STUDENT NAME: ____________________

STUDENT NUMBER: ____________________

TEACHER: ____________________

HURLSTONE AGRICULTURAL HIGH SCHOOL

PRO PATRIA

YEAR 11

EXTENSION 1 MATHEMATICS

2009

YEARLY EXAMINATION

EXAMINERS ~ S. HACKETT, S.GEE, S.FAULDS, G.RAWSON, P.BICZO

GENERAL INSTRUCTIONS

- *Reading time – 5 minutes.*
- *Working time – 1.5 hours.*
- *Attempt all 5 questions.*
- *Each question is worth 15 marks.*
- *Total marks – 75 marks*
- *All necessary working should be shown in every question.*
- *Marks may not be awarded for careless or badly arranged work.*
- *Board approved calculators and mathematical templates may be used.*
- *Each question is to be started in a new examination booklet.*
- *This assessment task must **NOT** be removed from the examination room.*

QUESTION ONE **(15 MARKS)** Use a SEPARATE Booklet.

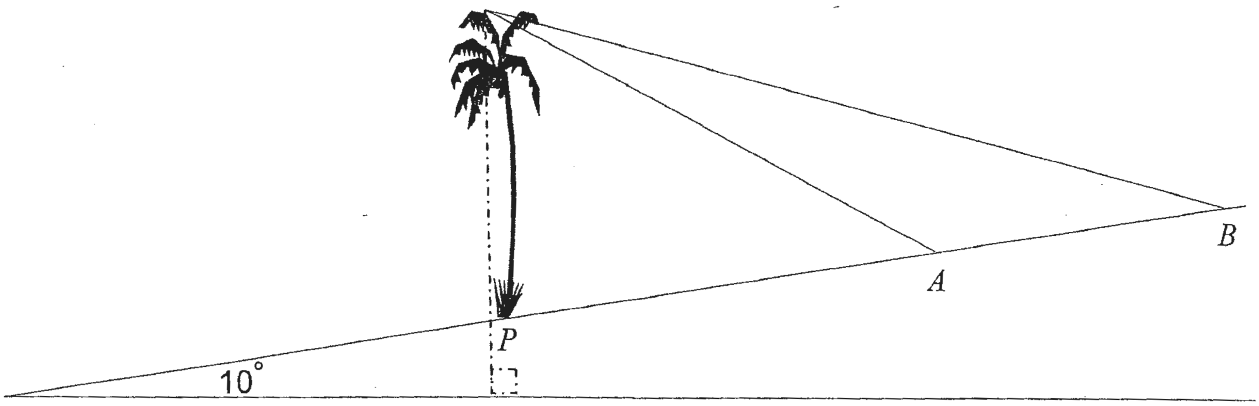
a) Solve $|2x-3|=4$. **2**

b) Factorise $2m^3-54$. **2**

c) Solve $3x-7 \geq 8x+18$ **2**

d) In an attempt to lower their increasing road toll, the Fijian government plans to widen some roads. In order to widen one particular road, they must first chop down a coconut tree. Before this, they must calculate the height of the tree.
Points P, A and B are in order on the straight road that is inclined at $10°$ to the horizontal.
The base of the tree is at point P and the angle of elevation of the top of the tree is $30°$ from the horizontal at point A and $5°$ from the horizontal at point B. Point A is 100 metres from point B.

(i) Copy or trace this diagram into your answer booklet and show all information given. **1**



(ii) Calculate the height of the coconut tree to the nearest centimetre. **3**

e) Consider the function $f(x)=1+\dfrac{3}{(x-2)}$.

(i) Give the equations of the horizontal and vertical asymptotes for $y=f(x)$. **2**

(ii) **Without** using calculus, sketch the graph of $y=f(x)$. **2**

(iii) Hence solve $\dfrac{3}{(x-2)}>-1$ **1**

QUESTION TWO **(15 MARKS)** Use a SEPARATE Booklet

a) (i) Using long division, prove that $2x-3$ is a factor of $P(x)=2x^3-5x^2-21x+36$ **2**

(ii) Hence, factorise $2x^3-5x^2-21x+36$ **1**

b) (i) Sketch the curve $y=x(1-x)(x+2)^2$ **2**

(ii) Hence, solve the inequality $x(1-x)(x+2)^2\le 0$ **1**

c) Show that the equation $2^{2x}-2^{x+2}=32$ can be expressed in the form $u^2-4u-32=0$, and hence solve for x. **3**

d) If α and β are the roots of the equation $2x^2+4x+7=0$, find the value of:

(i) $\alpha+\beta$ **1**

(ii) $\alpha^2+\beta^2$ **1**

e) Form the quadratic equation whose roots are $p-\sqrt{q}$ and $p+\sqrt{q}$. Write your answer in the form $Ax^2+Bx+C=0$ **2**

f) When the polynomial $P(x)$ is divided by x^2+x-2, the remainder is $3x-1$. What is the remainder when $P(x)$ is divided by $x-1$? **2**

QUESTION THREE **(15 MARKS)** Use a SEPARATE Booklet

(a) Evaluate (i) $\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}$ **1**

(ii) $\lim_{x \to \infty} \frac{x^2 + 5x - 3}{2x^2 - 3x + 11}$ **1**

(b) Given the formula for differentiation from first principles is $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$,

show from first principles that $\frac{d}{dx}\left(3x^2 + 2x\right) = 6x + 2$. **3**

(c) (i) Differentiate $y = x^3 - 2x^2$. **1**

(ii) Show that the point $A(-1,-3)$ lies on the curve $y = x^3 - 2x^2$. **1**

(iii) Find the equation of the tangent to the curve $y = x^3 - 2x^2$ at the point A. **2**

(d) Differentiate

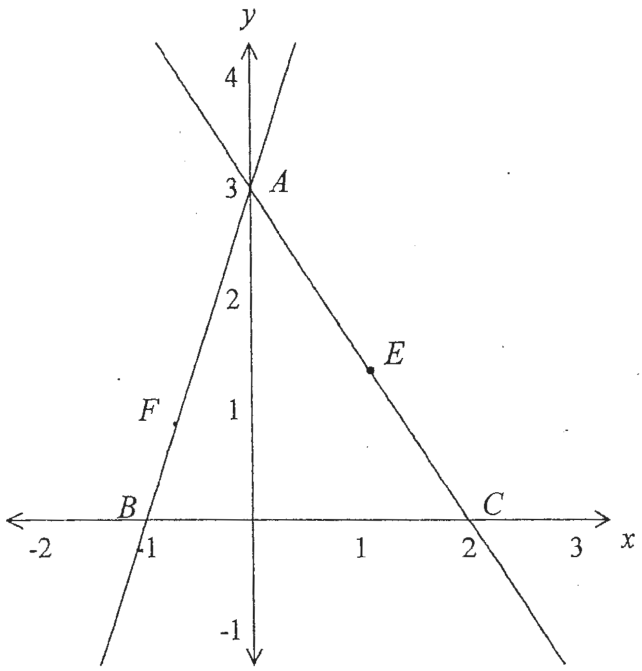
(i) $\frac{1}{x} + \sqrt{x}$ **2**

(ii) $\left(2x^3 + 1\right)^4$ **2**

(iii) $\frac{3x}{2x + 1}$ **2**

QUESTION FOUR **(15 MARKS)** Use a SEPARATE Booklet.

The points $A\ (0,3)$, $B\ (-1,0)$ and $C\ (2,0)$ are the vertices of a triangle.



(a) Find the gradient of the line AC. **1**

(b) Show that the equation of AC is $3x+2y-6=0$ **2**

(c) BE is the altitude from B to AC. Show that BE has equation $2x-3y+2=0$. **2**

(d) Calculate the length of the line segment BE. **2**

(e) Given that the altitude CF has equation $x+3y-2=0$, show that CF and BE intersect on the y axis. **2**

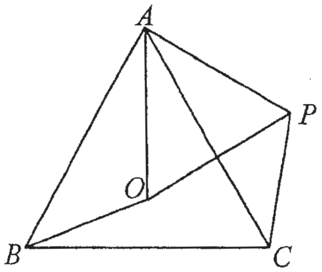
(f) Find the midpoint M of AC. **1**

(g) Find the coordinates of the point R which divides BM internally in the ratio 2:1. **2**

(h) Find the acute angle between the lines AB and AC. (Answer to the nearest degree) **3**

QUESTION FIVE **(15 MARKS)** Use a SEPARATE Booklet.

(a)



In the above figure ΔABC and ΔAPO are equilateral triangles. Copy the diagram into your answer booklet, including all given information.

(i) Explain why $\angle BAO = \angle PAC$ **2**

(ii) Prove $\Delta AOB \equiv \Delta APC$. **3**

(iii) Hence, prove $OB = CP$. **1**

(b) Prove the identity: $\dfrac{\cos A + \sin A}{\cos A - \sin A} \equiv \sec 2A + \tan 2A$ **3**

(c) Express as a single ratio: $\dfrac{\tan\frac{\alpha}{2} + \tan\frac{3\alpha}{2}}{1 - \tan\frac{\alpha}{2}\tan\frac{3\alpha}{2}}$ **1**

(d) (i) Write down the expansion for: $\cos(\alpha + \beta)$ **1**

(ii) Use the expansion in (i) above and your knowledge of identities and exact trigonometric ratios to write the expression:

$$\frac{7\sqrt{3}}{2}\cos\alpha - \frac{7}{2}\sin\alpha$$

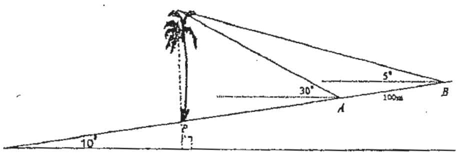
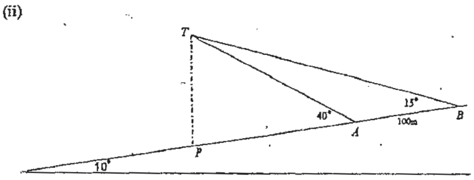
in the form: $A\cos(\alpha + \beta)$ clearly stating the values of A and β. **2**

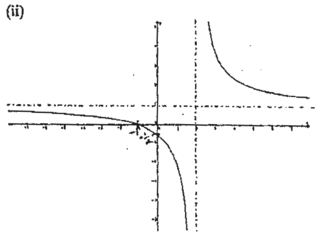
(iii) **Hence**, find the minimum value of $\frac{7\sqrt{3}}{2}\cos\alpha - \frac{7}{2}\sin\alpha$ and the smallest positive value of α for which this occurs. **2**

Year 11 Yearly 2009 – Extension 1 Mathematics
Solutions and Marking Guidelines

tion No: 1

tmes Addressed in this Question: P4

	Sample Solution	Marking Guidelines
(a)	$2x-3=4$ $2x-3=-4$ $2x=7$ $2x=-1$ $\therefore x=\frac{7}{2}, -\frac{1}{2}$	2 marks – both correct solutions from a correct method 1 mark – one correct solution from a correct method.
(b)	$2m^3-54 = 2(m^3-27)$ $= 2(m-3)(m^2+3m+9)$	2 marks – expression completely factorised 1 mark – some progress towards correct factorisation
(c)	$3x-7\geq 8x+18$ $-25\geq 5x$ $\therefore x\leq -5$	2 marks – 1 mark –
(d)(i)		1 mark– diagram correctly showing ALL given information
(ii)		
	In ΔABT, $\frac{AT}{\sin 15^\circ}=\frac{100}{\sin 25^\circ}$ $AT=\frac{100\sin 15^\circ}{\sin 25^\circ}$ In ΔAPT $\frac{PT}{\sin 40^\circ}=\frac{AT}{\sin 80^\circ}$ $PT=\frac{AT\sin 40^\circ}{\sin 80^\circ}$ $PT=39.97$ $\therefore$ Tree is 39.97 metres high.	3 marks – correct solution 2 marks – substantial progress towards correct solution 1 mark – some progress towards correct solution

			Solution	Marking Guidelines
P4	(e)	(i)	Horizontal asymptote: $y=1$ Vertical asymptote: $x=2$	2 marks – both asymptotes correct 1 mark – one asymptote correct
		(ii)		2 marks – correct graph clearly showing asymptotes and intercepts 1 mark – hyperbola in correct quadrants with asymptotes
		(iii)	$x<-1, x>2$	1 mark – correct solution from graph drawn in part (ii)

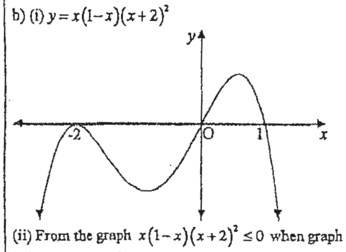
Year 11 Yearly — Mathematics Extension 1 — Examination 2009

Question No.2 — Solutions and Marking Guidelines

Outcomes Addressed in this Question

P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities

PE3 solves problems involving inequalities and polynomials

Outcome	Solutions	Marking Guidelines
PE3	a)(i) $2x-3\,)\overline{2x^3-5x^2-21x+36}$ quotient x^2-x-12 $2x^3-3x^2$ $-2x^2-21x$ $-2x^2+3x$ $-24x+36$ $-24x+36$ 0 Since remainder = 0, $2x-3$ is a factor	2 marks: correct solution 1 mark : significant progress towards correct solution
P3	(ii) $P(x)=2x^3-5x^2-21x+36$ $\therefore P(x)=(2x-3)(x^2-x-12)$	1 mark : correct answer or equivalent
PE3	b) (i) $y=x(1-x)(x+2)^2$ 	2 marks: correct graph 1 mark : significant progress towards correct graph
PE3	(ii) From the graph $x(1-x)(x+2)^2\leq 0$ when graph is below or on x axis. $\therefore x\leq 0$ and $x\geq 1$	1 mark : correct answer
P3	c) $2^{2x}-2^{x+2}=32$ $(2^x)^2-2^x.2^2=32$ $(2^x)^2-4.2^x-32=0$ Let $u=2^x$. $\therefore u^2-4u-32=0$ $(u-8)(u+4)=0$ $u=8$ and $u=-4$. $\therefore 2^x=8$ and $2^x=-4$. $x=3$ ($2^x=-4$ has no solution)	3 marks: correct justification, correct solution to quadratic and original equation 2 marks : substantial progress to above 1 mark : significant progress to above

	d) For $2x^2+4x+7=0$,	
P3	(i) $\alpha+\beta=\frac{-b}{a}=\frac{-4}{2}=-2$	1 mark : correct answer
P3	(ii) $\alpha\beta=\frac{c}{a}=\frac{7}{2}$ $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$ $(-2)^2=\alpha^2+\beta^2+2\times\frac{7}{2}$ $\alpha^2+\beta^2=-3$	1 mark : correct answer
P3	e) Sum of roots $=p-\sqrt{q}+p+\sqrt{q}=2p$ Product of roots $=(p-\sqrt{q})(p+\sqrt{q})=p^2-q$ Equation is $x^2-(\text{sum of roots})x+(\text{product of roots})=0$ $\therefore$ equation is $x^2-2px+p^2-q=0$	2 marks: correct answer 1 mark: significant progress towards correct answer
PE3	f) Using the division transformation $P(x)=(x^2+x-2).Q(x)+(3x-1)$ Remainder, when $P(x)$ is divided by $x-1$ is $P(1)$. $P(1)=(1^2+1-2).Q(1)+(3\times 1-1)$ $=0+2$ $\therefore$ remainder = 2	2 marks: correct solution 1 mark: significant progress towards correct answer

Year 11	Extension 1 Mathematics	Task 3 2009
Question No.3	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

P6 relates the derivative of a function to the slope of its graph
P7 determines the derivative of a function through routine application of the rules of differentiation
P8 understands and uses the language and notation of calculus

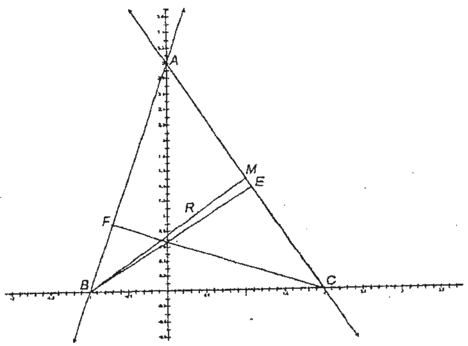
Outcome	Solutions	Marking Guidelines
P8	a (i) $\lim_{x\to 2}\frac{x^2-x-2}{x-2}$ $\lim_{x\to 2}\frac{(x-2)(x+1)}{x-2}$ $\lim_{x\to 2}(x+1)$ 3	1 mark correct solution
P8	(ii) $\lim_{x\to\infty}\frac{x^2+5x-3}{2x^2-3x+11}$ $\lim_{x\to\infty}\frac{\frac{x^2}{x^2}+5\frac{x}{x^2}-\frac{3}{x^2}}{2\frac{x^2}{x^2}-3\frac{x}{x^2}+\frac{11}{x^2}}$ $\lim_{x\to\infty}\frac{1+\frac{5}{x}-\frac{3}{x^2}}{2-\frac{3}{x}+\frac{11}{x^2}}$ $\frac{1}{2}$	1 mark correct solution
P6 P8	b $\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $\frac{d}{dx}(3x^2+2x)=6x+2$ $f(x+h)=3(x+h)^2+2(x+h)$ $=3(x^2+2xh+h^2)+2x+2h$ $=3x^2+6xh+3h^2+2x+2h$ $f(x)=3x^2+2x$ $\frac{dy}{dx}=\lim_{h\to 0}\frac{3x^2+6xh+3h^2+2x+2h-3x^2-2x}{h}$ $=\lim_{h\to 0}\frac{6xh+3h^2+2h}{h}$ $=\lim_{h\to 0}6x+3h+2$ $=6x+2$	3marks correct method leading to correct conclusion 2 mark substantially correct solution 1 mark elementary progress towards correct solution
P7	c i) $y=x^3-2x^2$ $y'=3x^2-4x$	1 mark correct solution
	ii) $y=x^3-2x^2$ $-3=(-1)^3-2\times(-1)^2$ $-3=-1-2$	1 mark correct solution
P6	iii) $y=x^3-2x^2$ $y'=3x^2-4x$ When $x=-1$ gradient of the tangent $m=3(-1)^2-4(-1)$ $m=7$ $y-y_1=m(x-x_1)$ $y+3=7(x+1)$ $y+3=7x+7$ $y=7x+4$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
P7	d i) $\frac{d}{dx}\left(\frac{1}{x}+\sqrt{x}\right)=\frac{d}{dx}\left(x^{-1}+x^{\frac{1}{2}}\right)$ $=-x^{-2}+\frac{1}{2}x^{\frac{-1}{2}}$ $=\frac{-1}{x^2}+\frac{1}{2\sqrt{x}}$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
P7	ii) $\frac{d}{dx}(2x^3+1)^4=4(2x^3+1)^3\times 6x^2$ $=24x^2(2x^3+1)^3$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution
P7	iii) $\frac{d}{dx}\left(\frac{3x}{2x+1}\right)=\frac{3(2x+1)-3x\times 2}{(2x+1)^2}$ $=\frac{6x+3-6x}{(2x+1)^2}$ $=\frac{3}{(2x+1)^2}$	2 marks correct method leading to correct conclusion 1 mark substantially correct solution

Year 11	Extension 1 Mathematics	Yearly Examination 2009
Question No. 4	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

Outcome	Solutions	Marking Guidelines



(a) $m_{AC}=\dfrac{3-0}{0-2}=-\dfrac{3}{2}$

1 mark – correct answer

(b) AC has $m=-\frac{3}{2}$, and passes through (0, 3) so it's equation is

$$y-y_1=m(x-x_1)$$
$$y-3=-\tfrac{3}{2}(x-0)$$
$$2y-6=-3x$$
$$3x+2y-6=0$$

2 marks – correct solution with steps clearly shown

1 mark – substantially correct method

(c) BE is altitude $\therefore BE\perp AC$

$$m_{BE}=-\frac{1}{m_{AC}}=\frac{2}{3}$$
$$y-y_1=m(x-x_1)$$
$$y-0=\tfrac{2}{3}(x-(-1))$$
$$3y=2x+2$$
$$2x-3y+2=0$$

2 marks – correct solution with steps clearly shown

1 mark – substantially correct method

(d) $3x+2y-6=0,\ (-1,0)$

$$d_{BE}=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$
$$=\frac{|3(-1)+2(0)-6|}{\sqrt{3^2+2^2}}$$
$$=\frac{9}{\sqrt{13}}$$

2 marks – correct solution

1 mark – substantially correct solution

Note: – **0 marks…** assuming E is the midpoint of AC is incorrect and simplifies the question.

(e)

$$x+3y-2=0\ \ ...(1)$$
$$2x-3y+2=0\ \ ...(2)$$

(1) + (2) gives $3x=0$

$x=0$

which is the y-axis

2 marks – correct solution

1 mark – substantially correct solution

(f) Midpoint of AC

$$M:\left(\frac{2+0}{2},\frac{0+3}{2}\right)$$

ie $\left(1,\frac{3}{2}\right)$

1 mark – correct answer

(g) $B:\left(\overset{x_1}{-1},\overset{y_1}{0}\right)$ $M:\left(\overset{x_2}{1},\overset{y_2}{\tfrac{3}{2}}\right)$ $\overset{k}{2}:\overset{l}{1}$

$$R:\left(\frac{lx_1+kx_2}{k+l},\frac{ly_1+ky_2}{k+l}\right)$$
$$=\left(\frac{1(-1)+2(1)}{2+1},\frac{1(0)+2(\frac{3}{2})}{2+1}\right)$$
$$=\left(\frac{1}{3},1\right)$$

2 marks – correct solution

1 mark – substantially correct solution

(h)

AB has $m_1=3$

AC has $m_2=-\frac{3}{2}$ (from (a))

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$
$$=\left|\frac{3-(-\frac{3}{2})}{1+3(-\frac{3}{2})}\right|$$
$$=\left|\frac{\frac{9}{2}}{-\frac{7}{2}}\right|=\frac{9}{7}$$
$$\theta=52°$$

3 marks – correct solution

2 marks – substantially correct solution

1 mark – partially correct solution

Question No. 5	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

PE2 uses multi-step deductive reasoning in a variety of contexts
P3 Performs routine arithmetic and algebraic manipulation involving trigonometric identities
P4 chooses and applies appropriate trigonometric and geometric techniques

Outcome	Solutions	Marking Guidelines
P4, PE2	(a) (i) Let $\angle OAC = x^\circ$ $\angle BAC = 60^\circ$ (angle in equilateral ΔBAC) $\therefore \angle BAO = 60^\circ - x^\circ$ Also, $\angle OAP = 60^\circ$ (angle in equilateral ΔOAP) $\therefore \angle PAC = 60^\circ - x^\circ$ $= \angle BAO$	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution
PE2, P4	(ii) In Δ's AOB and APC AO = AP (equal sides in equilateral ΔOAP) $\angle BAO = \angle PAC$ (shown above) AB = AC (equal sides in equilateral ΔBAC) $\therefore \Delta AOB \equiv \Delta APC$	**3 marks** Correct solution. **2 marks** Substantially correct solution where reasoning is incomplete or erroneous. **1 mark** Demonstrates some knowledge of how these two triangles may be proved to be congruent.
PE2, P4	(iii) Now, OB = CP (corresponding sides in congruent Δ's)	**1 mark** Correct solution
P3, P4	(b) $\text{L.H.S} = \dfrac{\cos A + \sin A}{\cos A - \sin A}$ $= \dfrac{\cos A + \sin A}{\cos A - \sin A} \times \dfrac{\cos A + \sin A}{\cos A + \sin A}$ $= \dfrac{\cos^2 A + 2\sin A\cos A + \sin^2 A}{\cos^2 A - \sin^2 A}$ $= \dfrac{1 + \sin 2A}{\cos 2A}$ $= \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$ $= \sec 2A + \tan 2A$ $= \text{R.H.S}$	**3 marks** Correct solution. **2 marks** Substantially correct solution mostly correct use of identities. **1 mark** Demonstrates some knowledge of trig. identities, making some progress towards correct solution.
P3, P4	(c) $\dfrac{\tan\frac{\alpha}{2} + \tan\frac{3\alpha}{2}}{1 - \tan\frac{\alpha}{2}\tan\frac{3\alpha}{2}} = \tan\left(\frac{\alpha}{2} + \frac{3\alpha}{2}\right)$ $= \tan\left(\frac{4\alpha}{2}\right)$ $= \tan 2\alpha$	**1 mark** Correct answer
P3, P4	(d) (i) $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	**1 mark** Correct answer
P3, P4	(ii) $\frac{7\sqrt{3}}{2}\cos\alpha - \frac{7}{2}\sin\alpha = 7\left(\frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha\right)$ $= 7(\cos\alpha\cos 30^\circ - \sin\alpha\sin 30^\circ)$ $= 7\cos(\alpha + 30^\circ)$ which is in the form $A\cos(\alpha+\beta)$ where $A = 7$ and $\beta = 30^\circ$.	**2 marks** Correct solution, demonstrating knowledge of both trig. identities and exact trig. ratios. **1 mark** Partially correct solution OR correct answer without reference to exact trig. ratios.
P3, P4	(iii) The minimum value of $\cos\theta$ is -1 when $\theta = 180^\circ$. $\therefore$ The minimum value of $A\cos(\alpha+\beta)$ is -A when $\alpha+\beta = 180^\circ$. Since $\frac{7\sqrt{3}}{2}\cos\alpha - \frac{7}{2}\sin\alpha = 7\cos(\alpha+30^\circ)$, the minimum value of the expression is -7 when $\alpha + 30^\circ = 180^\circ$ ie. when $\alpha + 30^\circ = 180^\circ$. $\therefore$ The expression $\frac{7\sqrt{3}}{2}\cos\alpha - \frac{7}{2}\sin\alpha$ has a minimum value of -7 when $\alpha = 150^\circ$.	**2 marks** Correct solution. **1 mark** Partially correct solution.