Name ____________________

Preliminary HSC Final - 2013

Mathematics Extension 1

Time Allowed - 1 hour 30 minutes + 5minutes reading

Instructions: Calculators may be used in any parts of the task. For 1 Mark Questions, the correct answer is sufficient to receive full marks. For Questions worth more than 1 Mark, necessary working MUST be shown to receive full marks.

Multiple Choice	/6
Question 7	/12
Question 8	/12
Question 9	/12
Question 10	/12
Total	/54

Fill in the correct answer on the answer sheet - Questions 1 - 6 are worth 1 mark each

1. $P(x)$ is an even polynomial. When $P(x)$ is divided by $(x-2)$ the remainder is 7. What is the remainder when $P(x)$ is divided by $(x+2)$

A -7

B $-7x$

C $7x$

D 7

2. By factorising $3^{n+1}+3^n$, rewrite $\frac{3^{1001}+3^{1000}}{4}$ as a power of 3

A 3^{1002}

B 3^{1000}

C 3^{1001}

D 3^{999}

3. If $(x-1)$ is a factor of $P(x)=2x^3+x^2+2x+a$, what is the value of a ?

A -6

B -5

C 5

D 6

4. Which of the following is an expression for $\sin(A+B)-\sin(A-B)$

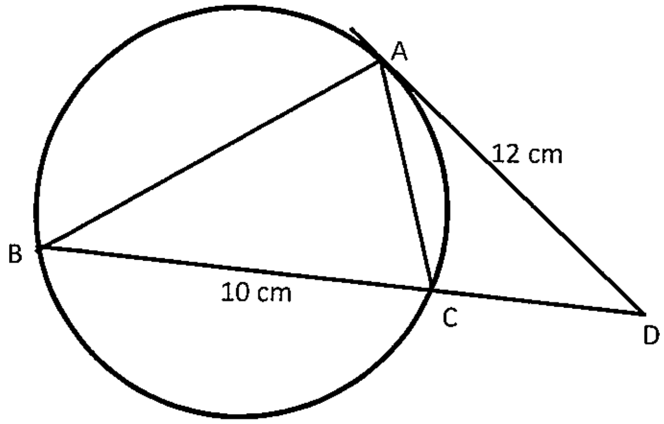
A $-2\sin A\cos B$

B $-2\cos A\sin B$

C $2\cos A\sin B$

D $2\sin A\cos B$

5.



ABC is a triangle inscribed in a circle. The tangent to the circle at A meets BC produced at D, where BC = 10 cm and AD = 12cm. What is the length of CD?

A 6 cm

B 7 cm

C 8 cm

D 9 cm

6. How many arrangements of the word GEOMETRY are possible if the word TRY appears in the new word?

A $2 \times 6!$

B $2 \times 8!$

C $\frac{8!}{3!}$

D $\frac{1}{2} \times 6!$

Question 7 12 Marks **(Begin a new sheet of paper)** **Marks**

a) The point P divides the interval AB joining A $(-3,-4)$ and B $(1,2)$ externally in the ratio 3:2. Find the coordinates of P. **3**

b) Find the acute angle between the lines $y = x + 2$ and $y = 2x - 3$. (Give your answer to the nearest minute.) **3**

c) Consider the word Q U A D R I L A T E R A L, leaving answers in unsimplified factorial form find:

(i) How many different arrangements of this word are possible? **1**

(ii) In how many of these arrangements will both the L's be together and the R's be together? **1**

(iii) In how many of the arrangements from part (i) will all the vowels be together? **1**

d) A committee of 4 boys and 3 girls is to be formed from a class of 15 boys and 10 girls.

(i) How many different committees can be formed? **1**

(ii) The class decides that Sue and Peter must not both be on the committee at the same time. How many committees are now possible? **2**

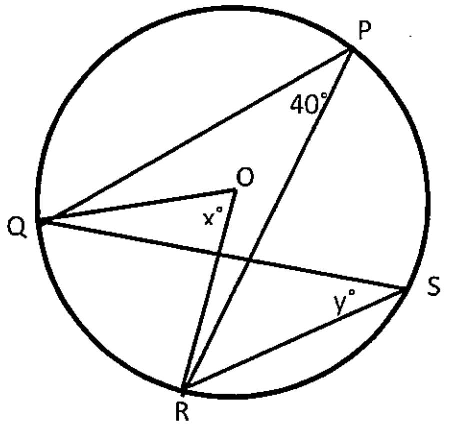
Question 8 12 Marks **(Begin a new sheet of paper)** **Marks**

a) (i) Using long division, prove that $(2x-3)$ is a factor of $P(x) = 6x^3 - 41x^2 + 24x + 36$. **2**

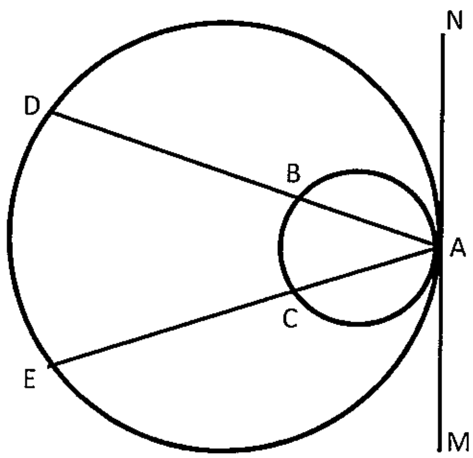
(ii) Hence completely factorise $P(x)$. **1**

b) Sketch the curve $y = (x+1)^3(3-2x)^2$ showing all intercepts. **3**

c) P, Q, R, and S are on the circle with centre O. Angle $QPR = 40°$.
Find the values of x and y giving a reason for each answer. **2**



d)



Two circles touch internally at A. MAN is the common tangent to the circles at A. ABD and ACE are two straight lines .

(i) Show that ΔABC is similar to ΔADE. **2**

(ii) Hence show that $\frac{AB \times AC}{AD \times AE} = \frac{BC^2}{DE^2}$. **2**

Question 9 12 Marks **(Begin a new sheet of paper)**

Marks

a) (i) Write down the expansion of $\cos(A + B)$. **1**

(ii) Hence find the exact value of $\cos 75°$ **2**

b) Use the substitution $t = \tan\frac{x}{2}$ to show that $\frac{1+\cos x+\sin x}{1-\cos x+\sin x} = \cot\frac{x}{2}$. **3**

c) Use the substitution $t = \tan\frac{\theta}{2}$ to solve $\sqrt{3}\sin\theta - \cos\theta = 1$ for $0 \leq \theta \leq 360°$ **3**

d) (i) Express $\cos x + \sqrt{3}\sin x$ in the form $A\cos(x - \alpha)$ where $A > 0$ and α is acute. **2**

(ii) Hence solve $\cos x + \sqrt{3}\sin x = 1$ for $0 \leq x \leq 360°$ **1**

Question 10 12 Marks **(Begin a new sheet of paper)**

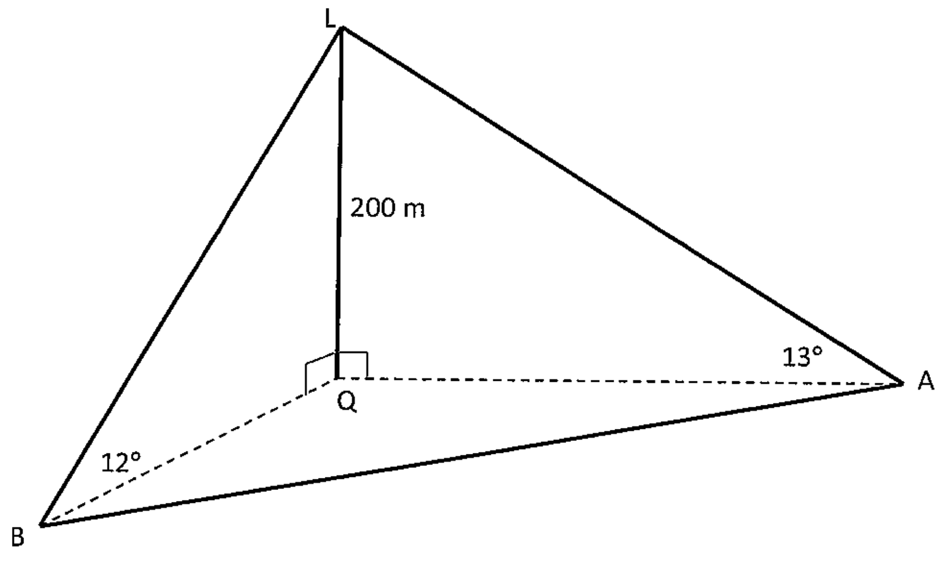
a) Solve the inequality $\frac{2x-1}{x} \geq x$ and graph your solution on a number line. **3**

b) Let α, β be the roots of $3x^2 + 5x - 1 = 0$.

(i) State the values of $\alpha + \beta,$ and $\alpha\beta$. **1**

(ii) Form the new quadratic equation with roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ **2**

c) The diagram shows a lighthouse at the top of a cliff. The combined height of the lighthouse and cliff is 200 m. A ship, A is due east of the lighthouse and the elevation of its top is 13°. A second ship, B finds that the lighthouse is on a bearing of 051°T, and the elevation of its top is 12°.



(i) Show that Angle AQB = 141° **1**

(ii) Show that AQ = 866.295 m (correct to 3 decimal places) **1**

(iii) Calculate how far apart the ships A and B are, giving your answer correct to the nearest metre. **2**

e) Simplify $\frac{1}{(n-1)!} + \frac{n^3+1}{(n+1)!}$ **2**

End of Test

Solutions to 2013 Extension 1 Preliminary Final

1. $P(2) = 7$ D

$\therefore P(-2) = 7$ as $P(x)$ is even

2. $3^{n+1} + 3^n = 3^n(3+1)$

$\therefore \dfrac{3^{1001} + 3^{1000}}{4} = \dfrac{3^{1000}(3+1)}{4}$ B

$= 3^{1000}$

3. $P(1) = 0 \quad \therefore 2+1+2+a = 0$ B

$a = -5$

4. $\sin(A+B) = \sin A\cos B + \cos A\sin B$ C

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\therefore \sin(A+B) - \sin(A-B) = 2\cos A\sin B$

5. Trial the answers

$18 \times 8 = 144$ C

6. (TRY) acts as one letter

$\therefore$ Arrange 6 things, 2 are alike

$\therefore \dfrac{6!}{2!}$ D

Q7 a) $x = \dfrac{kx_2 + lx_1}{k+l}$ $y = \dfrac{ky_2 + ly_1}{k+l}$ $\dfrac{k}{l} = \dfrac{3}{-2}$

$x = \dfrac{3\times 1 + -2\times -3}{3 + -2}$ $y = \dfrac{3\times 2 + -2\times -4}{3 + -2}$

$x = 9$ $y = 14$

$\therefore P(9, 14)$

b) $m_1 = 1$ $m_2 = 2$

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\dfrac{1-2}{1+2}\right|$

$\tan\theta = \frac{1}{3}$

$\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18^\circ 26'$

c) i) $\dfrac{13!}{3!\,2!\,2!}$ as there are 3 A's, 2 L's, 2 R's

ii) Put the L's together 1 way

Put the R's together 1 way

Arrange 11 things $\dfrac{11!}{3!}$

iii) Put the vowels together AAAIE

$\dfrac{5!}{3!}$ ways

Arrange: 9 things

$\dfrac{9! \times 5!}{2!\,2!\,3!}$

d) i) ${}^{15}C_4 \times {}^{10}C_3 = 163800$ ways

ii) Put them both in

${}^{14}C_3 \times {}^{9}C_2 = 13104$ ways

$\therefore$ Not both in is $163800 - 13104$

$= 150696$ ways

8 a)

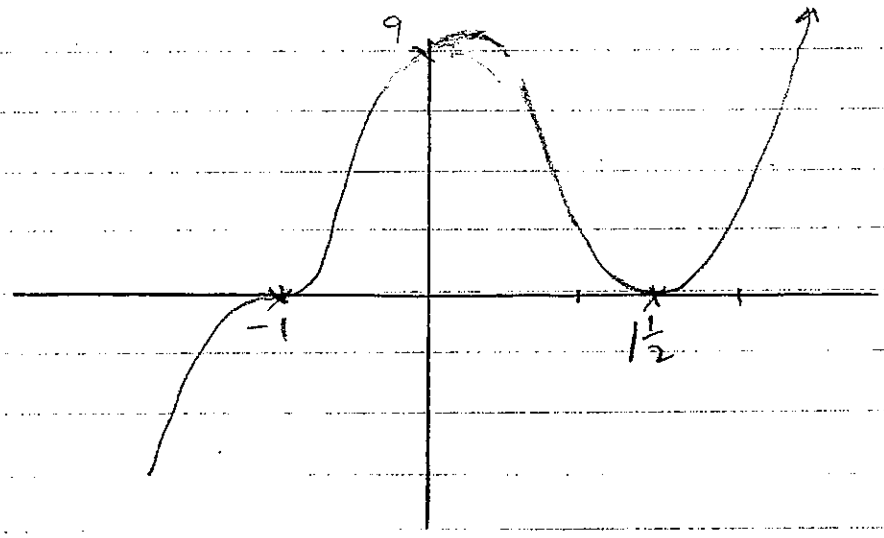
$$
\begin{array}{r}
3x^2 - 16x - 12 \\
2x - 3 \enclose{longdiv}{6x^3 - 41x^2 + 24x + 36} \\
-\underline{6x^3 \pm 9x^2} \quad\quad\quad\quad \\
-32x^2 + 24x + 36 \\
\underline{\pm 32x^2 \mp 48x} \quad\quad \\
-24x + 36 \\
\underline{-24x + 36} \\
\cdots
\end{array}
$$

$P(x) = (2x-3)(3x^2 - 16x - 12)$

$= (2x-3)(3x+2)(x-6)$

$$
\begin{array}{ccc}
3x & + & 2 \\
x & - & 6
\end{array}
$$

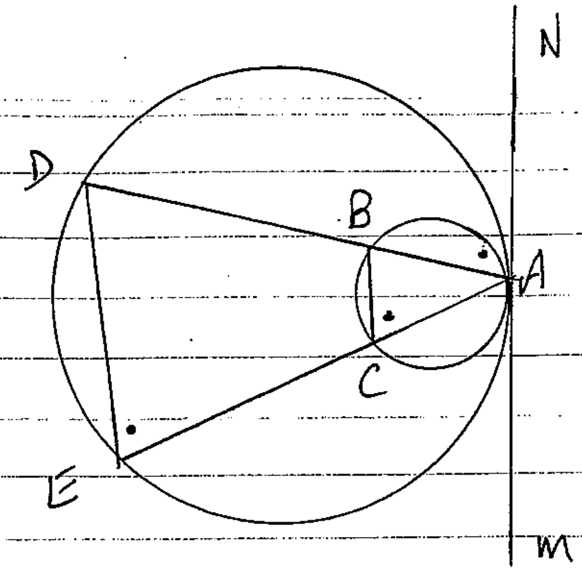
b)



c) $x = 80$ Angle at centre is twice angle at circumference standing on same arc.

$y = 40$ Angles in the same segment standing on arc QR are equal.

d)



i) In $\triangle$'s ABC, ADE

1. $\angle A$ is common

2. $\angle NAB = \angle ACB$ (Angle between tangent & chord equals angle in alternate segment)

$\angle NAB = \angle AED$ (Angle between tangent & chord equals angle in alternate segment)

$\therefore \angle ACB = \angle AED$ (Both $= \angle NAB$)

$\therefore \triangle ABC \parallel\!\mid \triangle ADE$ (2 angles test)

ii) $\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$ (corresponding sides in similar triangles)

or $\frac{AB}{AD} = \frac{BC}{DE}$ and (1)

$\frac{AC}{AE} = \frac{BC}{DE}$ (2)

(1) × (2) $\frac{AB}{AD} \times \frac{AC}{AE} = \frac{BC}{DE} \times \frac{BC}{DE}$

$$\frac{AB \times AC}{AD \times AE} = \frac{BC^2}{DE^2}$$

9 a) i) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

ii) Put $A = 45^\circ$ $B = 30^\circ$

$$\cos 75^\circ = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

b) $LHS = \dfrac{1+\cos x + \sin x}{1-\cos x + \sin x}$

$$= \frac{1 + \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2}}{1 - \frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2}} \times \frac{1+t^2}{1+t^2}$$

$$= \frac{1+t^2 + 1 - t^2 + 2t}{1+t^2 - 1 + t^2 + 2t}$$

$$= \frac{2+2t}{2t^2+2t}$$

$$= \frac{2(1+t)}{2t(t+1)} = \frac{1}{t}$$

$$= \cot \frac{x}{2}$$

c) $\sqrt{3}\sin\theta - \cos\theta = 1$

check does $\theta = 180^\circ$

$LHS = \sqrt{3}\sin 180 - \cos 180$

$0 - -1$

$= 1 = RHS.$

$\therefore \theta = 180^\circ$ is a solution.

$$\sqrt{3} \cdot \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} = 1$$

$$2\sqrt{3}t - 1 + t^2 = 1 + t^2$$

$$2\sqrt{3}t = 2$$

$$\sqrt{3}t = 1$$

$$t = \frac{1}{\sqrt{3}}$$

$$\therefore \tan\frac{\theta}{2} = \frac{1}{\sqrt{3}}$$

$$\frac{\theta}{2} = 30^\circ$$

$$\theta = 60^\circ$$

$\therefore \theta = 60^\circ$ or 180°

d) i) $\cos x + \sqrt{3}\sin x$

$\equiv A\cos(x - \alpha)$

$\equiv A\cos x\cos\alpha + A\sin x\sin\alpha$

$\therefore A\cos\alpha = 1$ + $A\sin\alpha = \sqrt{3}$

$\therefore \tan\alpha = \sqrt{3} \Rightarrow \alpha = 60^\circ$

$A^2 = 1^2 + (\sqrt{3})^2 \Rightarrow A = 2$

$2\cos(x - 60^\circ)$

ii) $2\cos(x - 60^\circ) = 1$

$\cos(x - 60^\circ) = \frac{1}{2}$

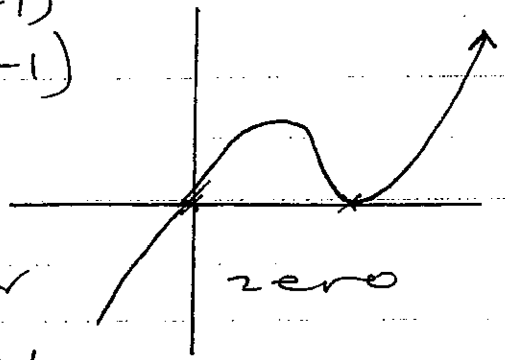
$x - 60^\circ = 60^\circ$ or 300° or -60°

$x = 120^\circ$ or 360° or 0°

10 a) $\dfrac{2x-1}{x} \geq x$ Note $x \neq 0$

Mult b/s by x^2 (which is positive)

$$x(2x-1) \geq x^3$$
$$0 \geq x^3 - x(2x-1)$$
$$0 \geq x(x^2 - 2x + 1)$$
$$0 \geq x(x-1)^2$$



Need graph negative or zero

$\therefore$ $x < 0$ or $x = 1$

b) $3x^2 + 5x - 1 = 0$

i) $\alpha + \beta = -\dfrac{5}{3}$ $\alpha\beta = -\dfrac{1}{3}$

ii) Sum of new roots

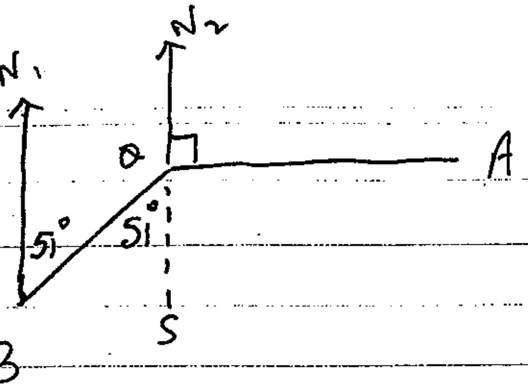
$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{(\alpha\beta)^2}$$
$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$
$$= \frac{\frac{25}{9} + \frac{2}{3}}{\frac{1}{9}} \times \frac{9}{9}$$
$$= 25 + 6 = 31$$

Product of new roots

$$\frac{1}{\alpha^2} \times \frac{1}{\beta^2} = \frac{1}{(\alpha\beta)^2} = 9$$

$\therefore$ Equation is $x^2 - 31x + 9 = 0$

10 c) i)

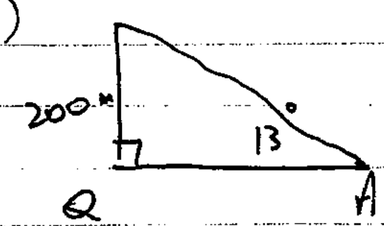


$\angle BQS = 51°$
(alternate $\angle$'s
$N_1B \parallel N_2S$)

$\angle SQA = 90°$
(A is due East)

$\therefore \angle BQA = 51 + 90$
$= 141°$

ii)



$\tan 13° = \dfrac{200}{QA}$ OR $\tan 77° = \dfrac{QA}{200}$

$QA = \dfrac{200}{\tan 13°}$ $QA = 200 \tan 77°$

$QA \doteq 866.295$ (to 3 dec places)

iii) $\tan 12° = \dfrac{200}{BQ}$ or $\tan 78° = \dfrac{BQ}{200}$

$BQ = \dfrac{200}{\tan 12°} \doteq 940.926$ m

$BA^2 = QA^2 + BQ^2 - 2.QA.BQ.\cos 141°$
$= 866.{..}^2 + 940.{..}^2 - 2 \times 866 \times 940. \cos 141°$
$= 2902742$
$BA = 1703.74$
$BA \doteq 1704$ m to nearest m

e) $\dfrac{1}{(n-1)!} + \dfrac{(n+1)(n^2-n+1)}{(n+1)!}$

$= \dfrac{1}{(n-1)!} + \dfrac{n^2-n+1}{n!}$

$= \dfrac{n + n^2 - n + 1}{n!}$

$= \dfrac{n^2+1}{n!}$