11 Extension 1 Yearly 2011 – Solutions

Question 1 (10 marks)

(a) C (b) B (c) B (d) D (e) C

(f) B (g) B (h) A (i) B (j) C

Question 2 (19 marks)

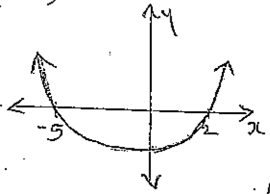
(a) $\frac{4x-1}{x-2} \leq 3$

Multiply by $(x-2)^2, x \neq 2$

$(x-2)(4x-1) \leq 3(x-2)^2$

$(x-2)[4x-1-3(x-2)] \leq 0$

$(x-2)(x+5) \leq 0$



(3)

$-5 \leq x < 2$

(b) $2x+3y-5=0$

$m_1 = \frac{-2}{3}$

$5x+2y+1=0$

$m_2 = \frac{-5}{2}$

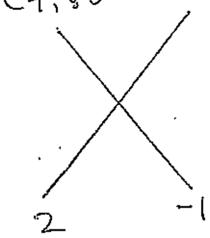
$\tan\alpha = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{\frac{-2}{3}+\frac{5}{2}}{1+\frac{10}{6}}\right| = \frac{11}{16}$

(3)

$\theta = 34^\circ 30'$

(c) A(4, 8) B(-3, 1)



$x = \frac{(2\times -3)+(-1\times 4)}{2-1} = -10$

$y = \frac{(2\times 1)+(-1\times 8)}{2-1} = -6$

(3)

P(-10, -6)

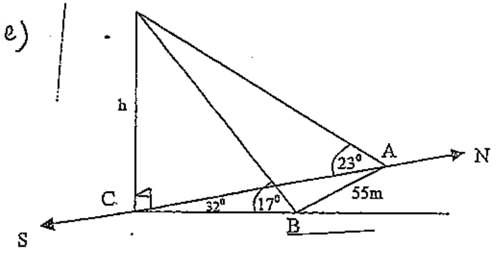
(d) $y = \frac{3x+1}{\sqrt{x+1}}$

$y' = \frac{\sqrt{x+1}\times 3 - (3x+1)\times\frac{1}{2\sqrt{x+1}}}{x+1}$

$= \frac{6(x+1)-3x-1}{2(x+1)^{\frac{3}{2}}} = \frac{3x+5}{2\sqrt{(x+1)^3}}$

(3)

(e)



(2)

(ii) $\tan 67^\circ = \frac{AC}{h}$

$AC = h\tan 67^\circ$

$\tan 73^\circ = \frac{BC}{h}$

(2)

$BC = h\tan 73^\circ$

By Cosine rule,

(iii) $55^2 = h^2\tan^2 67^\circ + h^2\tan^2 73^\circ - 2h\tan 67 \times h\tan 73^\circ \cos 32^\circ$

$= h^2(\tan^2 67^\circ + \tan^2 73^\circ - 2\tan 67 \cdot \tan 73 \cos 32)$

$h^2 = \frac{55^2}{\tan^2 67^\circ + \tan^2 73^\circ - 2\tan 67 \tan 73 \cdot \cos 32^\circ}$

$h = 31\text{m}$ (3)

Question 3 (19 marks)

(a) $P(2) = 8+3\times 4+14-k$

$= 8+12+14-k$

$= 34-k$

$34-k = 20$

$k = 14$ (2)

(b) $P(3)=0$ and $P(-2)=0$

$P(3) = -27+3p+q$

$P(-2) = -32-2p+q$

$3p+q = 27$

$-2p+q = 32$

$5p = -5$

$p = -1$

$q = 27-3p$

$= 27-3\times -1 = 30$

(4)

$p = -1;\ q = 30$

(c) (i) $\alpha+\beta+\gamma = -\frac{(-1)}{2} = \frac{1}{2}$ (1)

(ii) $\alpha\beta+\alpha\gamma+\beta\gamma = \frac{1}{2}$ (1)

(iii) $\alpha\beta\gamma = \frac{3}{2}$ (1)

(iv) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$= \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$

$= \frac{1}{2} \div \frac{3}{2} = \frac{1}{3}$ (2)

(v) $\alpha^2\beta\gamma+\alpha\beta^2\gamma+\alpha\beta\gamma^2$

$= \alpha\beta\gamma(\alpha+\beta+\gamma)$ (2)

$= \frac{3}{2}\times\frac{1}{2} = \frac{3}{4}$

(vi) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= \left(\frac{1}{2}\right)^2 - 2\times\frac{1}{2}$

$= \frac{1}{4} - 1 = \frac{-3}{4}$ (2)

(d) Let the roots be α, β and $\alpha+\beta$.

Sum of roots:

$\alpha+\beta+\alpha+\beta = -\frac{32}{4}$

$2\alpha+2\beta = \frac{-32}{4}$

$2(\alpha+\beta) = \frac{-32}{4}$

$\alpha+\beta = \frac{-32}{4}\times\frac{1}{2}$

$\alpha+\beta = -4$ —— (1)

Product of roots:

$\alpha\beta(\alpha+\beta) = \frac{-60}{4} = -15$

$-4\alpha\beta = -15$

$\alpha\beta = \frac{15}{4}$ —— (2)

From (1) $\beta = -4-\alpha$

Substitute in (2)

$\alpha(-4-\alpha) = \frac{15}{4}$

$-4\alpha-\alpha^2 = \frac{15}{4}$

$-16\alpha - 4\alpha^2 = 15$

$4\alpha^2+16\alpha+15 = 0$

$(2\alpha+5)(2\alpha+3) = 0$

$\alpha = -\frac{5}{2}$ or $\alpha = \frac{-3}{2}$

$\alpha = \frac{-5}{2}$

$\beta = -4-\alpha = \frac{-3}{2}$

$\alpha = \frac{-3}{2}$

$\beta = -4+\frac{3}{2}$

$= \frac{-5}{2}$

(4)

$\therefore$ the roots are

$-4, \frac{-3}{2}, \frac{-5}{2}$

Question 4 (21 marks)

(a) (i) $\cos 15^\circ = \cos(45-30)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2}$ (2)

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $2\cos^2 22\frac{1}{2}^\circ - 1$

$= \cos 2\times 22\frac{1}{2}^\circ$

$= \cos 45$ (2)

$= \frac{1}{\sqrt{2}}$

(b) $LHS = \frac{2\cos A}{\operatorname{cosec} A - 2\sin A}$

$= \frac{2\cos A}{\frac{1}{\sin A} - 2\sin A}$

$= \frac{2\cos A}{\frac{1-2\sin^2 A}{\sin A}}$

$= \frac{2\cos A \times \sin A}{1-2\sin^2 A}$ (4)

$= \frac{\sin 2}{\cos 2}$

$= \tan 2A = RHS$

(c) $5\sin 2\theta = 2\sin\theta$

$5\times 2\sin\theta\cos\theta = 2\sin\theta$

$10\sin\theta\cos\theta - 2\sin\theta = 0$

$2\sin\theta(5\cos\theta - 1) = 0$ (4)

$\sin\theta = 0$ or $\cos\theta = \frac{1}{5}$

$\theta = 0^\circ, 180^\circ, 360^\circ, 78^\circ 28', 281^\circ 32'$

(d) $2\sin\theta - 3\cos\theta = 3, \ 0 \le \theta \le 360^\circ$

$2\left(\frac{2t}{1+t^2}\right) - 3\left(\frac{1-t^2}{1+t^2}\right) = 3$

$4t - 3(1-t^2) = 3(1+t^2)$

$4t = 6$

$t = \frac{3}{2}$

$\tan\frac{\theta}{2} = \frac{3}{2}$

$\frac{\theta}{2} = 56.31^\circ$ $\quad \theta = 112.62^\circ$

Check for 180°

$LHS = 2\sin 180 - 3\cos 180$

$= 0 - 3\times -1 = 3$

$RHS = 3$ (4)

$\therefore \theta = 112.62^\circ, 180^\circ$

(e)(i) Let $3\cos x + 2\sin x \equiv A[\cos x\cos\alpha + \sin x\sin\alpha]$

$\equiv A\cos x\cos\alpha + A\sin x\sin\alpha$

$A\cos\alpha = 3$ $\quad A = \sqrt{13}$

$A\sin\alpha = 2$ (2)

$\sin\alpha = \frac{2}{\sqrt{13}}$ $\quad \alpha = 33°41'$

$\cos\alpha = \frac{3}{\sqrt{13}}$ $\quad 3\cos x + 2\sin x \equiv \sqrt{13}\cos(x - 33°41')$

(ii) $\sqrt{13}\cos(x - 33°41') = \sqrt{13}$

$\cos(x - 33°41') = 1$

$x - 33°41' = 0°, 360°$ (3)

$x = 33°41', 393°41'$

$= \underline{\underline{33°41'}}$

Question 5 (24 marks)

(a)(i) $7! = 5040$ (1)

(ii) $6! = 720$ (1)

(iii) $3 \times 2 \times 6! = 4320$ (2)

(b) 4 students in $10 \times 9 \times 8 \times 7$ ways

2 " " 10×9 ways

remaining 14 in $14!$ ways

Total number of arrangements =

$= \underline{\underline{10 \times 9 \times 8 \times 7 \times 10 \times 9 \times 14!}}$ (3)

(c)(i) $\frac{11!}{2! \times 2! \times 2!} = \underline{\underline{4989600}}$

(ii) $\frac{8!}{2! \times 2!} \times \frac{4!}{2!}$ (4)

$= \underline{\underline{120960}}$

(d) (i) ${}^{17}C_4 = \underline{\underline{2380}}$ (1)

(ii) ${}^{15}C_2 = \underline{\underline{105}}$ (1)

(iii) ${}^{15}C_4 = \underline{\underline{1365}}$ (1)

(iv) Number of committees in which Vivian is a member and Manraj is not a member

$= {}^{15}C_3 = \underline{\underline{455}}$

No. of committees in which either Vivian or Manraj is a member $= 2 \times 455$

$= \underline{910}$ (2)

(e)(i) $9! = \underline{\underline{362880}}$ (1)

(ii) $1 \times 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1$

$= \underline{\underline{2880}}$ (2)

(iii) $8! \times 2$ (2)

(iv) $8!$ (1)

(v) Number of ways in which the girls are not together

$= 9! - 8! \times 2$

$= \underline{\underline{282240}}$ (2)

Question 6 (15 marks)

(a) $x = 6t$

$y = 3t^2$ —— (2)

From (1) $t = \frac{x}{6}$. Substitute in (2)

$y = 3 \times \frac{x^2}{36} = \frac{x^2}{12}$ (2)

$\underline{\underline{x^2 = 12y}}$

(b) $x^2 = 2y$

$= 4 \times \frac{1}{2}y \quad \therefore a = \frac{1}{2}$

Parametric equations are

$x = 2at = 2 \times \frac{1}{2}t = \underline{\underline{t}}$

$y = at^2 = \underline{\underline{\frac{1}{2}t^2}}$ (3)

(c) (i) Gradient $= \dfrac{8-\frac{1}{2}}{-8-2}$

$= \dfrac{-3}{4}$

Equation of the chord

$y - 8 = \dfrac{-3}{4}(x+8)$

$4y - 32 = -3x - 24$ (2)

$3x + 4y - 32 + 24 = 0$

$3x + 4y - 8 = 0$

(ii) $x^2 = 8y$

$a = 2$

Focus $(0, 2)$

Substitute $(0,2)$ in

$3x + 4y - 8 = 0$

LHS $= 0 + 8 - 8 = 0$

RHS $= 0$ (2)

$\therefore 3x + 4y - 8 = 0$ is

a focal chord

(d) $x^2 = 4ay$

$x = 2at \quad y = at^2$

$\dfrac{dx}{dt} = 2a \quad \dfrac{dy}{dt} = a \times 2t$

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$= \dfrac{2at}{2a} = t$

Equation of tangent

$y - at^2 = t(x - 2at)$

$y - at^2 = xt - 2at^2$

$y - tx - at^2 + 2at^2 = 0$ (2)

$y - tx + at^2 = 0$

(ii) $x = 4t, \ y = 2t^2$

$a = 2$

tangent at p : $y - px + 2p^2 = 0$ — (1)

tangent at q : $y - qx + 2q^2 = 0$ — (2)

(1) − (2) $\quad -px + qx + 2p^2 - 2q^2 = 0$

$2(p^2 - q^2) = px - qx$

$2(p+q)(p-q) = x(p-q)$

$x = 2(p+q)$

(4)

$y = px - 2p^2$

$= 2p(p+q) - 2p^2$

$= 2p^2 + 2pq - 2p^2$

$= 2pq.$

Point of intersection

$= (2(p+q), 2pq)$