Student Number

Gosford High School

2017

Preliminary Final Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 8 – 11 show relevant mathematical reasoning and/or calculations

Total Marks – 55

Section I Questions 1 – 7 **7 marks**

Allow about 10 minutes for this section

Section II Questions 8 – 11 **48 marks**

Allow about 1 hour and 20 minutes for

Section 1

7 marks

Attempt Questions 1 – 7

Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 7.

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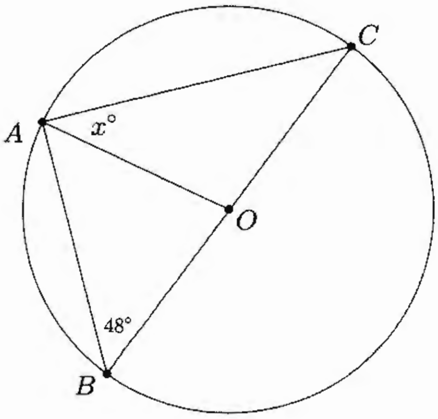
1 The diagram below shows a circle with centre O. Points A, B and C lie on the circle and $\angle OBA = 48°$.



If $\angle OAC = x°$, which of the following is the value of x?

(A) 48

(B) 42

(C) 45

(D) 52

2 Which of the following is equivalent to $\sqrt{3}\sin\theta-\cos\theta$?

(A) $2\sin\left(\theta+30\right)^{\circ}$

(B) $2\sin\left(\theta-30\right)^{\circ}$

(C) $2\sin\left(\theta+60\right)^{\circ}$

(D) $2\sin\left(\theta-60\right)^{\circ}$

3 The number of different arrangements of the letters of the word *SERVICES* which begin and end with the letter *S* is:

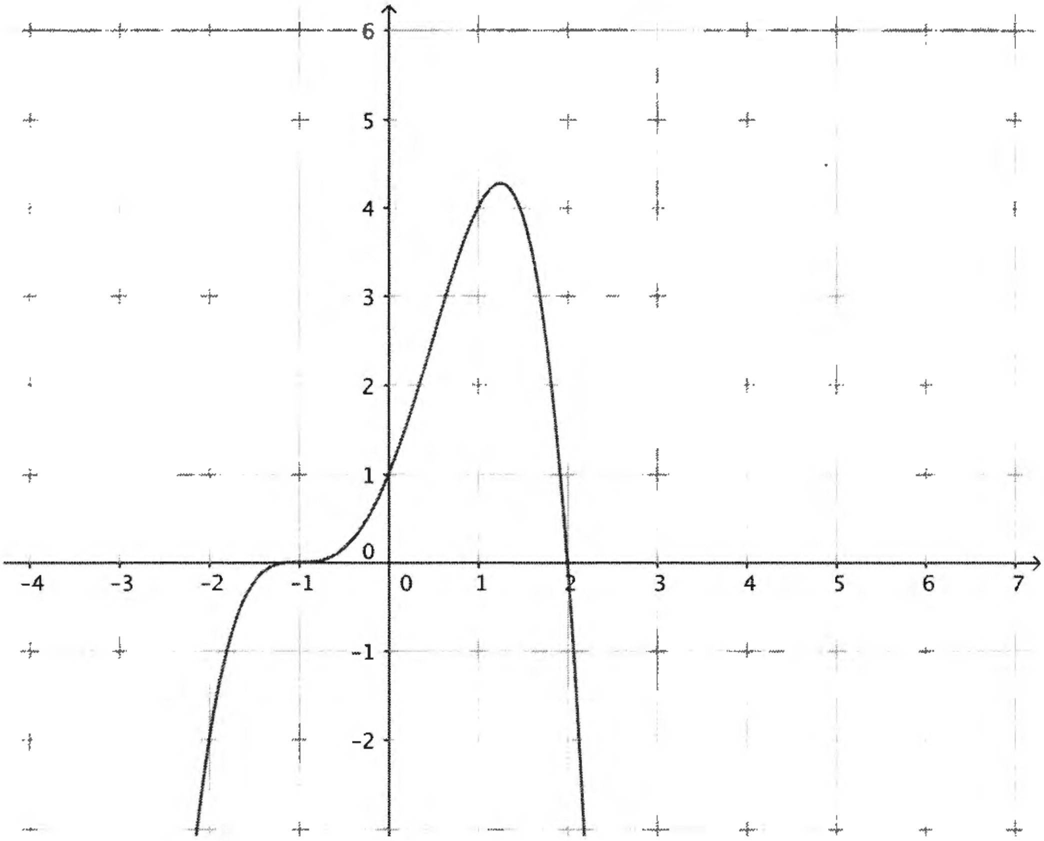
(A) $\dfrac{6!}{\left(2!\right)^{2}}$

(B) $\dfrac{8!}{\left(2!\right)^{2}}$

(C) $\dfrac{6!}{2!}$

(D) $\dfrac{8!}{2!}$

4



Which of the following could be the equation of the graph shown above?

(A) $y = \frac{1}{2}(x+1)^3(x-2)$

(B) $y = \frac{1}{2}(x+1)^3(2-x)$

(C) $y = (x+1)^3(x-2)$

(D) $y = (x+1)^3(2-x)$

5 What is the solution to $|2x-1| \geq |x+1|$?

(A) $0 \leq x \leq 3$

(B) $x \leq 0$ or $x \geq 3$

(C) $0 \leq x \leq 2$

(D) $x \leq 0$ or $x \geq 2$

6 If $\sin x = \frac{3}{5}$, $90^\circ \leq x \leq 180^\circ$, evaluate $\tan 2x$.

(A) $-\frac{7}{24}$

(B) $-\frac{24}{7}$

(C) $\frac{7}{24}$

(D) $\frac{24}{7}$

7 A root of the polynomial $P(x) = x^3 - 2x^2 - 5x + 6$ is

(A) 2

(B) -3

(C) 1

(D) -4

Section II

48 marks

Attempt Questions 8 – 11

Allow about 1 hour and 20 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In questions 8 – 11, your responses should include relevant mathematical reasoning and/or calculations.

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) The monic polynomial $P(x)$ has constant term 6 and is such that $P\left(x\right)=(x+3)(x-2)(ax+b)$. Find the values of a and b. **2**

(b) The lines $y=2mx+1$ and $y=mx+7$ intersect at an acute angle whose tangent is $\frac{1}{3}$. Show that the possible values of m are $m=\frac{1}{2}$, 1. **3**

(c) Solve the inequality $\frac{x+3}{2x}>1$. **2**

(d) The staff in an office consists of 6 males and 5 females. **2**

How many committees of 5 staff can be chosen which contain exactly 2 females?

(e) Solve the equation $\sin 2x=\tan x$ for $0^{\circ}\leq x\leq 180^{\circ}$. **3**

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) The equation $x^3 - 2x^2 + kx + c = 0$ has roots $x = -1, x = 1$ and $x = 2$. **2**

Show that the value of *k* is -1.

(b) If *P(3,4)* divides *AB* internally in the ratio 2:3 and *B* is (-2,1). **2**

Find the co-ordinates of *A*.

(c) Use the substitution $t = \tan\frac{x}{2}$ to show that the solutions to the equation **3**

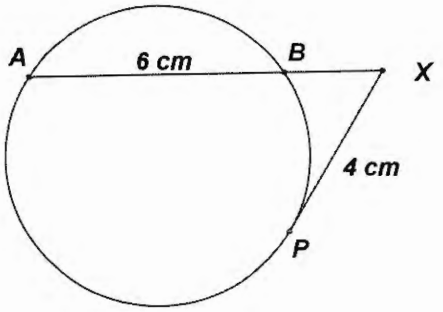
$6 + 4\cos x + 5\sin x = 0$, correct to the nearest degree where $0^\circ \le x \le 360^\circ$,

are $x = 211^\circ, 252^\circ$.

(d) Solve for x: **2**

$x + 2\sqrt{x+1} = 7$

(e)



The diagram above shows a circle and *AB* is a chord of length 6 *cm*.

The tangent to the circle at *P* meets *AB* produced at *X*. *PX* = 4 *cm*.

(i) Find the length of *BX*. **1**

(ii) Given that *AP* is a diameter of the circle find the exact length of the radius of the circle. **2**

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) Find the domain and range of the function $f(x)=\sqrt{3-\sqrt{x}}$ **3**

(b) Find the Cartesian equation of the parabola given **1**

$x=t+3,\ \ y=4-2t^2$

(c) Show that $\tan 2x+\tan x=\dfrac{\sin 3x}{\cos 2x\cos x}$ **2**

(d) In how many ways can 5 women and 6 men sit around a table.

(i) Without restrictions. **1**

(ii) If 2 particular men wish to sit next to a particular woman. **2**

(e) From the top of a lighthouse, *L*, 115 metres above sea level, a container ship, *C*, is seen on a bearing of 135°T, at an angle of depression of 10°. **3**

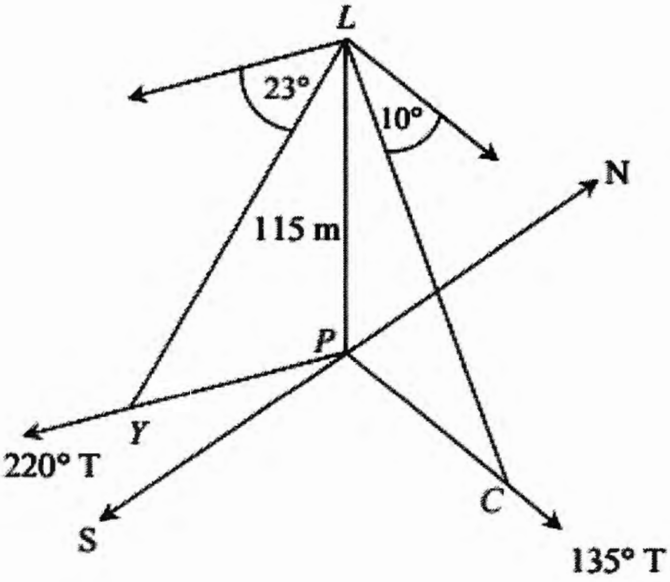
A yacht, *Y*, is also sighted on a bearing of 220°T, at an angle of depression of 23°.

This is illustrated in the diagram below.

Show that the square of the distance between the two vessels, $(CY)^2$, is given by:

$$(CY)^2 = 115^2\left(\tan^2 67^\circ + \tan^2 80^\circ - 2\tan 67^\circ \tan 80^\circ \cos 85^\circ\right)$$

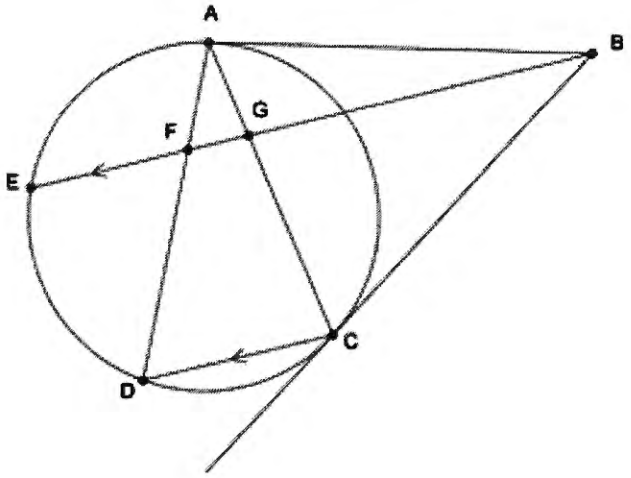
Give clear reasons for each step in your calculations.



Question 11 (12 marks) Use a SEPARATE writing booklet.

(a) Show that $A=\frac{1}{2}$ if $\frac{6^n+3^n}{2^{n+1}+2}=A\times 3^n$. **2**

(b) In the diagram *EB* is parallel to *DC*. Tangents from *B* meet the circle at *A* and *C*.



Copy or trace the diagram into your answer booklet.

(i) Prove that $\angle BCA=\angle BFA$. **2**

(ii) Prove *ABCF* is a cyclic quadrilateral. **1**

(c) If α , β and γ are the roots of the equation $2x^3-5x^2-2x+6=0$,

find the values of:

(i) $\alpha+\beta+\gamma$ **1**

(ii) $\alpha\beta+\beta\gamma+\alpha\gamma$ **1**

(iii) $\alpha^2+\beta^2+\gamma^2$ **2**

(iii) $\frac{1}{\alpha^2\beta^2}+\frac{1}{\beta^2\gamma^2}+\frac{1}{\alpha^2\gamma^2}$ **3**

Extension 1 Prelim 2017

1 $48 + x + 90 = 180$

$\therefore x = 42$ Ⓑ

2 $\sqrt{3}\sin\theta - \cos\theta$

$R = 2$

$\therefore 2\sin(\theta - \alpha) \equiv \sqrt{3}\sin\theta - \cos\theta$

$\therefore 2\cos\alpha = \sqrt{3}$

$2\sin\alpha = 1$

$\therefore \tan\alpha = \frac{1}{\sqrt{3}}$

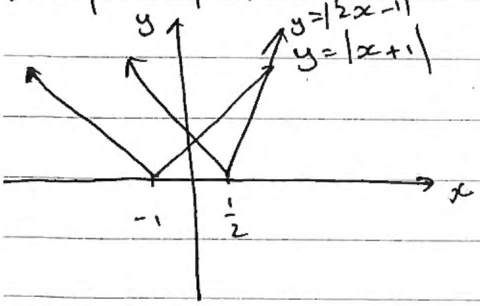
$\alpha = 30°$

$\therefore 2\sin(\theta - 30)°$ Ⓑ

3 $\frac{6!}{2!}$ Ⓒ

4 $y = \frac{1}{2}(x+1)^3(2-x)$ Ⓑ

5 $|2x-1| \geq |x+1|$



pts int: $-(2x-1) = x+1$

$3x = 0$ $\quad \underline{x = 0}$

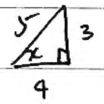
$2x - 1 = x + 1$

$\underline{x = 2}$

$= |2x-1|$ above $y = |x+1|$

$x \leq 0\ ,\ x \geq 2$ Ⓓ

6



Q2

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$= \frac{2 \cdot \frac{-3}{4}}{1 - \frac{9}{16}}$

$= \frac{\frac{-3}{\cancel{2}}}{\frac{7}{\cancel{16}\ 8}}$

$= \frac{-24}{7}$ Ⓑ

7 $P(x) = x^3 - 2x^2 - 5x + 6$

$P(2) \neq 0$ $\qquad P(1) = 0$

$P(-3) \neq 0$ $\qquad P(-4) \neq 0$ Ⓒ

8

a) $P(x) = (x+3)(x-2)(ax+b)$

$3(-2)(b) = 6$

$\therefore -6b = 6$ $\quad \therefore b = -1$

$(1)(1)a = 1$ $\quad \therefore a = 1$

b) $y = 2mx + 1$ $\qquad m_1 = 2m$

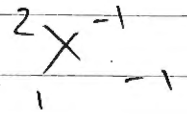
$y = mx + 7$ $\qquad m_2 = m$

$\tan\alpha = \frac{2m - m}{1 + 2m^2}$

$\therefore \frac{m}{1+2m^2} = \frac{1}{3}$

$3m = 1 + 2m^2$

$2m^2 - 3m + 1 = 0$



$(2m-1)(m-1) = 0$

$m = \frac{1}{2}, 1$

c) $\frac{x+3}{2x} > 1$

C.V at

$\boxed{x=0}$

$x+3=2x$

$\boxed{x=3}$

× | ✓ | ×

0   3

$x=-1$ $\frac{2}{-2} > 1$ ×

$x=1$ $\frac{4}{2} > 1$ ✓

$x=4$ $\frac{7}{8} > 1$ ×

$0 < x < 3$

d) 6m 5F (5 chosen 2F)

∴ $^6C_3 \times {}^5C_2 = 200$

e) $\sin 2x = \tan x$

$2\sin x\cos x = \frac{\sin x}{\cos x}$ $\cos x \neq 0$

$2\sin x\cos^2 x - \sin x = 0$

$\sin x(2\cos^2 x - 1) = 0$

$\sin x = 0$ | $2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

∴ $x = 0, 180°, 45°, 135°$

Q9

a) $x^3 - 2x^2 + kx + c = 0$

$x = -1, 1, 2$

$\Sigma\alpha\beta = -1-2+2$

$= -1$

∴ $-1 = \frac{k}{1}$ ∴ $k = -1$

b) $A(x,y)$ $B(-2,1)$

$2:3$

$3 = \frac{-4+3x}{5}$ $4 = \frac{2+3y}{5}$

$15 = -4+3x$ $20 = 2+3y$

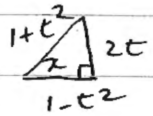
$3x = 19$ $3y = 18$

$x = \frac{19}{3}$ $y = 6$

$A\left(\frac{19}{3}, 6\right)$

c) $6 + 4\cos x + 5\sin x = 0$

$t = \tan\frac{x}{2}$



$6 + 4\left(\frac{1-t^2}{1+t^2}\right) + 5\left(\frac{2t}{1+t^2}\right) = 0$

$6(1+t^2) + 4(1-t^2) + 10t = 0$

$6 + 6t^2 + 4 - 4t^2 + 10t = 0$

$2t^2 + 10t + 10 = 0$

$t^2 + 5t + 5 = 0$

$t = \frac{-5 \pm \sqrt{25-20}}{2}$

$= \frac{-5 \pm \sqrt{5}}{2}$

∴ $\tan\frac{x}{2} = \frac{-5 \pm \sqrt{5}}{2} \doteqdot -1.38, -3.62$

∴ $x = 251°51', 210°53'$

d) $x + 2\sqrt{x+1} = 7$

$2\sqrt{x+1} = 7 - x$

$4(x+1) = (7-x)^2$

$4x + 4 = 49 - 14x + x^2$

$x^2 - 18x + 45 = 0$ $x=3$ only solutio

$(x-15)(x-3) = 0$

$x = 3, 15$

check $x = 3$ $3 + 2\sqrt{4} = 7$ ✓

$x = 15$ $15 + 2\sqrt{16} = 7$ ×

e) i) $BX.AX = PX^2$

$\therefore \quad x(6+x)=16$

$6x+x^2=16$

$x^2+6x-16=0$

$(x+8)(x-2)=0$

$x=-8,2 \qquad x>0$

$\therefore \; x=2$ ie $BX=2$ cm

ii) $AP^2+PX^2=AX^2$ (Pythagoras)

/ (∠ between radius & tangent at pt contact $=90°$)

$\therefore \; AP^2+16=8^2$

$AP^2=48$

$\therefore \; AP=4\sqrt{3}$ cm

10

a) $f(x)=\sqrt{3-\sqrt{x}} \qquad x\geq 0$

$3-\sqrt{x}\geq 0$

$\sqrt{x}\leq 3$

$x\leq 9$

domain: $0\leq x\leq 9$

range: $0\leq y\leq \sqrt{3}$ max $\sqrt{3}$

min 0

b) $x=t+3 \Rightarrow t=x-3$

$y=4-2t^2$

$y=4-2(x-3)^2$

$y=4-2(x^2-6x+9)$

$y=4-2x^2+12x-18$

$y=-2x^2+12x-14$

c) $\tan 2x+\tan x = \frac{\sin 3x}{\cos 2x\cos x}$

$LHS = \frac{\sin 2x}{\cos 2x} + \frac{\sin x}{\cos x}$

$= \frac{\sin 2x\cos x+\sin x\cos 2x}{\cos x\cos 2x}$

$= \frac{\sin(2x+x)}{\cos x\cos 2x}$

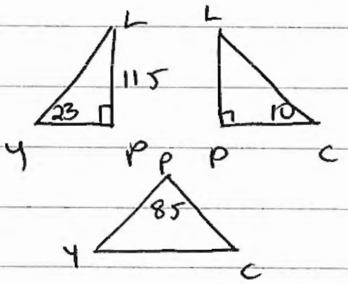
$= \frac{\sin 3x}{\cos x\cos 2x}$

$= RHS$ //

d) i) $10! = 3628800$

ii) $8!\times 2 = 80640$

e)



$\tan 23° = \frac{115}{PY} \qquad \tan 10° = \frac{115}{PC}$

$PY = \frac{115}{\tan 23°} \qquad PC = \frac{115}{\tan 10°}$

$\angle YPC = 220-135 = 85°$

$\therefore \; CY^2 = PY^2+PC^2-2.PY.PC.\cos 85°$

(cosine Rule)

$\therefore \; CY^2 = \frac{115^2}{\tan^2 23} + \frac{115^2}{\tan^2 10°} - \frac{2.115\;.115}{\tan 23\tan 10}$

$CY^2 = 115^2\left(\frac{1}{\tan^2 23}+\frac{1}{\tan^2 10} - \frac{2\cos 85}{\tan 23\tan 10}\right)$

$= 115^2(\cot^2 23° + \cot^2 10° - 2\cot 23°\cot 10$

$\cos 85$

$= 115^2(\tan^2 67° + \tan^2 80° - 2.\tan 67\tan 8$

$\cos 8$

$\cot\theta° = \tan(90-\theta)°$

Q11

a) $\frac{6^n+3^n}{2^{n+1}+2} = A \times 3^n$

LHS $= \frac{(2\times 3)^n + 3^n}{2.2^n+2}$

$= \frac{2^n.3^n+3^n}{2.2^n+2}$

$= \frac{3^n(2^n+1)}{2(2^n+1)}$

$= \frac{3^n}{2} = \frac{1}{2}\times 3^n$

$\therefore A = \frac{1}{2}$

b) i) ∠BCA = ∠CDA

(∠ between tangent & chord at pt of contact = ∠ in alternate segment)

∠CDA = ∠BFA

(Corresponding ∠'s =, EB||CD)

ii) ABCF is cyclic quad

∠BCA = ∠BFA (from above)

∴ ABCF is cyclic quad

(= ∠'s at circumference from same arc)

c) $2x^3-5x^2-2x+6=0$

i) $\alpha+\beta+\gamma = \frac{5}{2}$

ii) $\alpha\beta+\beta\gamma+\alpha\gamma = \frac{-2}{2} = -1$

iii) $\alpha^2+\beta^2+\gamma^2$:

$(\alpha+\beta+\gamma)^2 = \alpha^2+\alpha\beta+\alpha\gamma+\alpha\beta+\beta^2+\beta\gamma$
$+\alpha\gamma+\beta\gamma+\gamma^2$

$= \alpha^2+\beta^2+\gamma^2+2(\Sigma\alpha\beta)$ ①

$\therefore \alpha^2+\beta^2+\gamma^2 = (\Sigma\alpha)^2 - 2(\Sigma\alpha\beta)$

$= \left(\frac{5}{2}\right)^2 - 2(-1)$ ①

$= \frac{25}{4}+2$

$= \frac{33}{4}$

iii) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\alpha^2\gamma^2}$

$= \frac{\gamma^2+\alpha^2+\beta^2}{\alpha^2\beta^2\gamma^2}$ ①

$\alpha\beta\gamma = \frac{-6}{2} = -3$ ①

$= \frac{\frac{33}{4}}{(-3)^2}$

$= \frac{11}{12}$ ①