Student Name: ....

2010

YEAR 11 PRELIMINARY YEARLY EXAMINATION

MATHEMATICS
EXTENSION 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 1 1/2 hours
- Write using blue or black pen
- Calculators may be used.
- Begin each question on a new page.
- All necessary working should be shown.

Total marks – 60

- Attempt Questions 1-5
- All questions are of equal value

Marks

Question 1 **(12 Marks)** Use a Separate Sheet of paper

a) In how many ways can the letters of the word BEGINNING be arranged in a line? **2**

b) A charm bracelet has 7 charms which are all different. How many distinct arrangements of the charms on the bracelet are possible? **2**

c) There are 16 students in a class, of whom 9 are male and 7 female. Five students are to be chosen for a working group to plan an excursion.

(i) In how many different ways can the group be chosen if there are no restrictions on its' membership? **1**

(ii) If the group must consist of 3 males and 2 females, in how many ways can the group be chosen? **1**

d) (i) Solve for x: $\dfrac{x+1}{x-3} \leq 3$ **2**

(ii) Solve: $x^2 + 5x - 6 > 0$ **2**

(iii) Hence, find values of x for which the following inequalities hold **2**

$$\frac{x+1}{x-3} \leq 3 \text{ and } x^2 + 5x - 6 > 0$$

End of Question 1

Question 2 **(12 Marks)** Use a Separate Sheet of paper **Marks**

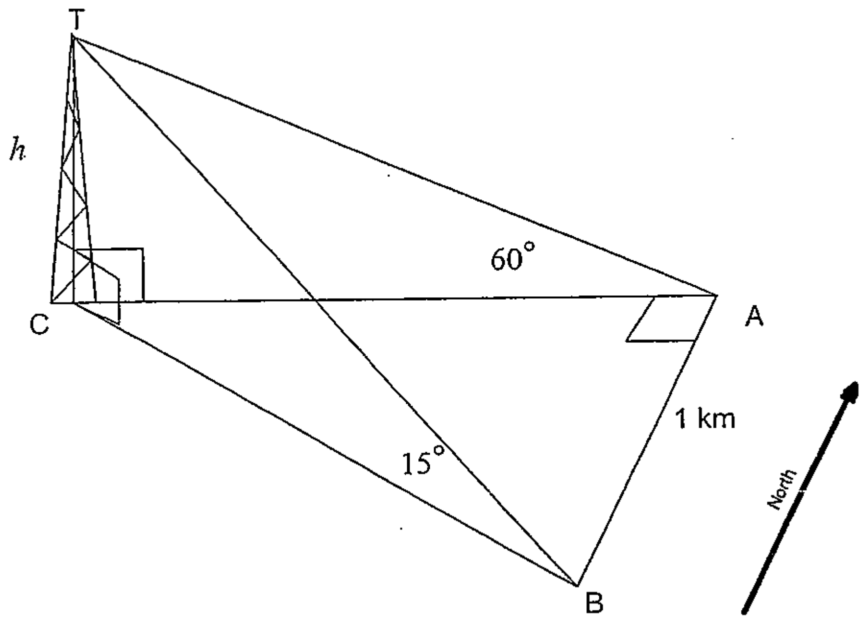
a) (i) By expressing $\cos 3\theta$ as $\cos(\theta + 2\theta)$ show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **2**

(ii) Hence solve the equation $\cos 3\theta = 2\cos^2\theta - \cos\theta$ for $0^\circ \leq \theta \leq 360^\circ$. **3**

b) (i) Express $4\sin\theta - 3\cos\theta$ in terms of $t = \tan\frac{\theta}{2}$. **2**

(ii) Hence solve $4\sin\theta - 3\cos\theta = 0$ for $-180^\circ \leq \theta \leq 180^\circ$. **2**

c) The angle of elevation of the top of a tower (T) from a point A due East of the tower is 60°. From a point *B* due South of *A*, the angle of elevation of T is 15°. *A* and *B* are at the same elevation as the base of the tower. If the distance *AB* = 1 km, find the height (*h*) of the tower. **3**

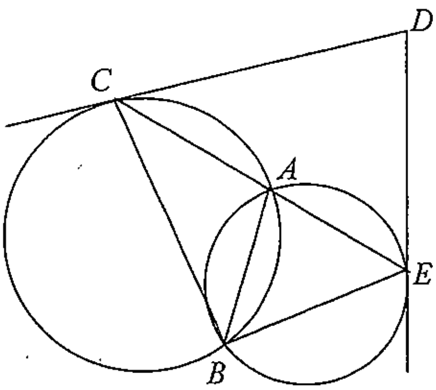


End of Question 2

Question 3 (12 Marks) Use a Separate Sheet of paper **Marks**

a) Sketch the regions $y < \sqrt{9 - x^2}$ and $y \geq x^2 - 9$ on the same graph, indicating where both regions hold simultaneously **2**

b) **3**



Two circles intersect at A and B. CAE is a straight line where C is a point on the first circle and E is a point on the second circle. The tangent at C to the first circle and the tangent at E to the second circle meet at D.

Prove that $BCDE$ is a cyclic quadrilateral.

c) The polynomial $P(x) = x^3 + a^2x^2 + ax + b$ leaves a remainder of 2 when divided by x and a remainder of 13 when divided by $(x+1)$.

i) Show that $b = 2$ **1**

ii) Find the value of a. **2**

d) i) Express $3\cos x + \sin x$ in the form $R\sin(x+\alpha)$ where $R > 0$ and $0 < \alpha < 90^\circ$ giving R in simplest form and α in degrees **2**

ii) Hence solve the equation $3\cos x + \sin x = -2$ for $0 \leq x \leq 360^\circ$ giving the solutions to the nearest degree. **2**

End of Question 3

Question 4 (12 Marks) Use a Separate Sheet of paper **Marks**

a) i) Expand $\left(a+\sqrt{b}\right)^2$ **1**

ii) Given that $\left(a+\sqrt{b}\right)^2 = 7+4\sqrt{3}$ find values for a and b. **2**

iii) Hence find $\sqrt{7+4\sqrt{3}}$ **1**

b) Calculate the acute angle between the lines

$y = 2x - 3$ and

$3x + 5y - 1 = 0$ to the nearest minute. **2**

c) Find the number of ways the letters of the word ANGLE can be arranged in a straight line so that

i) No two consonants are next to each other **1**

ii) The three consonants are side-by-side **1**

iii) Exactly 2 of the 3 consonants are side-by-side **1**

d) Solve for x: $x^2 - 5x + 2 + \dfrac{4}{5x - x^2 - 2} = 0$ **3**

End of Question 4

Question 5 (12 Marks) Use a Separate Sheet of paper **Marks**

a) (i) Express $x^3 - x^2 - 10x - 8$ as a product of three linear factors. **2**

(ii) Solve the inequality $\dfrac{x^3 - 10x}{x^2 + 8} \geq 1$. **2**

b) $A\left(8, \sqrt{50}\right)$ and $B\left(1, \sqrt{18}\right)$ are two points. Find in simplest exact form the coordinates of the point P which divides the interval AB externally in the ratio 3 : 1. **2**

c) The polynomial $P(x) = x^3 + 2x^2 - 4x - 1$ has zeros α, β and γ so that $P(x) = (x - \alpha)(x - \beta)(x - \gamma)$.

(i) Find the value of $(1 - \alpha)(1 - \beta)(1 - \gamma)$. **1**

(ii) Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$. **2**

d) A is the point $(5,0)$ and O is the origin. Given that the point B (x, y) lies on the line $y = 1 - 3x$ and that OB is perpendicular to AB, find all possible coordinates of B. **3**

End of Examination

2010 Year 11 Ext 1 Preliminary HSC

Question 1

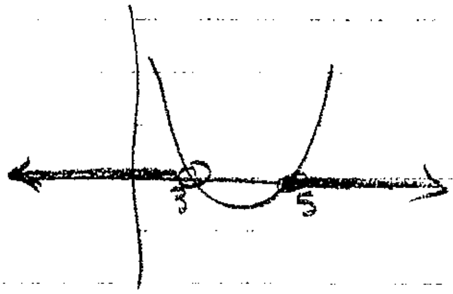
1 a) $\frac{9!}{3!2!2!} =$

b) $\frac{6!}{2}$

c) i) ${}^{11}C_5 =$

ii) ${}^{9}C_3 \times {}^{7}C_2 =$

d) i) $\frac{x+1}{x-3} \times (x-3)^2 \le 3(x-3)^2$



$(x+1)(x-3) \le 3(x-3)^2$

$0 \le 3(x-3)^2 - (x+1)(x-3)$

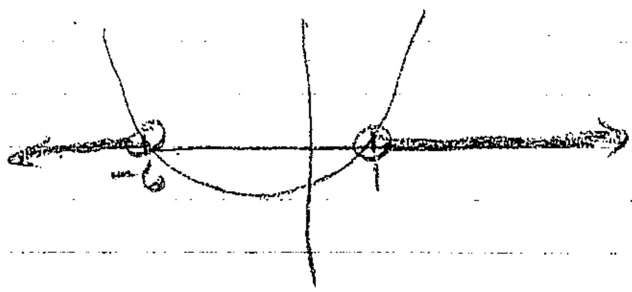
$0 \le (x-3)\{3x-9-x-1\}$

$0 \le (x-3)(2x-10)$

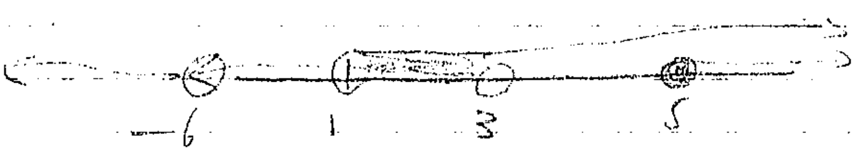
$0 \le 2(x-3)(x-5)$

$\therefore x < 3$ or $x \ge 5$

ii) $(x+6)(x-1) > 0$



iii) $x \ge 5$ or $x < -6$ or $1 < x < 3$



Question 2

a) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$\cos(\theta+2\theta) = \cos\theta\cos2\theta - \sin\theta\sin2\theta$$
$$= \cos\theta(2\cos^2\theta-1) - \sin\theta \cdot 2\sin\theta\cos\theta$$
$$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$$
$$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$$
$$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$$
$$\cos3\theta = 4\cos^3\theta - 3\cos\theta$$

ii)
$$\cos3\theta = 2\cos^2\theta - \cos\theta,$$
$$4\cos^3\theta - 3\cos\theta = 2\cos^2\theta - \cos\theta$$

$$4\cos^3\theta - 2\cos^2\theta - 2\cos\theta = 0$$
$$2\cos\theta\,(2\cos^2\theta - \cos\theta - 1) = 0$$

$2\cos\theta$ $+1$
$\cos\theta$ -1

$$2\cos\theta(2\cos\theta+1)(\cos\theta-1) = 0$$
$\cos\theta = 0$ or $\cos\theta = -\frac{1}{2}$ or $\cos\theta = 1$

$\theta = 90^\circ, 270^\circ,$ $\theta = 120^\circ, 240^\circ$ $\theta = 0^\circ$ or 360°

b) $\sin\theta = \dfrac{2t}{1+t^2}$ $\cos\theta = \dfrac{1-t^2}{1+t^2}$

$$4\sin\theta - 3\cos\theta = \frac{8t}{1+t^2} - \frac{3(1-t^2)}{1+t^2}$$

$$4\sin\theta - 3\cos\theta = 0$$

Check $\theta = 180^\circ$?
$4\times0 - 3\times-1 \neq 0$

$$\frac{8t}{1+t^2} - \frac{3(1-t^2)}{1+t^2} = 0$$

$3t$ -1
t $+3$

$$8t - 3 + 3t^2 = 0$$
$$3t^2 + 8t - 3 = 0$$
$$(3t-1)(t+3) = 0$$
$$t = \tfrac{1}{3} \text{ or } -3$$

$\tan\frac{\theta}{2} = \frac{1}{3}$ or $\tan\frac{\theta}{2} = -3$

$\frac{\theta}{2} = 18.43\ldots$ or $\frac{\theta}{2} = -71.565\ldots$

$\theta = 36^\circ\ 52'$ $\qquad$ $\theta = -143^\circ\ 8'$

c) $\frac{AC}{h} = \tan 30^\circ \implies AC = h\tan 30^\circ$

$\frac{BC}{h} = \tan 75^\circ \implies BC = h\tan 75^\circ$

$BC^2 = AC^2 + AD^2$

$h^2\tan^2 75^\circ = h^2\tan^2 30 + 1$

$h^2(\tan^2 75 - \tan^2 30) = 1$

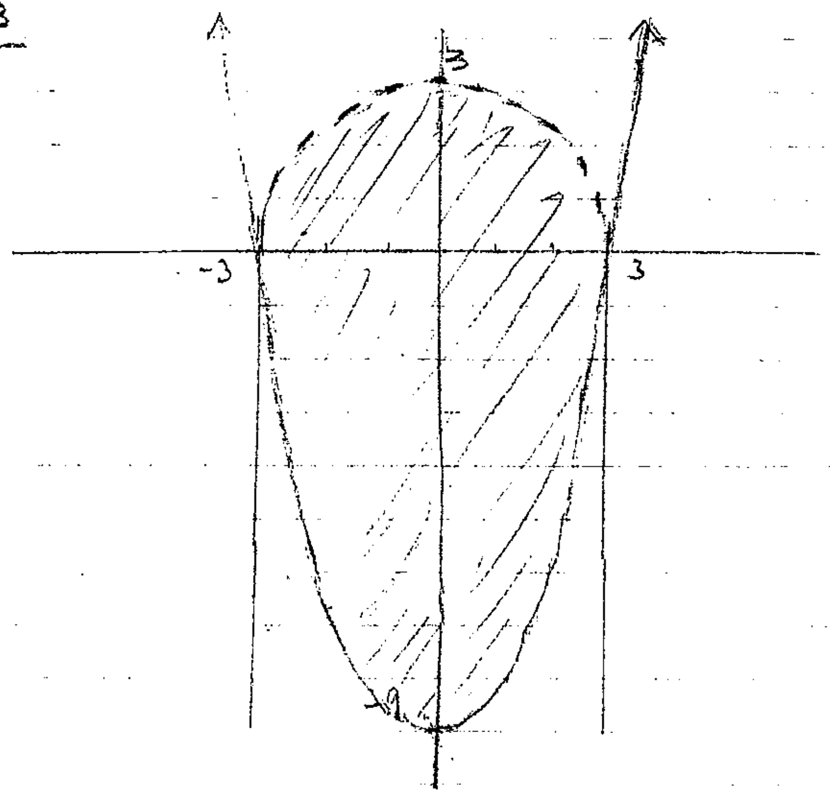
$$h^2 = \frac{1}{\tan^2 75 - \tan^2 30}$$

$h = \sqrt{0.073557}$ Km

$h = 73.557$ m

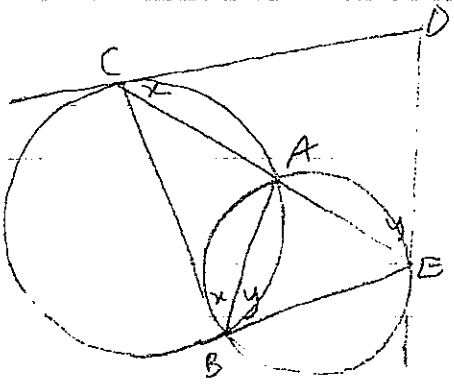
Question 3

a)



2

b)



$\angle DCE = \angle CBA = x$

(angle between tangent & chord equals angle in alternate segment)

$\angle DEC = \angle ABE = y$

(alternate segment theorem)

$\angle CDE = 180 - (x+y)$ (angle sum of $\triangle CDE$)

Also $\angle CBE = x + y$

$\therefore \angle CDE + \angle CBE = 180 - (x+y) + x + y$

$= 180°$

$\therefore$ BCDE is a cyclic quadrilateral

(one pair of opposite angles are supplementary)

3

c) $P(x) = x^3 + a^2x^2 + ax + b$

$P(0) = 2$ 1

$2 = 0 + 0 + 0 + b$

$\therefore P(x) = x^3 + a^2x^2 + ax + 2$

$P(-1) = 13$

$13 = -1 + a^2 - a + 2$

$0 = a^2 - a - 12$ 2

$0 = (a-4)(a+3)$

$a = 4$ or -3

$P(x) = x^3 + 16x^2 + 4x + 2$ check $P(-1) = 13$

or $P(x) = x^3 + 9x^2 - 3x + 2$ $P(-1) = 13$

d) i) $3\cos x + \sin x = R\sin(x+\alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$\therefore R\cos\alpha = 1$ ——— ①

$R\sin\alpha = 3$ ——— ②

$①^2 + ②^2$ $R^2 = 3^2 + 1^2$

$R = \sqrt{10}$

2

$\tan\alpha = 3 \implies \alpha = \tan^{-1}(3) \doteqdot 71°34'$

$\sqrt{10}\sin(x + 71°34') = -2$

$\sin(x + 71°34') = -\frac{2}{\sqrt{10}}$

$x + 71°34' = 180 + 39°14'$ or $360 - 39°14'$

$x = 147°40'$ or $249°12'$

$x \doteqdot 148°$ or $249°$ 2

to nearest degree

4

a) i) $(a+\sqrt{b})^2 = a^2 + 2a\sqrt{b} + b$

ii) $7 + 4\sqrt{3} = a^2 + b + 2a\sqrt{b}$

$\therefore \quad a^2 + b = 7$

$2a\sqrt{b} = 4\sqrt{3} \implies 4a^2 b = 48$

$a^2 b = 12$

$a^2 = 7 - b$

$(7-b)b = 12 \implies 7b - b^2 = 12$

$b^2 - 7b + 12 = 0 \qquad (b-3)(b-4) = 0$

$b = 3$ or 4

$\therefore \quad 3a^2 = 12$ or $4a^2 = 12$

$a^2 = 4 \qquad a^2 = 3$

$a = \pm 2 \qquad a = \pm\sqrt{3}$

$\therefore \quad \underline{a = \pm 2 \quad b = 3}$ or $\underline{a = \pm\sqrt{3} \quad b = 4}$

iii) $\sqrt{7 + 4\sqrt{3}} = 2 + \sqrt{3}$ for positive square root

or $\sqrt{7+4\sqrt{3}} = \sqrt{3} + \sqrt{4}$ gives same answer

b) $y = 2x - 3 \qquad m_1 = 2$

$3x + 5y - 1 = 0 \qquad m_2 = -\frac{3}{5}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{2 + \frac{3}{5}}{1 - 2\times\frac{3}{5}}\right| = \left|\frac{13}{-1}\right|$$

$\tan\theta = 13$

$\theta \doteqdot 85^\circ 36'$

c) ANGLE

i) No two consonants CVCVC

$3! \times 2! =$

ii) CCC VV

group consonants $3!$ ways

then arrange (CCC) VV $3!$ ways

$\therefore 3! \times 3!$

iii) Exactly 2 is complement of (i) + (ii)

$\therefore$ Total arrangements $= 5!$

Exactly 2 consonants $= 5! - 3! \times 2! - 3!3!$

$= 120 - 12 - 36$

$= 72$ ways

d) $x^2 - 5x + 2 - \dfrac{4}{x^2 - 5x + 2} = 0$

Put $u = x^2 - 5x + 2$

$u - \dfrac{4}{u} = 0$

$u^2 - 4 = 0$

$u = \pm 2$

$\therefore x^2 - 5x + 2 = 2$ or $x^2 - 5x + 2 = -2$

$x(x - 5) = 0$

$x = 0$ or 5

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

$x = 4$ or 1

$x = 0, 5, 4, 1$

Question 5

a) i) $P(x) = x^3 - x^2 - 10x - 8$

$P(1) = 1 - 1 - 10 - 8 \neq 0$

$P(-1) = -1 - 1 + 10 - 8 = 0$

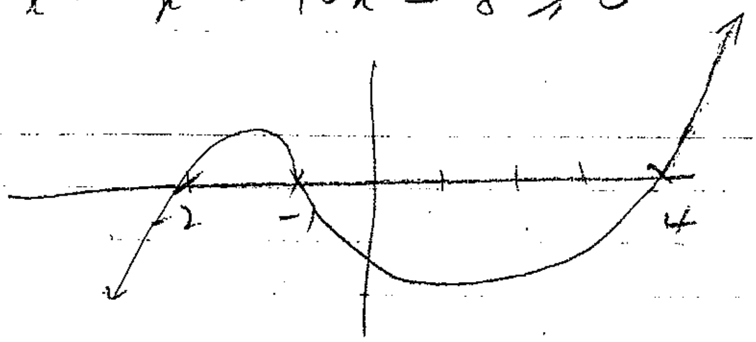
$$
\begin{array}{r}
x^2 - 2x - 8 \\
x+1 \enclose{longdiv}{x^3 - x^2 - 10x - 8} \\
\underline{x^3 + x^2 \qquad\qquad} \\
-2x^2 - 10x - 8 \\
\underline{-2x^2 - 2x \qquad} \\
-8x - 8 \\
-8x - 8
\end{array}
$$

$P(x) = (x+1)(x^2 - 2x - 8) = (x+1)(x-4)(x+2)$

ii) $\dfrac{x^3 - 10x}{x^2 + 8} \geq 1$ Note $x^2 + 8$ is always +

$x^3 - 10x \geq x^2 + 8$

$x^3 - x^2 - 10x - 8 \geq 0$



$-2 \leq x \leq -1$ or $x \geq 4$

b) $A(\overset{x_1}{8}, \overset{y_1}{5\sqrt{2}})$ $B(\overset{x_2}{1}, \overset{y_2}{3\sqrt{2}})$ $\dfrac{k}{l} = \dfrac{3}{-1}$

$x = \dfrac{3 \times 1 + -1 \times 8}{3 - 1}$ $y = \dfrac{3 \times 3\sqrt{2} - 1 \times 5\sqrt{2}}{3 - 1}$

$x = -\dfrac{5}{2}$ $= \dfrac{4\sqrt{2}}{2} = 2\sqrt{2}$

$P\left(-\dfrac{5}{2}, 2\sqrt{2}\right)$

c) $P(x) = x^3 + 2x^2 - 4x - 1$

$= (x-\alpha)(x-\beta)(x-\gamma)$

i) For $(1-\alpha)(1-\beta)(1-\gamma)$ Sub $x=1$

$P(1) = 1 + 2 - 4 - 1 = -2$

ii) For $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$

$P(x) = (x-\alpha)(x-\beta)(x-\gamma)$

Sub $x = \alpha+\beta+\gamma$ $\quad P(\alpha+\beta+\gamma) = (\beta+\gamma)(\gamma+\alpha)(\alpha-\beta)$

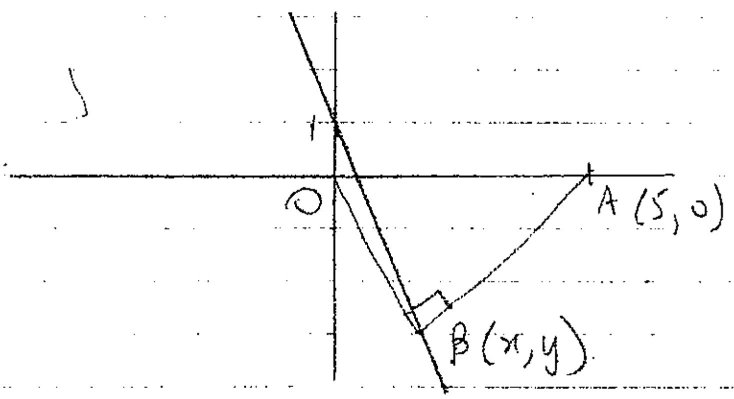
$= (\alpha+\beta+\gamma)^3 + 2(\alpha+\beta+\gamma)^2 - 4(\alpha+\beta+\gamma) - 1$

But $(\alpha+\beta+\gamma) = -2$

$P(\alpha+\beta+\gamma) = (-2)^3 + 2(-2)^2 - 4(-2) - 1 = -8 + 8 + 8 - 1$

$\underline{= 7}$

d)



For OB, $m_1 = \frac{y}{x}$

For AB, $m_2 = \frac{y-0}{x-5}$

$m_1 m_2 = -1$

$\frac{y}{x} \cdot \frac{y}{x-5} = -1$

$y^2 = -x(x-5) \qquad = -x^2 + 5x$

$x^2 - 5x + y^2 = 0$

$y = 1 - 3x$

$x^2 - 5x + (1-3x)^2 = 0$

$x^2 - 5x + 1 - 6x + 9x^2 = 0$

$10x^2 - 11x + 1 = 0$

$(10x - 1)(x - 1) = 0$

$10x \quad -1$

$x \quad -1$

$\left.\begin{array}{l} x = \frac{1}{10} \\ y = \frac{7}{10} \end{array}\right\}$ or $\left.\begin{array}{l} 1 \\ -2 \end{array}\right\}$

B is $\left(\frac{1}{10}, \frac{7}{10}\right)$ or $(1, -2)$