Name:_____

Teacher:_____

GOSFORD HIGH SCHOOL

2012
PRELIMINARY YEARLY EXAMINATION.

EXTENSION 1 Mathematics

General Instructions:

- Reading time – 5 minutes
- Working time – 1½ hours
- Write using black or blue pen.
- Board-approved calculators may be used.

Total Marks – 55

Section I – 7 marks
Answer on the sheet provided
Allow about 15 minutes for this section

Section II – 48 marks
Attempt Questions 8 – 11
Allow about 1¼ hours for this section

Question	Marks
1-7	/ 7
8	/ 12
9	/ 12
10	/ 12
11	/ 12
TOTAL	/ 55

This paper MUST NOT be removed from the examination room

Section I

7 marks
Attempt Questions 1-7
Allow about 15 minutes for this section
Use the multiple choice answer sheet for questions 1 – 7.

---

1. Factorise $54x^3 + 16$

   (A) $2(3x+2)(9x^2+6x+4)$

   (B) $2(3x+2)(9x^2-6x+4)$

   (C) $2(3x+2)(9x^2-12x+4)$

   (D) $2(3x+2)(3x^2-6x+4)$

2. Simplify $\dfrac{\sin(180^\circ+\theta)}{\cos(90^\circ-\theta)}$.

   (A) $\tan\theta$

   (B) 1

   (C) -1

   (D) $\tan(90^\circ-\theta)$

3. Which of the following is an expression for $\frac{1}{1-\tan x} - \frac{1}{1+\tan x}$

   (A) $\frac{2\tan x}{\sec^2 x}$

   (B) $\frac{\tan 2x}{\tan x}$

   (C) $\tan 2x$

   (D) $\tan x \tan 2x$

4. Find the number of ways the letters of the word RHOMBUS can be arranged in a straight line so that the vowels are together.

   (A) 720

   (B) 5040

   (C) 1440

   (D) 360

5. Given that $y = \frac{1}{t-1}$ and $x = t^2 - 2t + 1$, which of the following equations satisfy:

(A) $x = \frac{1}{y^2}$

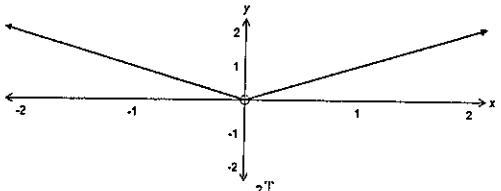
(B) $y = \frac{1}{x}$

(C) $x = \frac{1}{y}$

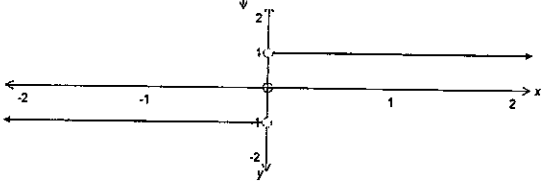
(D) $y = \frac{1}{x^2}$

6. The sketch of the graph $f(x) = \frac{|x|}{x}$ *for* $x \neq 0$ is:

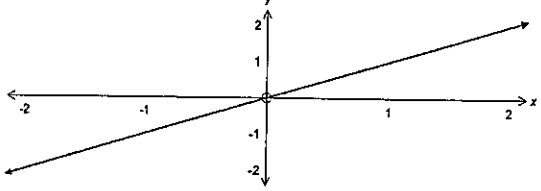
(A)



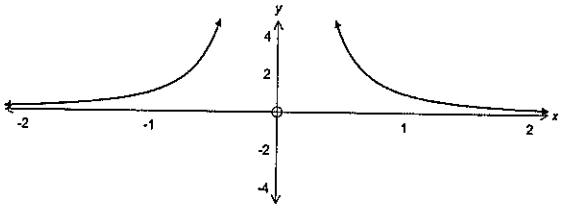
(B)



(C)



(D)



7. What is the value of $\cos 75^0$?

(A) $\frac{\sqrt{6}+\sqrt{2}}{2}$

(B) $\frac{\sqrt{6}-\sqrt{2}}{2}$

(C) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(D) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

End of Section 1

Section II

Total Marks (48)
Attempt Questions 8 - 11
Allow about 1¼ hours for this section.

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.
All necessary working should be shown in every question.

Question 8 (12 marks) Start a new writing booklet. **Marks**

(a) Solve $16^{4-x} = \frac{1}{8^x}$. **2**

(b) Find the perpendicular distance from the point (1,4) to the line $4y = 3x - 2$ **2**

(c) Solve $\frac{x^2-4}{x} > 0$. **3**

(d) P(a,b) and Q (b,a) are two points. The point R(5 ,-3) divides PQ externally in the ratio 3 : 1. Find the values of a and b **2**

(e) Use the substitution of $m = \frac{1}{x} + x$ to solve:

$$\frac{x^4+2x^2+1}{x^2} - \frac{5}{x} - 5x + 6 = 0$$ **3**

End of Question 8

Question 9 (12 marks) Start a new writing booklet. **Marks**

(a) (i) How many different committees of 8 people can be chosen from 16? **1**

(ii) How many different committees of 8 people with equal numbers of men and women can be chosen if there are 6 women and 10 men? **1**

(iii) How many different committees of 8 people with equal numbers of men and women can be chosen if there are 6 women and 10 men and Sue was chosen and Peter was not chosen? **1**

(b) Ben and Gaby go to the picture theatre with 3 other couples. They sit together as a group in a single row. Outside the theatre they have an argument and decided they do not want to sit together. **2**
How many arrangements are possible if the other couples are still sitting together?

(c) (i) Find a pair of values of a and b such that $x^4 + 4 \equiv (x^2 + a)^2 - (bx)^2$. **2**

(ii) Hence express $x^4 + 4$ as a product of two quadratic factors. **1**

(d) When $P(x) = x^3 + x^2 - a$ is divided by $x - 2$ the remainder is 4. Find the remainder when $P(x)$ is divided by x. **2**

(e) The equation $x^3 + ax^2 + bx + ab = 0$, where $a \neq 0$ and $b \neq 0$, has three real roots α, β *and* γ. Show that $(\alpha + \beta + \gamma)\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right) = 1$ **2**

Question 10 (12 marks) Start a new writing booklet. **Marks**

(a) Find in exact form, the value of $tan\ 15^0$ **2**

(b) (i) Show that $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$. **2**

(ii) Hence or otherwise, show that the exact value of $\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$. **2**

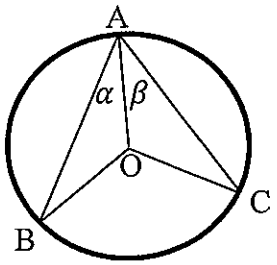
(c) Express $4\sin\theta - 3\cos\theta$ in terms of $t = \tan\frac{\theta}{2}$. **2**

(d) If $t = \tan\frac{\theta}{2}$ and using the expansion for $\tan(A+B)$ show that:

$$\tan\left(45^\circ + \frac{\theta}{2}\right) = \frac{1+\sin\theta}{\cos\theta}.$$ **4**

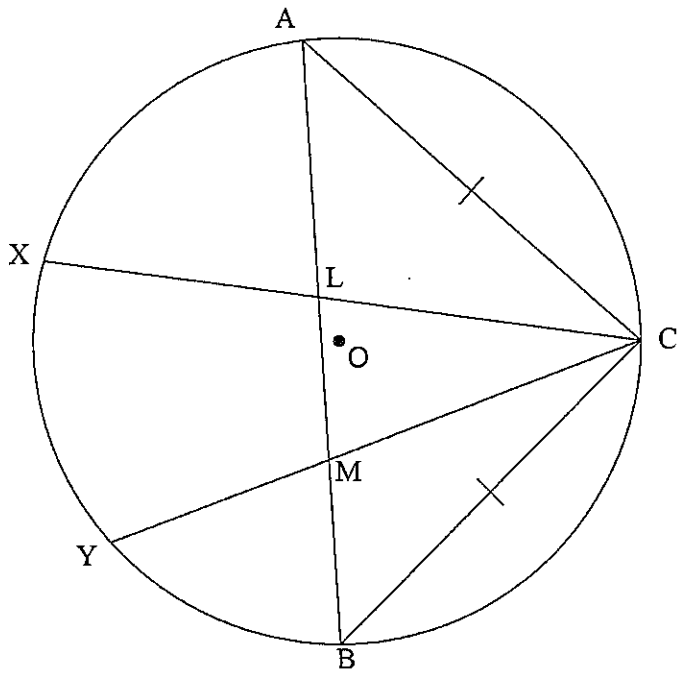
Question 11 (12 marks) Start a new sheet of writing booklet. **Marks**

(a)



O is the centre of the circle, find the size of angle BOC, giving reasons. **1**

(b) In the diagram A, B and C are 3 points on the circle. CX and CY are chords cutting AB at L and M respectively. AC=CB, $\angle CAB = \theta$ and $\angle ACX = \alpha$.



NOT TO SCALE

Copy the diagram into your answer booklet.

(i) State why $\angle CLB = \theta + \alpha$. **1**

(ii) Explain why $\angle AYC = \theta$ **2**

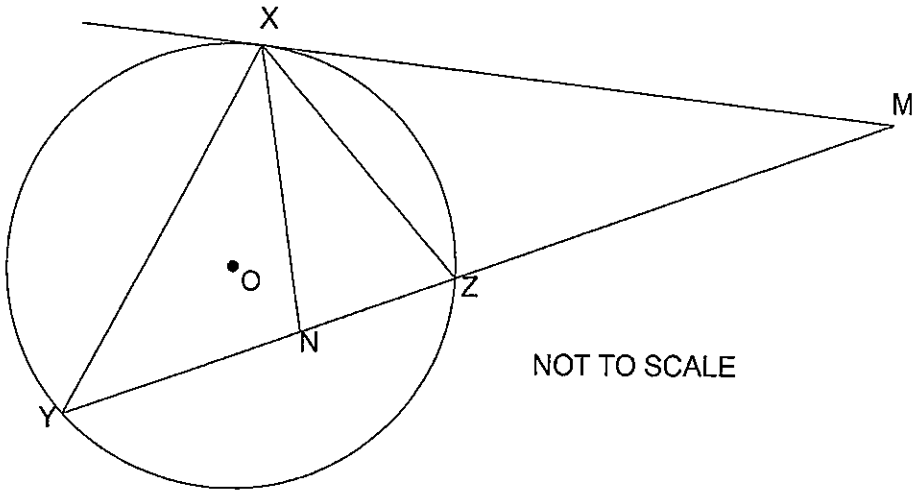
(iii) Explain why $\angle AYX = \alpha$. **1**

(iv) Prove that XYML is a cyclic Quadrilateral. **1**

Question 11 continued on next page

Question 11 continued

(c) In the diagram X, Y and Z are 3 points on the circle. The tangent to the circle at X meets YZ produced at M. N is a point on YZ such that XN bisects $\angle ZXY$.



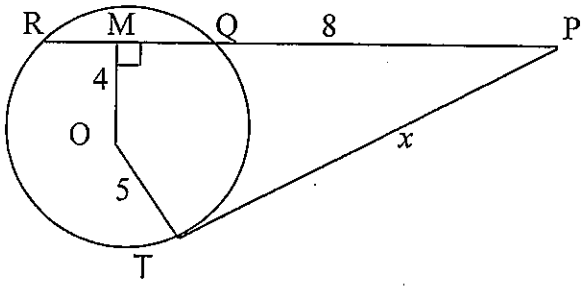
Marks

3

Copy the diagram into your answer booklet.

Show that $MX = MN$, giving reasons.

(d)



Copy the diagram into your answer booklet.

PT is a tangent to the circle, centre O. OM is perpendicular to the chord RQ.

OT is the radius and PQ is 8 units.

(i) Give **REASONS** why RQ is equal to 6 units. **1**

(ii) Find the value of x, giving reasons. **2**

Preliminary Examination – Extension 1 Mathematics 2012

Multiple Choice Answer Sheet

Name _____

Teacher _____

Completely fill the response oval representing the most correct answer.

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○

GOSFORD HIGH SCHOOL

PRELIM EXTENSION 1 YEARLY 2012

1 B
2 C
3 C
4 C
5 A
6 B
7 D
8

QUESTION 8

a) $16^{4-x} = \frac{1}{8^x}$

$(2^4)^{4-x} = 2^{-3x}$ (1)

$2^{16-4x} = 2^{-3x}$

$16-4x = -3x$

$x = 16$ (1)

b) $4y = 3x-2$

$0 = 3x-4y-2$

$a=3 \quad b=-4 \quad c=-2$

$(x,y) = (1,4)$

$\therefore d = \left|\frac{3.1-4.4-2}{\sqrt{3^2+(-4)^2}}\right|$

$= \left|\frac{3-16-2}{\sqrt{25}}\right|$

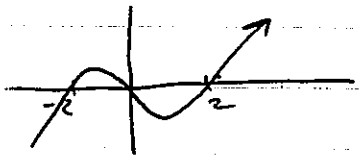
$= \left|\frac{-15}{5}\right|$

$= 3$

c) $\frac{x^2-4}{x} > 0$

$\frac{x^2(x^2-4)}{x} > 0$

$x(x-2)(x+2) > 0$



$-2 < x < 0$ and $x > 2$

d) $-a+3b = 10$ (1)

$3a-b = -6$ (2)

(1) + 3×(2) $\Rightarrow 8a = -8 \quad \therefore a = -1$

(2) + 3×(1) $\Rightarrow 8b = 24 \quad b = 3$

$\left(\frac{3b-a}{3-1}, \frac{3a-b}{3-1}\right) = (5,-3)$

e) $\frac{x^4+2x^2+1}{x^2} - \frac{5}{x} - 5x + 6 = 0$

(1) $\frac{(x^2+1)^2}{x^2} - \frac{5}{x} - 5x + 6 = 0$

$m = \frac{1}{x} + x = \frac{x^2+1}{x}$

$\therefore$ (1) $\Rightarrow m^2 - 5(m) + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2 \quad m = 3$

ie $\frac{x^2+1}{x} = 2 \qquad \frac{x^2+1}{x} = 3$

$x^2-2x+1 = 0 \qquad x^2-3x+1 = 0$

$(x-1)^2 = 0 \qquad x = \frac{+3 \pm \sqrt{(-3)^2-4\cdot}}{2\cdot}$

$x = 1$

$x = 1$ or $x = \frac{3 \pm \sqrt{5}}{2}$

RTO

Q9 a) i) $^{16}C_8 = 12870$

ii) $^6C_4 \times {}^{10}C_4 = 3150$

iii) $^5C_2 \times {}^9C_4 = 1260$

b) Couples sitting together
$= 4! \times 2^4 = 384$

Total arrangements of Ben & Gaby not as a couple $= 5! \times 2^3 = 960$

Ben & Gaby apart $= 960 - 384$
$= 576$

8) i) $x^4 + 4 = (x^2+a)^2 - (bx)^2$
$= x^4 + (2a - b^2)x^2 + a^2$

$2a - b^2 = 0$ ①

$a^2 = 4 \quad a > 0 \therefore a = 2$

from (i) $b^2 = 2a$

$b^2 = 4$

$b = \pm 2$

ii) $x^4 + 4 = (x^2+2)^2 - (2x)^2$
$= ((x^2+2)+2x)((x^2+2)-2x)$
$= (x^2+2x+2)(x^2-2x+2)$

d) $P(2) = 2^3 + 2^2 - a = 4$

$12 - a = 4$

$a = 8$

$P(x) = x^3 + x^2 - 8$

$P(0) = -8$

e) $\alpha + \beta + \gamma = \frac{-9}{1} = -a$

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{b}{-ab} = \frac{-1}{a}$

$\therefore (\alpha+\beta+\gamma)\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) = -a \times -\frac{1}{a} = 1$

Question 10

a) $\tan 15 = \tan(45 - 30)$

$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \times \tan 30}$

$= \frac{1 - 1/\sqrt{3}}{1 + 1 \times 1/\sqrt{3}} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{2}$

b) i) $\frac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$

LHS $= \frac{1 - (1 - 2\sin^2 x)}{1 + (2\cos^2 x - 1)}$

$= \frac{2\sin^2 x}{2\cos^2 x}$

$= \tan^2 x = \text{RHS}$

ii) $\tan^2 22\frac{1}{2}^\circ = \frac{1 - \cos 45^\circ}{1 + \cos 45^\circ}$

$= \frac{1 - \frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}}$

$= \frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{\frac{\sqrt{2}+1}{\sqrt{2}}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{(\sqrt{2}-1)^2}{2-1}$

$= (\sqrt{2}-1)^2$

$\tan 22\frac{1}{2} = \sqrt{2} - 1$

c) $4\sin\theta - 3\cos\theta$
where $t = \tan\frac{\theta}{2}$

$= 4\left(\frac{2t}{1+t^2}\right) - 3\left(\frac{1-t^2}{1+t^2}\right)$

$\frac{8t}{1+t^2} - \frac{3-3t^2}{1+t^2}$

$= \frac{8t-3+3t^2}{1+t^2}$

$= \frac{3t^2+8t-3}{1+t^2}$

d) $\tan\left(45° + \frac{\theta}{2}\right) = \frac{1+\sin\theta}{\cos\theta}$

$LHS = \frac{\tan 45 + \tan\frac{\theta}{2}}{1 - \tan 45 \tan\frac{\theta}{2}}$

$= \frac{1+t}{1-t}$ $\tan\frac{\theta}{2} = t$

$RHS = \frac{1+\sin\theta}{\cos\theta} = \frac{1+\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}}$

$= \frac{\frac{1+t^2+2t}{1+t^2}}{\frac{1-t^2}{1+t^2}}$

$= \frac{1+t^2+2t}{1-t^2}$

$= \frac{(t+1)^2}{(t+1)(t-1)}$

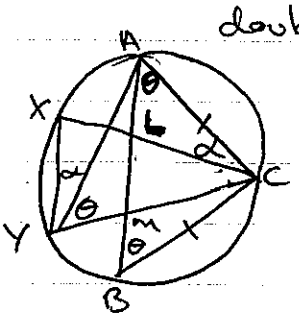
$= \frac{t+1}{t-1} = \frac{1+t}{1-t}$

$= LHS$

$\therefore \tan\left(45 + \frac{\theta}{2}\right) = \frac{1+\sin\theta}{\cos\theta}$

Question 11 a) $\angle BOC = 2(\alpha+\beta)$
(angle at centre is double angle at circumference)

b)



i) The exterior angle of a triangle (ΔALC) is equal to the sum of the opposite interior angles.

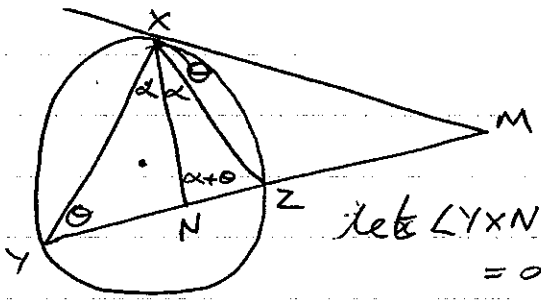
ii) $\angle ABC = \angle CAB = \theta$ (base angles of isosceles Δ)
$\therefore \angle AYC = \angle ABC = \theta$ (angles standing on same arc are equal)

iii) similarly $\angle ACX = \angle AYC = \alpha$
(angles on same arc are equal)

iv) $\angle XTM = \alpha - \theta$ (as above)
$= \angle CLB$
$= \alpha + \theta$
$\therefore$ XYML is a cyclic quadratic
(exterior angle of cyclic quad is equal to opposite interior angle)

Q. 11

c)



let $\angle YXN = \angle ZXN = \alpha$

let $\angle MXZ = \theta$

let $\angle YXN = \angle ZXN = \alpha$ (given)

let $\angle MXZ = \theta$

$\angle MXZ = \angle XYN$ ($\angle$ in alternate segment equal)

$= \theta$ between chord and tangent

$\angle XNZ = \theta + \alpha$ (exterior $\angle$ of $\triangle XYN$ equal to sum of interior opp $\angle$s)

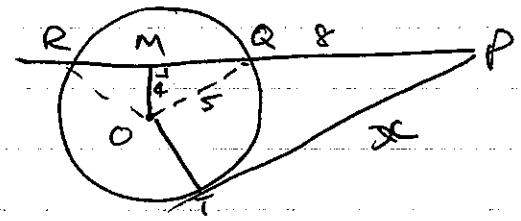
$\angle MXN = \angle MXZ + \angle ZXN$ (adjacent $\angle$)

$= \theta + \alpha$

$\angle MXN = \angle XNZ$ (base angles isosceles $\triangle MXN$)

$\therefore MX = MN$ (sides of isoseeles triangle.)

d)



i) Via Pythagoras theorem

$OQ = OR = 5$ (radius of circle)

$\therefore RM = \sqrt{5^2 - 4^2}$

$= 3$

$\therefore$ $RM = MQ$ (perpendicular bisector of OM)

$\therefore RQ = 2 \times 3$

$= 6$

ii) $x^2 = PQ.PR$

$x^2 = 8 \times 14$

$x^2 = 112$

$x = \sqrt{112}$ (measurement)

or

$x = 10.6$ (1d.p)