HURLSTONE AGRICULTURAL HIGH SCHOOL

YEARLY EXAMINATION 2008

PRELIMINARY COURSE

ASSESSMENT TASK 3

Mathematics Extension 1

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General Instructions

- Reading time – 5 minutes.
- Working time – 1½ hours.
- This test has 5 questions. Attempt all questions.
- Each question is worth 12 marks. Total: 60 marks.
- All necessary working should be shown in each question. Marks may not be awarded for careless or badly arranged work.
- **Start each question in a new booklet.** Write your student number on every sheet.
- This test must **NOT** be removed from the examination room.
- Board approved calculators and mathematical templates may be used.

QUESTION 1: Start a new booklet. **Marks**

(a) Find $\lim\limits_{x\to 3} \dfrac{x^2+2x-15}{x-3}$ **2**

(b) Use the definition of the derivative, $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, to find $f'(x)$ when $f(x)=x^2-2x+5$ **2**

(c) Find the derivative of $y=5x^4-3x^2+2-\dfrac{1}{x^2}$ **2**

(d) Differentiate with respect to x:

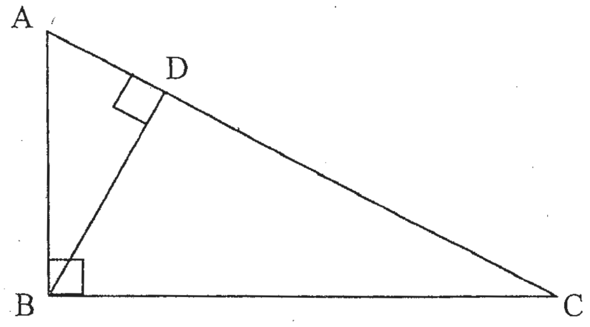
(i) $y=(x+2)(x-5)^2$ **2**

(ii) $y=\dfrac{3x+2}{5x-1}$ **2**

(e) Find the equation of the tangent to $y=x\sqrt{x}$ at the point $P(4,\, 8)$ **2**

QUESTION 2: Start a new booklet. **Marks**

(a) In the diagram, ΔABC is right angled at B, $BD \perp AC$ and $\Delta ABC ||| \Delta BDC$.



(i) Prove $\Delta ABC ||| \Delta ADB$. **2**

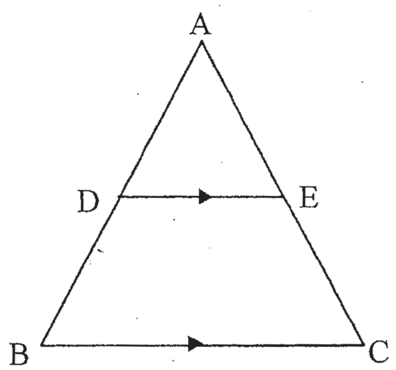
(ii) Given also that $\Delta BDC ||| \Delta ABC$, Show, giving reasons using similar triangles, that

$$AD + DC = \frac{AB^2 + BC^2}{AC}.$$

(Note: You must **not** use Pythagoras to answer part (ii)) **2**

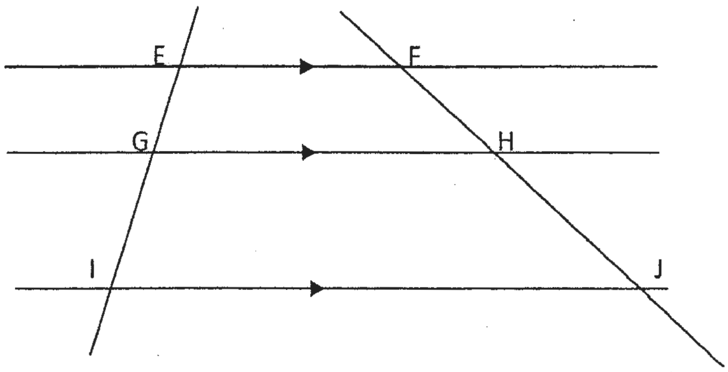
(iii) Use the result obtained in (ii) to prove Pythagoras' Theorem for ΔABC. **1**

(b)



The diagram shows ΔABC where D and E are midpoints of AB and AC respectively and $DE || BC$. Find, giving a reason, DE : BC. **1**

(c) **Marks**



In the diagram above, EF||GH||IJ. If EG = 3, GI = 5 and FJ = 20, find FH. Show all reasoning in your answer. **2**

(d) Solve the inequality:

$$\frac{|x-1|}{x} < 2$$ **4**

QUESTION 3: Start a new booklet. **Marks**

(a) Which of the points $D(-3, 9)$ or $E(8, -5)$ is nearer the origin? Justify your answer by using mathematical calculations. **2**

(b) Find to the nearest degree, the acute angle formed by the lines $5x - y + 1 = 0$ and $x - 3y - 2 = 0$ **3**

(c) For the points $A(-5, -1)$ and $B(1, 0)$:

(i) Write down the coordinates of the P, the point that divides AB in the ratio $k : 1$. (i.e. k : one) **2**

(ii) If P lies on $xy = 1$, show that $k^2 + 3k - 4 = 0$ **2**

(iii) Hence, or otherwise, find the coordinates of the points where the line AB meets $xy = 1$ **3**

QUESTION 4: Start a new booklet. **Marks**

(a)

(i) Write down the expansion of $\cos(\alpha+\beta)$. **1**

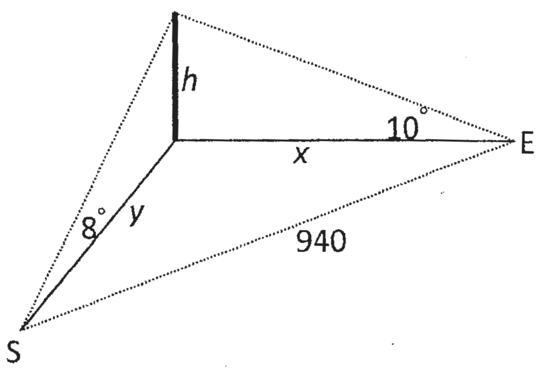
(ii) Hence find the exact value of $\cos 75°$. **2**

(b) Solve the equation $\sin 2x = \tan x$ for $0 \le x \le \pi$ **3**

(c) If $\cos A = \frac{7}{9}$ and $\sin B = \frac{1}{3}$ where A and B are acute angles, show that A = 2B by proving that $\cos 2B = \cos A$. **2**

(d) A surveyor who is y metres south of a tower sees the top of it with an angle of elevation 8°. A second surveyor is x metres east of the tower. From his position the angle of elevation is 10° to the top of the tower.

The two surveyors are 940m apart.



NOT TO SCALE

(i) Show that $y = h \tan 82°$ **1**

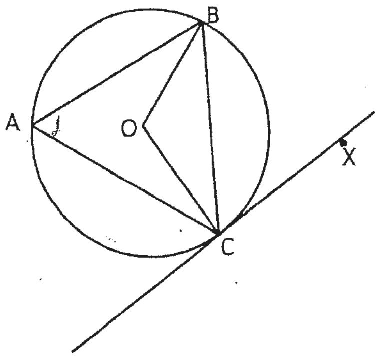
(ii) Show that the height of the tower, h, is given by

$$h = \frac{940}{\sqrt{\tan^2 80° + \tan^2 82°}}$$ **2**

(iii) Hence find the height of the tower to the nearest metre. **1**

QUESTION 5: Start a new booklet **Marks**

a) CX is tangent to the circle centre O. Let $\angle CAB = \alpha$.



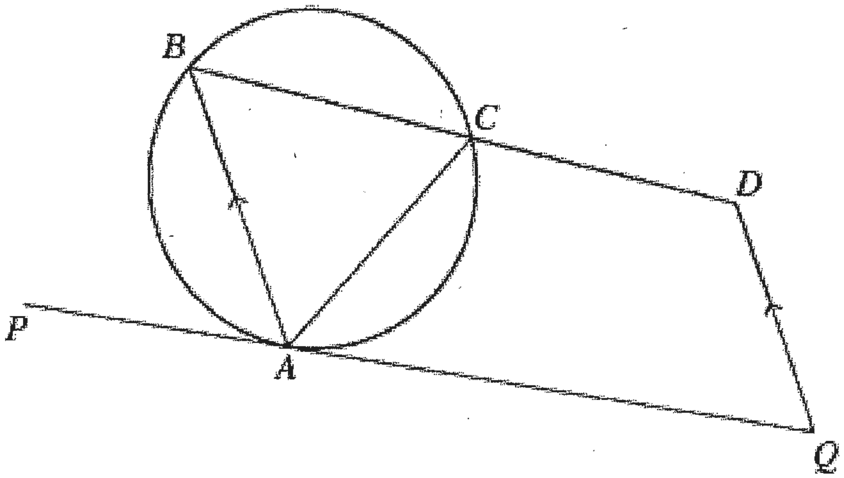
i) Find $\angle COB$ with reasons **1**

ii) Find $\angle OCB$ with reasons **1**

iii) Show that $\angle BCX = \angle BAC$ **1**

NOT TO SCALE

b)



NOT TO SCALE

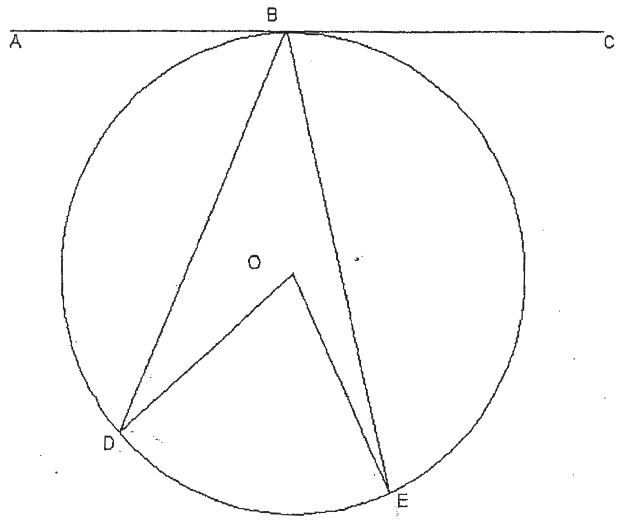
In the diagram, points *A*, *B* and *C* lie on the circle.

Line *PQ* is a tangent to the circle at *A*.

Line *QD* is parallel to *AB*, meeting *BC* produced at *D*.

Prove that *ACDQ* is a cyclic quadrilateral. **3**

c) **Marks**



NOT TO SCALE

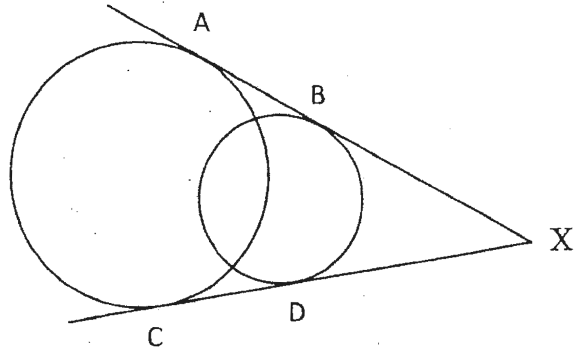
In the diagram, O is the centre of the circle.

AC is a tangent at B.

D and E are points on the circumference.

If $\angle ABD = 80^\circ$ and $\angle DBE = 40^\circ$, find the size of $\angle BEO$, giving reasons. **3**

d)



In the diagram, AB and CD are common tangents to the two circles.

The two tangents meet externally at X.

Explain why $AC \parallel BD$. **3**