YEAR 11 Mathematics Ext 1
Preliminary Course
Assessment Task 4
2009

Time Allowed – 1Hour 30 Minutes

1. There are 5 questions, NOT of equal value.
2. Answer each question in the booklets provided.
3. Start each Question in a new booklet.
4. Show all necessary working
5. Use the ruled side of the page only.
6. Calculators may be used

Topic	Mark
1. Question 1 (Assorted topics)	/14
2. Question 2 (Polynomials)	/12
3. Question 3 (Counting Theory)	/12
4. Question 4 (Circle Geometry)	/8
5. Question 5 (Trigonometry)	/16

2009 – Extension 1 – Preliminary Higher School Certificate

Question 1 14 Marks **Marks**

a) Differentiate $(x^2+1)\sqrt{x^2+1}$ **2**

b) Find the obtuse angle between the lines $y=2x$ and $x+y-3=0$ giving the answer correct to the nearest minute. **3**

c) A(–2,3) and B(6,–1) are two points. Find the coordinates of the point P which divides the interval AB externally in the ratio 3 : 2. **3**

d) Solve and graph your solution on a number line: **3**

$$\frac{x^2-4}{x+3} \leq x-4$$

e) Simplify $$\frac{2^{4n}\times 3^{2n}}{8^n\times 6^n} + 3^n + 3^n$$ **3**

Question 2 (12 Marks) Begin a New Booklet **Marks**

a) Find the remainder when $P(x) = x^3 - 3x^2 + 3x - 5$ is divided by $x - 2$ **2**

b) The equation $x^3 + 3x^2 - 5x + 6 = 0$ has roots α, β and γ. Find the value of **2**

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

c) (i) Use the Factor Theorem to show that $x^2 - 4$ is a factor of the polynomial **2**

$$P(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$$

(ii) Use long division to show that $P(x)$ has a **repeated** zero of $x = -1$ **2**

(iii) Hence factorise $P(x)$ completely into linear factors. **1**

(iv) Sketch the graph of $y = P(x)$ **2**

(v) Hence or otherwise, solve $x^4 + 2x^3 - 3x^2 - 8x - 4 \geq 0$ **1**

Question 3 (12 Marks) Begin a New Booklet **Marks**

a) (i) From a group of 7 girls and 6 boys, 3 girls and 2 boys are chosen. How many different groups of 5 are possible? **2**

(ii) If the group of 5 stands in a line, how many arrangements will have the boys standing together? **2**

b) (i) Bob chooses six numbers from the numbers 1 to 40 inclusive. In how many ways can he do this? **1**

(ii) A machine then chooses six numbers from the numbers 1 to 40 inclusive. In how many ways can the machine's numbers all be different from any of Bob's numbers? **1**

c) The letters from the word KURRARONG are cut out, and placed in a hat.

(i) All of the nine letters are drawn out one at a time and placed in a line, to form a "word". How many different "words" are possible? **1**

The letters are then replaced in the hat and mixed well.

(ii) Four letters are drawn out at random and placed in a line, to form a "word". How many different four letter "words" can be made in this way? **3**

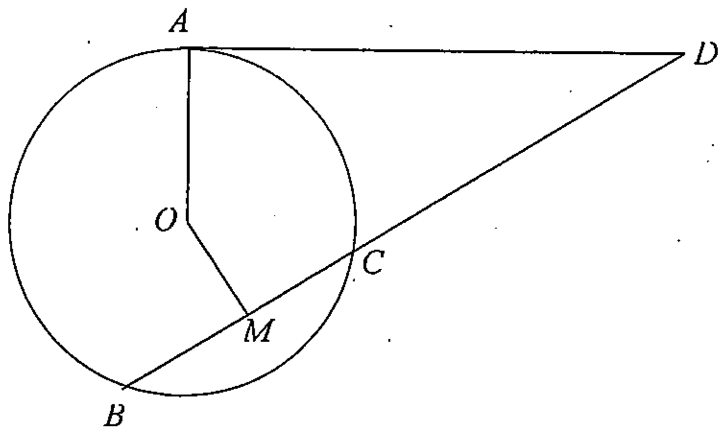
d) Show that $\dfrac{1}{(n-1)!} - \dfrac{1}{n!} + \dfrac{1}{(n+1)!} = \dfrac{n^2}{(n+1)!}$ **2**

Question 4 (8 Marks) Begin a New Booklet **Marks**

a) A, B and C are points on a circle with centre O. The tangent to the circle at A meets BC produced at D. M is the midpoint of BC.

(i) Copy the diagram and show that AOMD is a cyclic quadrilateral. **3**

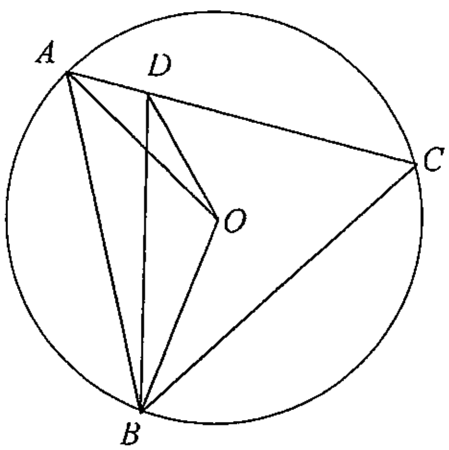
(ii) Give a reason why OD is a diameter of the circle through A, O, M and D. **1**



b) ABC is a triangle inscribed in a circle with centre O. D is a point on CA such that ABOD is a cyclic quadrilateral.

(i) Copy the diagram. Give a reason why $\angle BOA = \angle BDA$. **1**

(ii) Hence show that BD = CD. **3**



Question 5 (16 Marks) Begin a New Booklet **Marks**

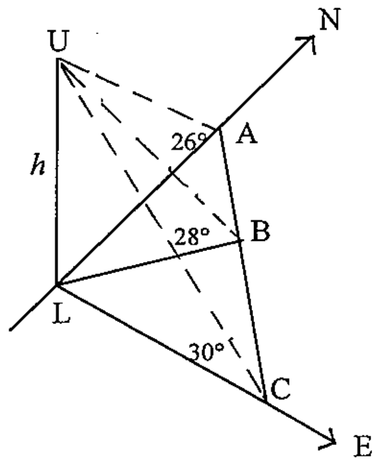
a) Express $\tan(90° + \theta)$ in terms of $\tan\theta$ **1**

b) Solve the equation $\cos 2x + \sin x = 0$ for $0° \le x \le 360°$ **4**

c) Prove that $$\frac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$$ **3**

d) Use the substitution $t = \tan\frac{x}{2}$ to show that $\dfrac{\sin x}{1 - \cos x} = \cot\dfrac{x}{2}$ **2**

e) Uluru is a large rock h metres high, on flat ground in Central Australia.
Three tourists A, B, and C are observing Uluru from the ground.
A is due north of Uluru, C is due east of Uluru, and B is on the line-of-sight from A to C and between them. The angles of elevation to the summit of Uluru from A, B, and C are 26°, 28°, and 30°, respectively.



(i) Show that the distance of A from the base L of Uluru is $h\tan 64°$ **1**

(ii) Give similar expressions for each of the distances of B and C from the base L. **1**

(iii) Show that $\angle LAC$ is $40°11'$ to the nearest minute. **1**

(iv) Hence, determine the bearing of B from the base of Uluru (to nearest degree). **3**

Solutions to 2009 Preliminary HSC Ext 1

1 a) $y = (x^2+1)^{3/2}$

$y' = \frac{3}{2}(x^2+1)^{\frac{1}{2}} \cdot 2x$

$y' = 3x\sqrt{x^2+1}$

b) $m_1 = 2, \quad m_2 = -1 \qquad (y = -x+3)$

$\tan\theta = \left|\frac{m_1 - m_2}{1+m_1 m_2}\right| = \frac{2+1}{1-2} = \left|\frac{3}{-1}\right| = 3$

$\tan\theta = 3 \qquad \therefore \theta = 180 - 71^\circ 34'$

$\theta = 108^\circ 26'$

c) $\overset{x_1 \, y_1}{A(-2, 3)} \qquad \overset{x_2 \, y_2}{B(6, -1)} \qquad k : l = 3 : -2$

$x = \frac{3\times 6 + -2\times -2}{3-2} \qquad y = \frac{3\times -1 + -2\times 3}{3-2}$

$x = 22, \qquad y = -9 \qquad \therefore P \text{ is } (22, -9)$

d) $\frac{x^2-4}{x+3} < x - 4$

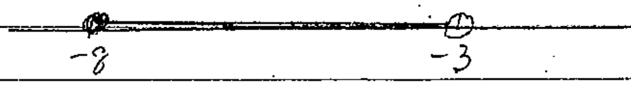
For $x > -3$ Solve $\cancel{x \neq -3}$ For $x < -3$ solve

$x^2 - 4 \leq (x-4)(x+3) \qquad x^2 - 4 \geq (x-4)(x+3)$

$x^2 - 4 \leq x^2 - x - 12 \qquad x^2 - 4 \geq x^2 - x - 12$

$x \leq -8 \qquad x \geq -8$

No Solution



$\therefore -8 < x \leq -3$

e) $\frac{2^{4n}\times 3^{2n}}{8^n \times 6^n} + 3^n + 3^n$

$\frac{2^{4n}\times 3^{2n}}{2^{3n}\times 2^n \times 3^n} + 3^n + 3^n$

$3^n + 3^n + 3^n$

$= 3\times 3^n$

$= 3^{n+1}$

Question 2

a) $P(x) = x^3 - 3x^2 + 3x - 5$

$P(2) = 8 - 12 + 6 - 5 = \underline{\underline{-3}}$

b) $x^3 + 3x^2 - 5x + 6 = 0$

$\Sigma\alpha = -3 \qquad \Sigma\alpha\beta = -5 \qquad \Sigma\alpha\beta\gamma = -6$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma}$

$= \frac{-5}{-6} = \underline{\underline{\frac{5}{6}}}$

c) $x^2 - 4 = (x-2)(x+2)$

$P(2) = 16 + 16 - 12 - 16 - 4 = 0 \quad \therefore (x-2)$ is factor

$P(-2) = 16 - 16 - 12 + 16 - 4 = 0 \quad \therefore (x+2)$ is factor

$\therefore (x-2)$ & $(x+2)$ are factors ie (x^2-4)

ii) Repeated zero of $x=-1$ $\therefore$ repeated factor of $(x+1)$ ie $(x+1)^2$

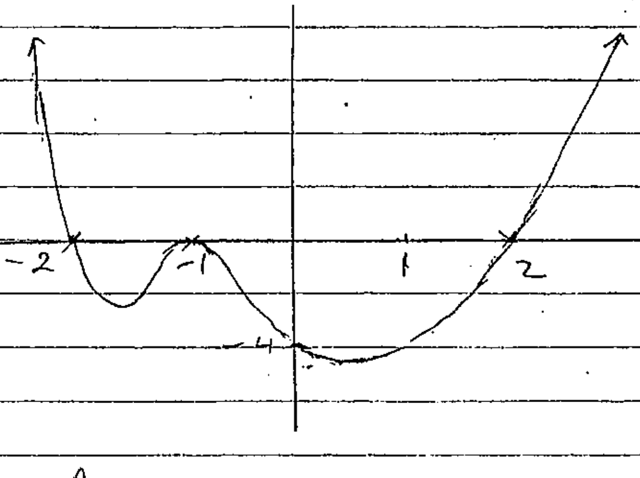
$$\begin{array}{r} x^2 \qquad - 4 \\ x^2+2x+1 \enclose{longdiv}{x^4 + 2x^3 - 3x^2 - 8x - 4} \\ x^4 + 2x^3 + x^2 \qquad\qquad \\ -4x^2 - 8x - 4 \\ \underline{-4x^2 - 8x - 4} \end{array}$$

Remainder is zero $\therefore (x+1)^2$ is a factor

$\therefore x = -1$ is a repeated zero.

iii) $P(x) = (x-2)(x+2)(x+1)^2$

iv)



v) $y \geq 0$ for $x \leq -2$, $x = -1$, $x \geq 2$

$\therefore$ Solution is $x \leq -2$ or $x = -1$ or $x \geq 2$

Question 3

a) i) $^7C_3 \times {^6C_2} = 525$

ii) Put the boys together 2 ways
Arrange 4 things in a line $4!$
$\therefore 2 \times 4! = 48$

b) i) $^{40}C_6$

ii) $^{34}C_6$

c) ii) 3 R's 1 other : Choose $^6C_1$ & arrange: 4 $= {^6C_1} \times 4$

2 R's 2 other : choose $^6C_2$ & arrange $\frac{4!}{2!}$ $= {^6C_2} \times \frac{4!}{2!}$

4 different letters: choose & arrange $^7P_4$

$= 24 + 180 + 840 = 1044$ words

d) $\frac{1}{(n-1)!} - \frac{1}{n!} + \frac{1}{(n+1)!}$

$= \frac{n(n+1) - (n+1) + 1}{(n+1)!}$

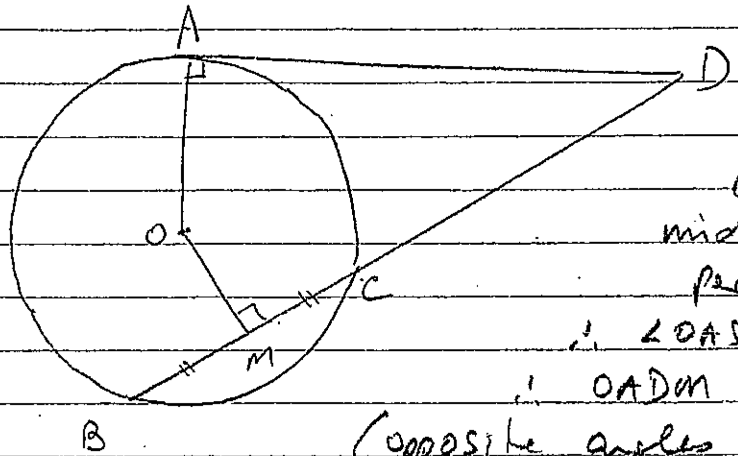
$= \frac{n^2 + n - n - 1 + 1}{(n+1)!}$

$= \frac{n^2}{(n+1)!}$

c) i) $\frac{9!}{3!}$

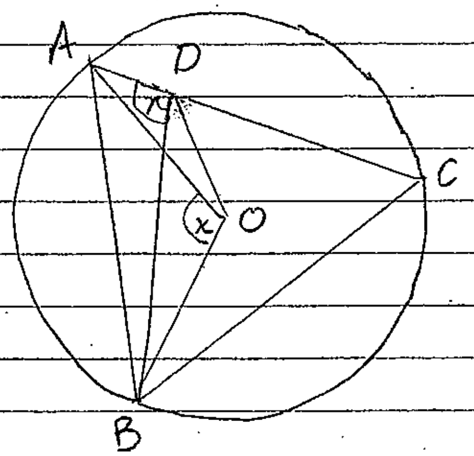
Question 4

a)



$\angle OAD = 90$
(Angle between tangent + radius)
$\angle OMC = 90°$
(line from centre to mid point of chord is perpendicular to chord)
$\therefore \angle OAD + \angle OMC = 180°$
$\therefore$ OADM is a cyclic quadrilateral
(opposite angles are supplementary)

ii) OD is diameter because the angle in a semi-circle is a right angle.



i) $\angle BOA = \angle BDA = x$ Angles at circumference standing on minor arc AB of circle ABOD

ii) $\angle BDC = 180 - x$ (straight $\angle$)
$\angle ACB = \frac{x}{2}$ Angle at circumference is half angle at centre standing on arc AB of circle ABC

$\therefore \angle DBC = x - \frac{x}{2}$ (exterior angle equals sum of interior opposite angles)

$\therefore \angle DBC = \frac{x}{2}$

$\therefore \angle DBC = \angle DCB$ ($\angle ACB$)

$\therefore BD = DC$ ($\triangle DBC$ is isosceles, sides opposite equal angles are equal)

Question 5

a) $$\tan(90+\theta) = \frac{\sin(90+\theta)}{\cos(90+\theta)} = \frac{\sin 90 \cos\theta + \cos 90 \sin\theta}{\cos 90 \cos\theta - \sin 90 \sin\theta}$$

$$= \frac{\cos\theta + 0}{0 - \sin\theta} = -\cot\theta$$

$$= \frac{-1}{\tan\theta}$$ (other methods)

b) $$\cos 2x + \sin x = 0$$

$$1 - 2\sin^2 x + \sin x = 0$$

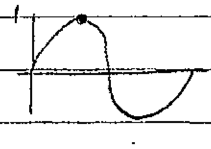
$$2\sin^2 x - \sin x - 1 = 0$$

$2\sin x \quad +1$

$\sin x \quad -1$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \quad \text{or} \quad \sin x = 1$$



$$x = 210^\circ \text{ or } 330^\circ \text{ or } 90^\circ$$

c) $$\frac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \frac{\frac{2\tan\theta}{1-\tan^2\theta} - \tan\theta}{\frac{2\tan\theta}{1-\tan^2\theta} + \frac{1}{\tan\theta}}$$

$$= \frac{2\tan\theta - \tan\theta + \tan^3\theta}{(1-\tan^2\theta)} \times \frac{(1-\tan^2\theta)\tan\theta}{2\tan^2\theta + 1 - \tan^2\theta}$$

$$= \frac{\tan\theta(1+\tan^2\theta)}{\tan^2\theta + 1} \times \tan\theta$$

$$= \tan^2\theta$$

d) $$\frac{\sin x}{1-\cos x} = \frac{2t}{1+t^2} \div \left(1 - \frac{1-t^2}{1+t^2}\right)$$

$$= \frac{2t}{1+t^2} \div \left(\frac{1+t^2-1+t^2}{1+t^2}\right)$$

$$= \frac{2t}{1+t^2} \times \frac{1+t^2}{2t^2}$$

$$= \frac{1}{t}$$

$$= \cot\frac{x}{2}.$$

5 e)

i) $\tan 26^\circ = \dfrac{h}{AL}$ in $\triangle ALU$

$\therefore AL = \dfrac{h}{\tan 26^\circ} = \dfrac{h}{\cot 64^\circ}$

$AL = h \tan 64^\circ$

ii) $BL = h \tan 62^\circ$

$CL = h \tan 60^\circ$

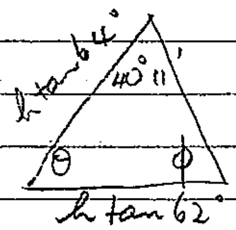
iii) In $\triangle ALC$

$\tan \angle LAC = \dfrac{LC}{LA} = \dfrac{h \tan 60^\circ}{h \tan 64^\circ}$

$\angle LAC = \tan^{-1}\left(\dfrac{\tan 60^\circ}{\tan 64^\circ}\right) = \tan^{-1} 0.8447776..$

$\angle LAC = 40^\circ\ 11'$ to nearest minute.

iv) In $\triangle ALB$



$\dfrac{\sin \phi}{h \tan 64^\circ} = \dfrac{\sin 40^\circ\ 11'}{h \tan 62^\circ}$

$\sin \phi = \dfrac{\tan 64^\circ \sin 40^\circ\ 11'}{\tan 62^\circ}$

$\sin \phi = 0.703516$

$\angle \phi = 44^\circ\ 43'$ or $180 - 44^\circ\ 43'$
$= 135^\circ\ 17'$

$\therefore \angle \theta = 180 - 44^\circ 43' - 40^\circ 11'$ or $180 - 135^\circ 17' - 40^\circ 11'$

$\angle \theta = 95^\circ\ 6'$ .. 4.533°
$4^\circ\ 32'$

But B is between A + C $\therefore$ obtuse angle is not possible

$\therefore$ Bearing is $4^\circ 32'$ or $005^\circ T$
or $N 5^\circ E$ to nearest degree.