Gosford High School

Year 11

2008
Preliminary
Higher School Certificate

Mathematics
Extension 1
Assessment Task 4

Time Allowed – 1 hour 30 mins + 5 mins reading

Remember to start each new question on a new page
Students must answer questions using a blue/black pen, and a sharpened B or HB pencil for graphs.
Approved scientific calculators may be used
Students need to be aware that

- 'bald' answers may not gain full marks.
- untidy and/or poorly organised solutions may not gain full marks.

Mathematics Extension 1 – Preliminary Higher School Certificate – 2008

Marks

Question 1 (12 Marks) Review

a) Solve $\dfrac{2}{x-1} \leq 1$ **2**

b) A is the point $(1, 2)$ and B is the point $(-1, -4)$. Find the coordinates of the point P which divides AB externally in the ratio $1 : 2$. **3**

c) Find the obtuse angle between the lines

$3x - y = 1$ and $x + 2y = 4$. (Answer to nearest minute.) **3**

d) Solve $\dfrac{x^2-6}{x} \leq 1$ **4**

Question 2 (15 Marks) Calculus (Begin a new sheet of paper)

a) Differentiate:

(i) $\sqrt{2}x - \sqrt{x} + \dfrac{x^2}{\sqrt{x}} - \dfrac{1}{2x}$ **4**

(ii) $x(3x-2)^5$ and simplify your answer **3**

(iii) $\dfrac{2x+1}{x-3}$ and simplify your answer **2**

b) Differentiate from first principles: $f(x) = 2x - 3x^2$ **3**

c) Find the equation of the normal to the curve $y = 3x^2 - 4x + 7$ at the point where $x=1$. **3**

Question 3 (14 Marks) Quadratic Function & Polynomials (Begin a new sheet of paper) **Marks**

a) Find the values of m for which the expression $x^2 + (m-2)x + 4$ is positive definite. **3**

b) Find the values of a, b, and c if $3x^2 - 5x + 2 \equiv a(x+1)^2 + b(x+1) + c$ **3**

c) (i) Show that $x+1$ is a factor of $P(x) = -x^3 + 3x + 2$ **1**

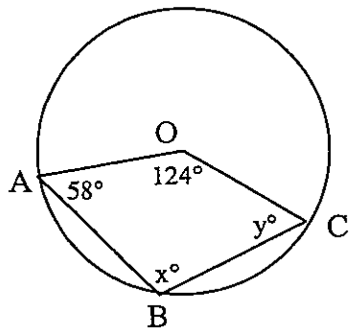
(ii) Fully factorise $P(x)$ **4**

(iii) Hence sketch the graph of $y = P(x)$ **3**

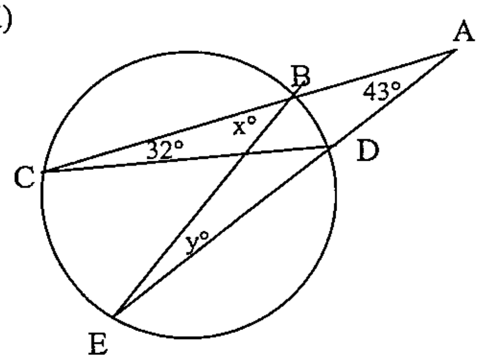
Question 4 (12 Marks) Circle Geometry (Begin a new sheet of paper)

a) Find the value of the pronumerals x and y, giving all reasons

(i) O is the centre **3**

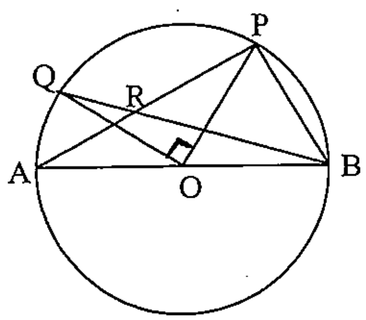


(ii) **3**



b)

(i) **3**



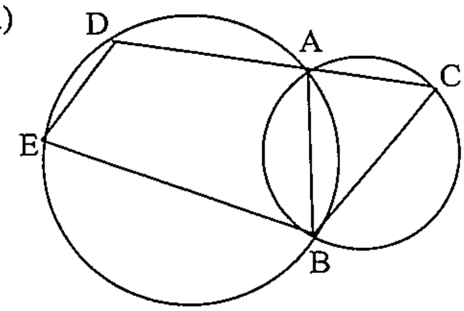
O is the centre.

$\angle POQ = 90°$

Find the size of $\angle PBQ$

Hence, prove that PB=PR

(ii) **3**



EB is a tangent to circle ABC

Prove that BC $\parallel$ ED

Question 5 (12 Marks) **(Begin a new sheet of paper)** **Marks**

a) (i) Write $\sqrt{3}\sin\theta + \cos\theta$ in the form $R\cos(\theta - \alpha)$ **3**

(ii) Solve $\sqrt{3}\sin\theta + \cos\theta = 1$ for $-180° \le \theta \le 360°$ **3**

b) The roots of $x^3 - x^2 - 5x + 2 = 0$ are $\alpha,\ \beta$ and γ.

(i) Write down the values of $\alpha\beta\gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$ **2**

(ii) Show that $\beta + \gamma = 1 - \alpha$ **1**

(iii) **Hence** evaluate $\dfrac{\alpha+\beta}{\gamma} + \dfrac{\beta+\gamma}{\alpha} + \dfrac{\gamma+\alpha}{\beta}$ **3**

Question 6 (13 Marks) Extension (Begin a new sheet of paper)

a) If α and β are the roots of $2x^2 + 5x - 3 = 0$, form the equation with roots α^2 and β^2 **3**

b) Express the derivative of $y = \dfrac{2x+1}{\sqrt{3x-2}}$ in simplest form. **3**

c) The point P(x, y) divides the join of A and B in the ratio $k : 1$

If A is the point (1, 2) and B is the point (–1, –4) ,

(i) Write expressions for x and y in terms of k. **2**

(ii) **Hence** find the value of k, such that the line $3x - 4y - 5 = 0$ divides AB in the ratio $k : 1$ **2**

d) Find as a relation between k, l, and m, the condition for the quadratic equation in x

$$(k^2 + l^2)x^2 + 2l(k+m)x + (l^2 + m^2) = 0$$ **3**

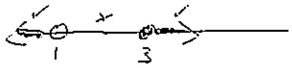
to have real roots. Simplify your answer as far as possible.

Q2/ $\frac{2}{x-1} \le 1$ CP at $x=1, x\neq 1$

Make equation to find other CP's

$2 = x-1$

$3 = x$



$x \geq 3$ and $x < 1$ (2)

(b) $(1,2)$ $(-1,-4)$

$(-1 : 2)$

$\left(\frac{(-1)(-1)+(2)(1)}{1}, \frac{(-1)(4)+(2)(2)}{1}\right)$

$(3, 8)$ (3)

(c) $y = 3x-1$ $2y = -x+4$

$m_1 = 3$ $m_2 = -\frac{1}{2}$

$\tan\theta = \left|\frac{3+\frac{1}{2}}{1+(3)(-\frac{1}{2})}\right|$

$\tan\theta = |-7|$

$\theta = 81°52'$ but obtuse

$\therefore \theta = 98°8'$ (3)

(d) Solve $\frac{x^2-6}{x} \le 1$ CP at $x=0, x\neq 0$

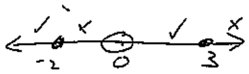
Make equation to find other CP's

$x^2-6 = x$

$x^2-x-6=0$

$(x-3)(x+2)=0$

CP's at $x=3, -2$



$0 < x \le 3$ and $x \le -2$ (4)

a) i) $\frac{d}{dx}\left(\sqrt{2}x - \sqrt{x} + \frac{x^2}{\sqrt{x}} - \frac{1}{2x}\right)$

$= \frac{d}{dx}\left(\sqrt{2}x - x^{1/2} + x^{3/2} - \frac{x^{-1}}{2}\right)$

$= \sqrt{2} - \frac{1}{2}x^{-1/2} + \frac{3}{2}x^{1/2} + \frac{x^{-2}}{2}$

$= \sqrt{2} - \frac{1}{2\sqrt{x}} + \frac{3\sqrt{x}}{2} + \frac{1}{2x^2}$

ii) $\frac{d}{dx}\, x(3x-2)^5$

$= (3x-2)^5 + 5x(3x-2)^4 . 3$

$= (3x-2)^5 + 15x(3x-2)^4$

$= (3x-2)^4(3x-2+15x)$

$= (3x-2)^4(18x-2)$

$= 2(3x-2)^4(9x-1)$

iii) $\frac{d}{dx}\,\frac{2x+1}{x-3}$

$= \frac{(x-3).2-(2x+1)}{(x-3)^2}$

$= \frac{2x-6-2x-1}{(x-3)^2}$

$= \frac{-7}{(x-3)^2}$

$\lim_{h\to 0} \frac{2(x+h)-3(x+h)^2-(2x-3x^2)}{h}$

$= \lim_{h\to 0} \frac{2x+2h-3x^2-6xh-3h^2-2x+3x^2}{h}$

$= \lim_{h\to 0} \frac{2h-6xh-3h^2}{h}$

$= \lim_{h\to 0} \frac{h(2-6x-3h)}{h}$

$= \lim_{h\to 0} 2-6x-3h$

$= 2-6x$

c) $y = 3x^2-4x+7$

$\frac{dy}{dx} = 6x-4$

When $x=1$ $\frac{dy}{dx} = 6-4 = 2$

and $y = 3-4+7 = 6$

so $M_T = 2$ and $M_N = -\frac{1}{2}$ at $(1,6)$

$y-y_1 = m(x-x_1)$

$y-6 = -\frac{1}{2}(x-1)$

$2y-12 = -x+1$

$x+2y-13=0$

QUESTION 3

$\Delta < 0$

$b^2-4ac < 0$ (1)

$(m-6)(m+2)<0$ (1)

$-2 < m < 6$ (1)

$\le$ (-1) $-2 \le m = 6$ (2)

3// $\Delta < 0$ must

$3x^2-5x+2 \equiv a(x^2+2x+1)+b(x+1)+c.$

$\equiv ax^2+2ax+a+bx+b+c$

-1 each error

$a=3$ $2a+b=-5$, $b=-11$ $a+b+c=2$, $3-11+c=2$, $c=10$ (3)

or sub $x=-1$ gives $c=10$

$P(-1) = -(-1)^3+3x-1+2$

$= 0$ $\therefore$ factor $x+1$ is (1) (1)

) $x+1$) $-x^2+x+2$

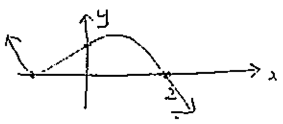
$(x+1)(-x^2+x+2)$ (2)

$-(x+1)(x^2-x-2)$ (4)

$-(x+1)(x-2)(x+1)$ (1)

$-(x+1)^2(x-2)$ (1)

iv)



cuts $x=2$ (-1)

turns $x=-1$ (-1)

start under (-1)

y intercept 2 $-\frac{1}{2}$

no axes label $-\frac{1}{2}$

(2)

QUESTION 4

a) $x = \frac{1}{2} \times 236$ $\angle$ centre $= 2\times$ angle circle (1)

$= 118$ (1)

$y = 60°$ $\angle$ sum quad (1)

124 non $\angle$ sum quad $y=$ (5+)

ii) $y = 32°$ $\angle$ on same arc. (1)

$\angle CDE = 75°$ exterior angle

$\angle x = 75°$ $\angle$ on same arc (2)

(4)

b) $\angle APB = 90°$ ($\angle$ in semicircle)

$\angle PBQ = 45°$ ($\angle$ centre $2\times$ angle circum) (1) (3)

$\angle PRB = 45°$ ($\angle$ sum $\triangle$)

$\therefore$ PRB isosceles $\triangle$ 2 base $\angle$s equal

ii) $\angle EBA = \angle BCA$ (angle in alternate seg (1)

$\angle EDA = 180-x$ opp $\angle$ in cyclic quad supp (1)

$\angle EDA + \angle DCB = 180°$ (cointerior $\angle$'s) (1)

$\therefore$ BE $\parallel$ DE

$\cos\theta + \sqrt{3}\sin\theta$

$R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$\therefore R\cos\alpha = 1$

$R\sin\alpha = \sqrt{3}$

$\tan\alpha = \sqrt{3}$

$\alpha = 60°$

$R = \sqrt{3+1}$

$= 2$

$\cos\theta + \sqrt{3}\sin\theta = 2\cos(\theta - 60°)$

$2\cos(\theta - 60°) = 1$

$\cos(\theta - 60°) = \frac{1}{2}$

$\theta - 60° = 60°, 300°$ or $-60°$

$\theta = 120°, 360°$ or $0°$

) $\alpha\beta\gamma = -2$

$\alpha\beta + \beta\gamma + \gamma\alpha = -5$

) $\alpha + \beta + \gamma = -\frac{b}{a}$

$= 1$

$\therefore \beta + \gamma = 1 - \alpha$

$\frac{\alpha+\beta}{\gamma} + \frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta}$

$\frac{1-\gamma}{\gamma} + \frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}$

$\frac{\alpha\beta - \alpha\beta\gamma + \beta\gamma - \alpha\beta\gamma + \alpha\gamma - \alpha\beta\gamma}{\alpha\beta\gamma}$

$\frac{\alpha\beta + \beta\gamma + \alpha\gamma - 3\alpha\beta\gamma}{\alpha\beta\gamma}$

$\frac{-5+6}{-2}$

$-\frac{1}{2}$

7(a)

$(\alpha + \beta) = \frac{-5}{2}$ $\alpha\beta = -\frac{3}{2}$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $(\alpha\beta)^2 = \frac{9}{4}$

$= \frac{25}{4} + \frac{6}{2}$

$= \frac{37}{4}$

$\therefore x^2 - (\alpha^2+\beta^2)x + (\alpha\beta)^2 = x^2 - \frac{37}{4}x + \frac{9}{4}$ 3

$= 4x^2 - 37x + 9$

$\therefore$ required equation is

$4x^2 - 37x + 9 = 0$

(b) $y = \frac{2x+1}{(3x-2)^{\frac{1}{2}}}$

$y' = \frac{(3x-2)^{\frac{1}{2}} \cdot 2 - [(2x+1) \cdot \frac{1}{2}(3x-2)^{-\frac{1}{2}} \cdot 3]}{(3x-2)}$

$= \frac{(3x-2)^{-\frac{1}{2}}[2(3x-2) - \frac{3}{2}(2x+1)]}{(3x-2)}$

$= \frac{\frac{4(3x-2) - 3(2x+1)}{2}}{(3x-2)^{\frac{3}{2}}}$

$= \frac{12x - 8 - 6x - 3}{2\sqrt{(3x-2)^3}}$?

$= \frac{6x - 11}{2\sqrt{(3x-2)^3}}$

26 (c) (i)   A $(1,2)$ B $(-1,-4)$

$k : 1$

$\frac{(k)(-1) + (1)(1)}{k+1}, \frac{(k)(-4) + (1)(2)}{k+1}$

$\left(\frac{1-k}{k+1}, \frac{2-4k}{k+1}\right)$ (2)

(ii) $\frac{3(1-k)}{k+1} - \frac{4(2-4k)}{(k+1)} - \frac{5(k+1)}{(k+1)} = 0$

$3 - 3k - 8 + 16k - 5k - 5 = 0$

$-10 + 8k = 0$

$8k = 10$

$k = \frac{10}{8}$

$k = \frac{5}{4}$ (2)

d) Real roots if $b^2 - 4ac \geq 0$

$[2l(k+m)]^2 - 4(k^2+l^2)(l^2+m^2) \geq 0$

$4l^2(k^2 + 2km + m^2) - 4(k^2l^2 + k^2m^2 + l^4 + l^2m^2) \geq 0$

$4k^2l^2 + 8kl^2m + 4l^2m^2 - 4k^2l^2 - 4k^2m^2 - 4l^4 - 4l^2m^2 \geq 0$

$-4l^4 + 8kl^2m - 4k^2m^2 \geq 0$

$-4(l^4 - 2kl^2m + k^2m^2) \geq 0$

$l^4 - 2kl^2m + k^2m^2 \leq 0$

l^2 $-km$

l^2 $-km$ (3)

$(l^2 - km)^2 \leq 0$

As left hand side must be positive or zero

then $l^2 = km$