STUDENT NUMBER/NAME: ……………………..

GOSFORD HIGH SCHOOL

2014

PRELIMINARY COURSE

FINAL EXAMINATION

Mathematics Extension 1

- **General Instructions**
- Reading time – 5 minutes
- Working time – 1½ hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- In Questions 6-9, show relevant mathematical reasoning and/or calculations

Total Marks – 53

Section I

5 marks

- Attempt Questions 1 – 5
- Allow about 10 minutes for this section

Section II

48 marks

- Attempt Questions 6 – 9
- Allow about 1hour and 20 minutes for this section

Section I

5 marks

Attempt Questions 1 – 5.

Allow about 10 minutes for this section.

Use the separate multiple-choice answer sheet for Questions 1 – 5.

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1. Which of the following is an expression for $\dfrac{9^n - 6^n}{9^n - 4^n}$?

(A) $\dfrac{6^n}{4^n}$

(B) $\dfrac{3^n}{5^n}$

(C) $\dfrac{3^n}{3^n - 2^n}$

(D) $\dfrac{3^n}{3^n + 2^n}$

2. If $R\cos(x+\alpha) \equiv \sin x - \cos x$, what are the values of $R\cos\alpha$ and $R\sin\alpha$?

(A) $R\cos\alpha = -1,\quad R\sin\alpha = -1$

(B) $R\cos\alpha = -1,\quad R\sin\alpha = 1$

(C) $R\cos\alpha = 1,\quad R\sin\alpha = -1$

(D) $R\cos\alpha = 1,\quad R\sin\alpha = 1$

3. What is the value of $\frac{^nC_3}{^nP_3}$?

(A) 6

(B) 3

(C) $\frac{1}{3}$

(D) $\frac{1}{6}$

4. If $7\sqrt{x^5} - \sqrt{4x} + 3\sqrt{x^6} = a + b\sqrt{x}$, find a and b.

(A) $a = 3x^3,\ b = 7x^2 - 4$

(B) $a = 3x^3,\ b = 7x^2 - 2$

(C) $a = 3x^2,\ b = 7x^2 - 2$

(D) $a = 3x^6,\ b = 7x^4 - 4$

5.

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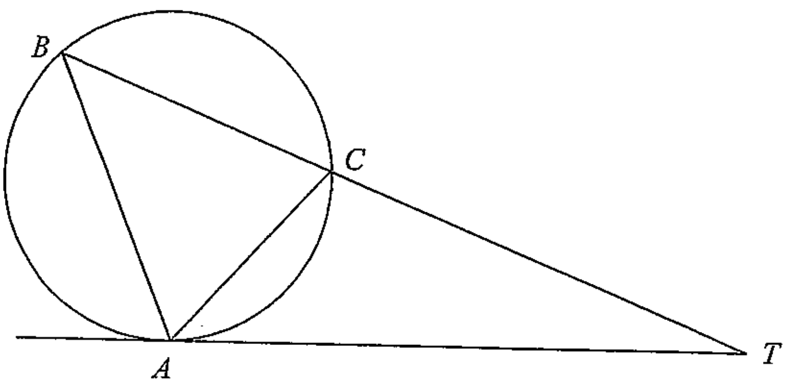


In the diagram, ABC is a triangle inscribed in a circle. The tangent to the circle at A meets BC produced at T. $TA = 10$ cm and $TC = 8$ cm. What is the length of BC?

(A) 3·5 cm

(B) 4 cm

(C) 4·5 cm

(D) 5 cm

END OF SECTION I

SECTION II

48 marks
Attempt Questions 6 – 9
Allow about 1 hour and 20 minutes for this section.

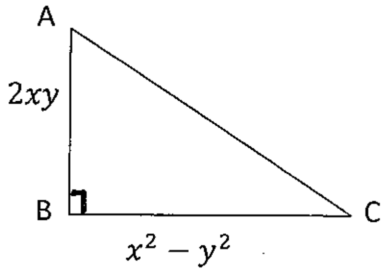
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 6 – 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 6: (12 marks) (Use a SEPARATE writing booklet)

a. Solve the inequality $x - \frac{4}{x} \geq 0$ 2

b. Given the right angled triangle ABC 2



find an expression, in its simplest form, for AC.

c. The point A(4, 2) divides the interval XY internally in the ratio K : 1. If X is the point (10, –4) and Y is the point (0, 6), find the value of K, in its simplest form. 2

d. The angle between the lines

$y = mx + 3$ and $x - 3y + 9 = 0$ is 45°. 2

Find the exact value(s) of m.

e. Simplify completely the expression

$$\frac{(n+1)! - n!}{(n-1)!}$$ 2

f. There are 8 people in a dinner group, 4 men and 4 women. In how many ways can they be seated around a circular table if a particular man Carl refuses to sit next to a certain woman Amanda. 2

END OF QUESTION 6

Question 7: (12 marks) (Use a SEPARATE writing booklet)

a. If each distinct arrangement of the letters of **1**

D E L E T E D

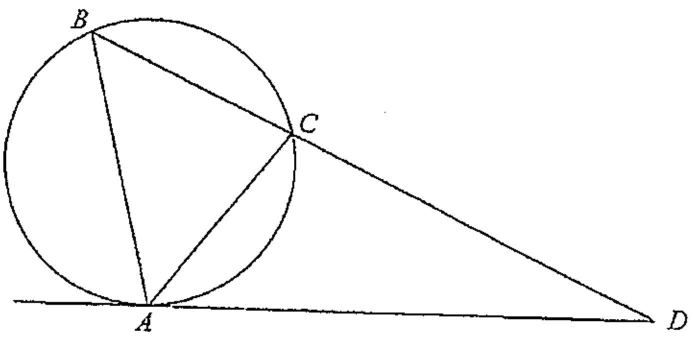
is called a word, then how many words are possible?

b. Find the number of ways in which the 7 letters of the word PYRAMID can be arranged in a row.

i. without restriction **1**

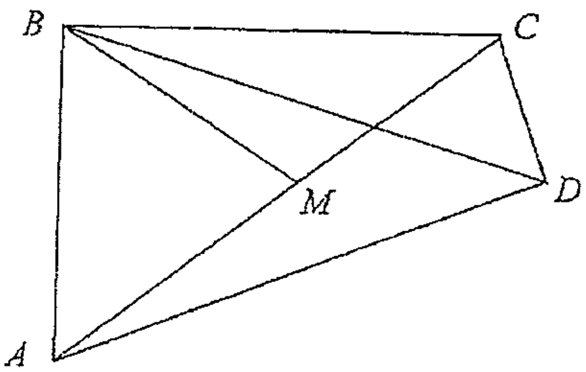
ii. so that the 5 consonants are in alphabetical order from left to right. **1**

c. **2**



In the diagram, ABC is a triangle inscribed in a circle with $BC = AC$. The tangent to the circle at A meets BC produced at D. Show that AC *bisects* $\angle DAB$.

d.



In the diagram, $ABCD$ is a quadrilateral in which $\angle ABC = \angle ADC = 90°$. M is the midpoint of AC.

i. Copy the diagram and explain why $ABCD$ is a cyclic quadrilateral. **1**

ii. Show that $\angle BMC = 2\angle BDC$ **2**

(Question 7 continues over the page)

Question 7 continued:

e. i. Use the substitution $t = \tan\frac{x}{2}$

to show that $\sec x + \tan x = \frac{1+t}{1-t}$ 2

ii. Hence, solve the equation 2

$secx + \tan x = -1$ for $0° \leq x° \leq 360°$

END OF QUESTION 7

Question 8: (12 marks) (Use a SEPARATE writing booklet)

a. Find the number of ways a committee can be chosen from 6 students, 5 parents and 4 teachers.

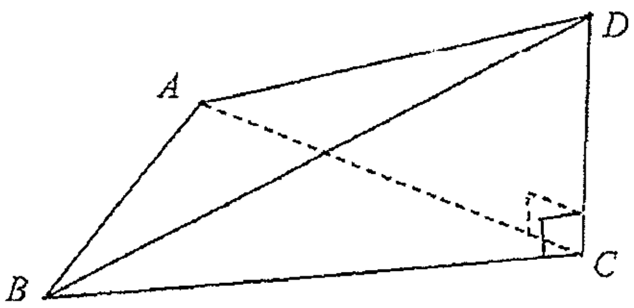
i. without restriction. 1

ii. so that the committee contains a majority of students. 2

b. Solve the equation: 2

$$2\cos^2 x = -3\sin x \text{ for } 0° \le x° \le 360°$$

c.



In the diagram a vertical tower *CD* of height *h* metres stands with its bottom C on horizontal ground. *A* and *B* are two points on the ground such that *AB = 28 metres.*

From *B* the bearing of the tower is 050° and the angle of elevation of the top D of the tower is 30°. From A the bearing of the tower is 110° and the angle of elevation of the top D of the tower is 60°. Find the exact height of the tower.

i. By first drawing a "bird's eye view" diagram indicating ΔABC show that $\angle ACB = 60°$. 1

ii. Hence, find the exact height of the tower. 3

d. i. Express tan (A + B) in terms of tan A and tan B. 1

ii. Hence, show that, if A, B and C are the vertex angles of a triangle then, 2

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

END OF QUESTION 8

Question 9: (12 marks) (Use a SEPARATE writing booklet)

a. Factorise fully $54x^3 + 16$ 1

b. i. Show that 2

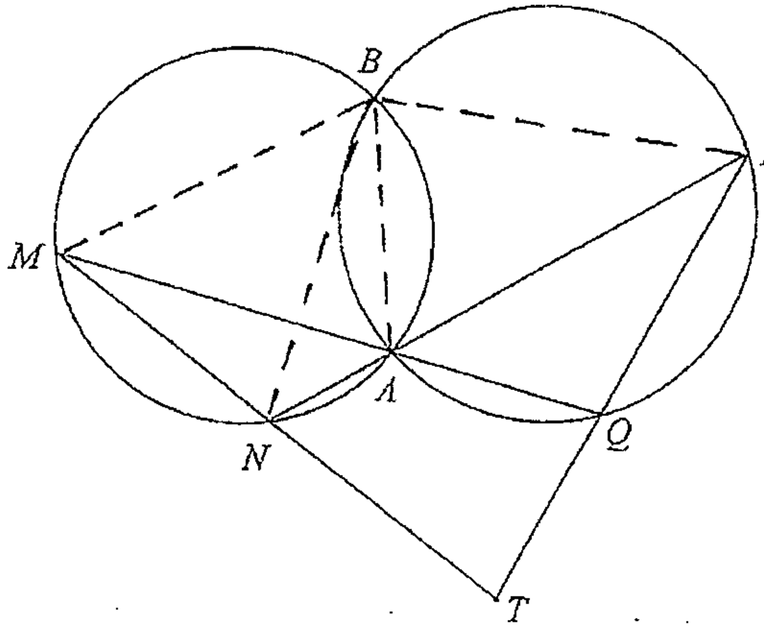
$$\frac{1 + tan\,x}{1 - tan\,x} - \frac{1 - tan\,x}{1 + tan\,x} = 2tan2x$$

ii. Hence, find the value of: 1

$$\frac{1 + tan\,75°}{1 - tan\,75°} - \frac{1 - tan\,75°}{1 + tan\,75°}$$

in simplest exact form.

c. 3



In the diagram, two circles intersect at *A* and *B*. Points *M* and *N* lie on one circle and points *P* and *Q* lie on the other circle such that *MAQ* and *NAP* are straight lines. *MN* produced and *PQ* produced meet at *T*.

(Also, lines *MB, NB, BA* and *BP* are constructed for you.)

Show that *TNBP* is a cyclic quadrilateral.

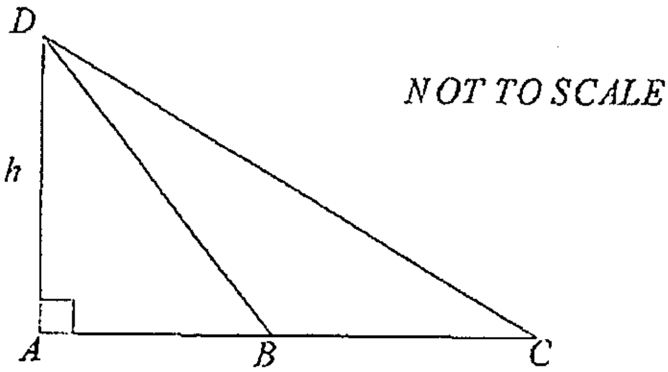
(Question 9 continues over the page)

Question 9 continued:

d.

(i) Prove that $\sin(45° + \theta)\sin(45° - \theta) = \frac{1}{2}\cos 2\theta$ 2

(ii) In the diagram, D is the top of a tower h metres high. The angles of depression of the points B and C are $(45° + \theta)$ and $(45° - \theta)$ respectively 3



Using the result in (i) show that $BC = 2h\tan 2\theta$

END OF EXAMINATION

MATHS EXT 1 YR 11 YEARLY 2014.

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SECTION I

1. $\frac{9^n-6^n}{9^n-4^n} = \frac{3^n(3^n-2^n)}{(3^n-2^n)(3^n+2^n)}$

$= \frac{3^n}{3^n+2^n}$ (D)

2. $R\cos(x+\alpha)$

$= R(\cos x\cos\alpha - \sin x\sin\alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

$\therefore R\cos\alpha = -1$ (A)

$R\sin\alpha = -1$

3. $\frac{^nC_3}{^nP_3} = \frac{\frac{n!}{(n-3)!\,3!}}{\frac{n!}{(n-3)!}}$

$= \frac{1}{3!}$

$= \frac{1}{6}$ (D)

4. $7\sqrt{x^5} - \sqrt{4x} + 3\sqrt{x^6}$

$= 7\sqrt{x^4 x} - 2\sqrt{x} + 3x^3$

$= 7x^2\sqrt{x} - 2\sqrt{x} + 3x^3$

$= 3x^3 + (7x^2-2)\sqrt{x}$ (B)

$\therefore a = 3x^3 \quad b = 7x^2-2$

5. $AT^2 = BT \times CT$

$10^2 = BT \times 8$

$100 = 8 \times BT$

$12.5 = BT$ (C)

$\therefore BC = 12.5-8$

$= 4.5$ cm.

Answers.

1. D
2. A
3. D
4. B
5. C

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SECTION II

Q6 a) $x - \frac{4}{x} \geq 0$

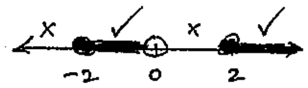
Consider $x - \frac{4}{x} = 0 \quad x \neq 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

Test



$-2 \leq x < 0$ or $x \geq 2$

b) Using Pythagoras

$(2xy)^2 + (x^2-y^2)^2 = AC^2$

$4x^2y^2 + x^4 - 2x^2y^2 + y^4 = AC^2$

$x^4 + 2x^2y^2 + y^4 = AC^2$

$(x^2+y^2)^2 = AC^2$

$\therefore AC = x^2+y^2$

(positive since a length)

c) $(10,-4) \quad (0,6)$

$k:1$

$\downarrow$

$(4,2)$

using the x co-ord:

$\frac{1(10)+k(0)}{k+1} = 4$

$10 = 4k+4$

$6 = 4k$

$\frac{6}{4} = k$

$\therefore k = \frac{3}{2}$

(d) Using $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\tan 45° = \left|\frac{m-\frac{1}{3}}{1+\frac{1}{3}m}\right|$

$1 = \left|\frac{\frac{3m-1}{3}}{\frac{3+m}{3}}\right|$

$1 = \left|\frac{3m-1}{3+m}\right|$

$\therefore \frac{3m-1}{3+m} = 1$ or $\frac{3m-1}{3+m} = -1$

$3m-1 = 3+m$ or $3m-1 = -3-m$

$2m = 4 \qquad 4m = -2$

$m = 2 \qquad m = -\frac{1}{2}$

$\therefore m = 2$ or $-\frac{1}{2}$

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(e) $\frac{(n+1)! - n!}{(n-1)!}$

$= \frac{n!(n+1-1)}{(n-1)!}$

$= \frac{n!(n)}{(n-1)!}$

$= n \times n$

$= n^2$

(f) Total seatings $= 7!$

Ways together $= 6! \times 2$

$\therefore$ Ways apart $= 7! - 6! \times 2$

$= 3600$

Q7 a) Arrangements $= \frac{7!}{3!\,2!}$

$= 420$

b) (i) $7! = 5040$

(ii) Ways the consonants can be arranged $= 5!$

(alphabetical order is only 1 of these!)

Ways within the 7 letter arrangement

$= \frac{7!}{5!}$

$= 42$

(c) let $\hat{CAD} = \theta$

$\therefore \hat{ABC} = \theta$ (alternate segment theorem)

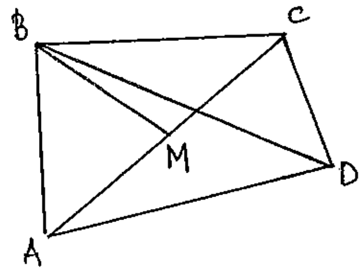
also $\hat{BAC} = \theta$ (equal base $\angle$s in isosceles Δ)

$\therefore \hat{ABC} = \hat{BAC} = \theta$

$\therefore$ AC bisects $\hat{DAB}$

(d)

(i)



ABCD is a cyclic quadrilateral since $\hat{ABC} = \hat{ADC} = 90°$ (given in data)

and opposite angles in a cyclic quadrilateral are equal

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(ii) Since $\hat{ABC} = 90°$

AC must be a diameter

(angle in a semicircle $= 90°$)

$\therefore$ M is the centre of the circle (midpoint of diameter)

$\therefore \hat{BMC} = 2 \times \hat{BDC}$

(angle at the centre is twice the angle at the circumference standing on the same arc)

(e)(i) $\cos x = \frac{1-t^2}{1+t^2}$ $\therefore \sec x = \frac{1+t^2}{1-t^2}$

$\tan x = \frac{2t}{1-t^2}$

$\therefore \sec x + \tan x = \frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}$

$= \frac{1+2t+t^2}{1-t^2}$

$= \frac{(1+t)^2}{(1-t)(1+t)}$

$= \frac{1+t}{1-t}$

as req.

(ii) $\therefore \frac{1+t}{1-t} = -1$

$1+t = -1+t$

$0 = -2$

$\therefore$ no solutions

but we must test $x = 180°$

LHS $=$

$\sec 180° + \tan 180°$

$= -1 + 0$

$= -1$

$\therefore 180°$ is a solution

(the only solution)

Q8

(a) (i) ${}^{15}C_3 = 455$

(ii) there can be 3 students or 2.

$\therefore {}^6C_3 + {}^6C_2 \times {}^9C_1$

$= 20 + 15 \times 9$

$= 155$

(b) $2\cos^2 x = -3\sin x$

$2(1-\sin^2 x) = -3\sin x$

$0 = 2\sin^2 x - 3\sin x - 2$

$0 = (2\sin x + 1)(\sin x - 2)$

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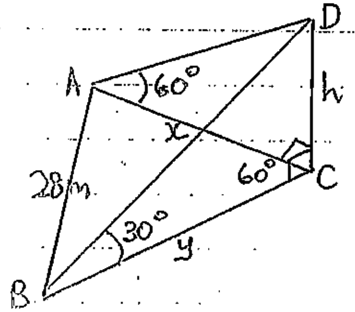
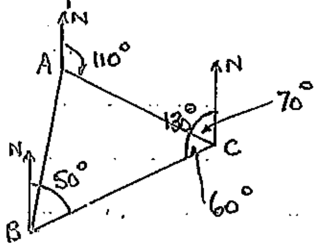
$\therefore \sin x = -\frac{1}{2}$ or $\sin x = -2$ (no solns)

$\therefore x = 210^\circ, 330^\circ$

(c)



This diagram would also help.



From the top diagram

$\tan 60^\circ = \frac{h}{x}$

$x = \frac{h}{\tan 30^\circ}$

$x = \frac{h}{\sqrt{3}}$

Similarly $y = \sqrt{3}h$

Using the cosine rule

$28^2 = x^2 + y^2 - 2xy\cos 60^\circ$

$784 = \frac{h^2}{3} + 3h^2 - 2h^2 \times \frac{1}{2}$

$784 = \frac{10h^2}{3} - h^2$

$784 = \frac{7h^2}{3}$

$2352 = 7h^2$

$336 = h^2$

$\sqrt{336} = h$

$\therefore h = 4\sqrt{21}$ m.

(d) (i) $\tan(A+B)$

$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$

(ii) if A, B, C are the 3 angles of a Δ then $A+B+C = 180^\circ$

$\therefore A+B = 180^\circ - C$

$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan(180^\circ - C)$

$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\tan A + \tan B = -\tan C(1 - \tan A \tan B)$

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$\therefore \tan A + \tan B = -\tan C + \tan A \tan B \tan C$

$\therefore \tan A + \tan B + \tan C = \tan A \tan B \tan C$ as req.

Q9

(a) $54x^3 + 16$

$= 2(27x^3 + 8)$

$= 2(3x+2)(9x^2 - 6x + 4)$

(b) (i) LHS $= \frac{(1+\tan x)^2 - (1-\tan x)^2}{(1-\tan x)(1+\tan x)}$

$= \frac{1 + 2\tan x + \tan^2 x - 1 + 2\tan x - \tan^2 x}{(1-\tan x)(1+\tan x)}$

$= \frac{4\tan x}{1 - \tan^2 x}$

$= 2 \times \frac{2\tan x}{1 - \tan^2 x}$

$= 2 \times \tan 2x$

$= 2\tan 2x$ as req.

(ii) $\therefore x = 75^\circ$

LHS $= 2\tan 150^\circ$

$= 2 \times -\frac{1}{\sqrt{3}}$

$= -\frac{2}{\sqrt{3}}$

$= -\frac{2\sqrt{3}}{3}$

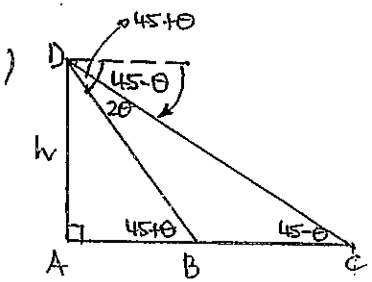
(c) $M\hat{A}B = M\hat{N}B$ ($\angle$'s at the circumference standing on the same arc are equal)

$M\hat{A}B = B\hat{P}Q$ (exterior $\angle$ of cyclic quadrilateral is equal to interior opposite $\angle$)

and $\therefore M\hat{N}B = B\hat{P}Q$

and $\therefore$ TNBP is a cyclic quadrilateral since exterior $\angle$ of cyclic quadrilateral = interior opposite angle.

(d)



(i) $\sin(45+\theta)\sin(45-\theta)$

$= (\sin 45^\circ \cos\theta + \sin\theta \cos 45^\circ) \times (\sin 45^\circ \cos\theta - \sin\theta \cos 45^\circ)$

$= \sin^2 45^\circ \cos^2\theta - \sin^2\theta \cos^2 45^\circ$

$= \frac{1}{2} \cdot \cos^2\theta - \frac{1}{2}\sin^2\theta$

You may ask for an extra Writing Booklet if you need more space.

Start here.

$= \frac{1}{2}(\cos^2\theta - \sin^2\theta)$

$= \frac{1}{2}\cos 2\theta$ as req.

(ii) In ΔADB

$\sin(45+\theta) = \frac{h}{DB}$ — (1)

In ΔBDC, using sine rule

$\frac{BC}{\sin 2\theta} = \frac{DB}{\sin(45-\theta)}$ —(2)

From (1)

$DB = \frac{h}{\sin(45+\theta)}$

In (2) $\frac{BC}{\sin 2\theta} = \frac{h}{\sin(45+\theta)\sin(45-\theta)}$

$\frac{BC}{\sin 2\theta} = \frac{h}{\frac{1}{2}\cos 2\theta}$

$BC = 2h\frac{\sin 2\theta}{\cos 2\theta}$

$= 2h\tan 2\theta$

as req.