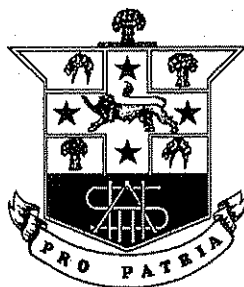


# HURLSTONE AGRICULTURAL HIGH SCHOOL



# MATHEMATICS

# EXTENSION 1

2011

YEAR 11

# YEARLY (TASK 3) EXAMINATION

EXAMINERS ~ S. FAULDS, J. DILLON, P. BICZO, D. CRANCHER, G. RAWSON,  
S. HACKETT

## GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
  - Working Time – 1 hour 30 minutes.
  - Attempt **all** questions.
  - Each question is worth 10 marks.
  - **All** necessary working should be shown in every question.
  - This paper contains eight (8) questions.
  - Board approved calculators and MathAids may be used.
- **Each question is to be started in a new answer booklet.** Write the question number and your student number at the top of each answer booklet.
  - You **must** hand in an answer booklet for **each question** even if a question has not been attempted.
  - This examination must **NOT** be removed from the examination room

STUDENT NAME: \_\_\_\_\_

TEACHER: \_\_\_\_\_

**QUESTION 1**      **10 marks**      Start a NEW answer booklet

**Marks**

- (a) Evaluate  $\log_2 5$  correct to four decimal places. 2
- (b) Find  $3 \times 2^{2.5}$  correct to three significant figures. 1
- (c) Express with a rational denominator  $\frac{1+\sqrt{3}}{2-\sqrt{3}}$ . 2
- (d) Solve the inequality  $\frac{3-2x}{x-3} < 4$ . 3
- (e) Prove that  $a + \frac{1}{a} \geq 2$  when  $a > 0$ . 2

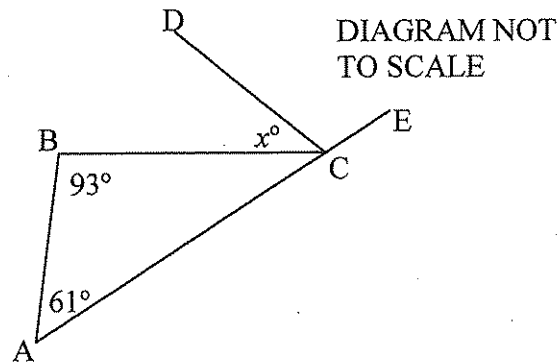
**QUESTION 2**

**10 marks**

Start a NEW answer booklet

**Marks**

(a)



**2**

Find  $\angle BCD$  in the diagram above, given DC bisects  $\angle BCE$ . Justify your answer by showing all reasoning.

(b) ABC is a triangle, right-angled at B. X is a point on BC such that  $AX=XC$  and  $\angle BAX = 2\angle XAC$ .

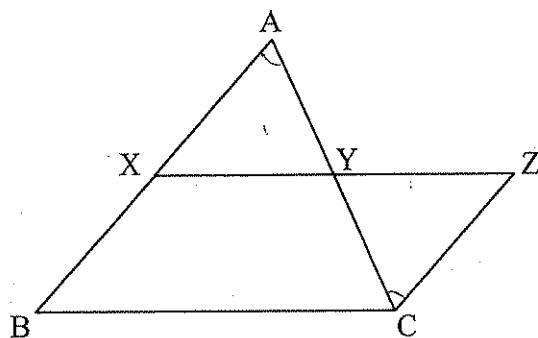
(i) Draw a neat sketch of  $\triangle ABC$ , showing all of the given information.

**1**

(ii) Find the size of  $\angle ACB$ , giving reasons.

**2**

(c) The diagram below shows parallelogram BXZC with BX produced to A. Y is the midpoint of AC.



(i) Prove  $\triangle AXY \equiv \triangle CZY$ .

**3**

(ii) Hence, show that X is the midpoint of AB.

**2**

**QUESTION 3**

**10 marks**

Start a NEW answer booklet

**Marks**

- (a) Without using a calculator, show that:

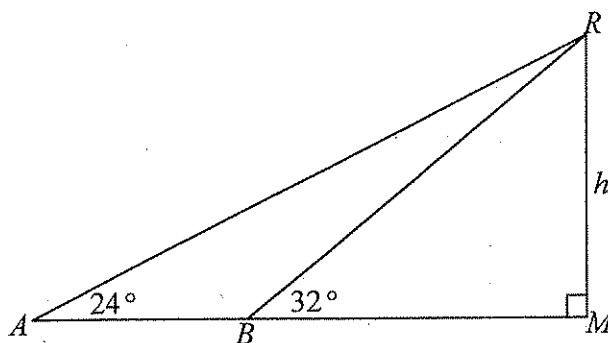
2

$$\frac{2 \tan \frac{\pi}{6}}{1 - \tan^2 \frac{\pi}{6}} = \tan \frac{\pi}{3}$$

- (b) Show that  $\sin 75^\circ = \frac{1 + \sqrt{3}}{2\sqrt{2}}$ .

2

- (c) A walker walks along the ground directly towards a distant high rocky cliff  $R$ . At a point  $A$  the angle of elevation of the cliff is  $24^\circ$ . At point  $B$ , a kilometre closer to the cliff, the angle of elevation is  $32^\circ$ .



- (i) Find the horizontal distance from  $B$  to the cliff, to the nearest metre.

2

- (ii) Find the height of the cliff above ground level, to the nearest metre.

1

- (d) Prove that  $\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \sec^2 \theta \operatorname{cosec}^2 \theta$ .

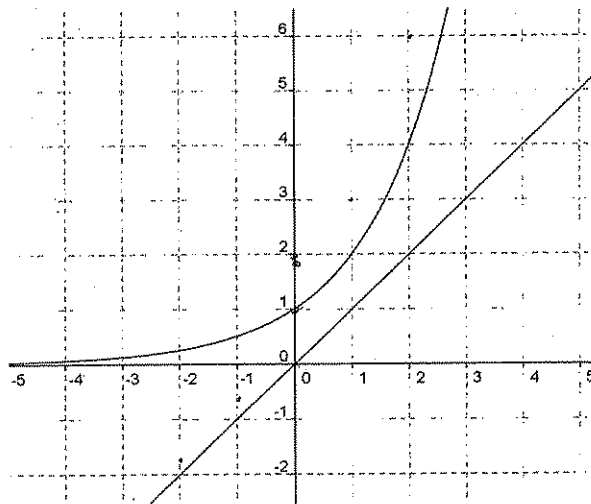
3

**QUESTION 4****10 marks**

Start a NEW answer booklet

**Marks**

- (a) (i) State the definition of an odd function. **1**
- (ii) Show that  $f(x) = 2x^3$  is an odd function. **1**
- (b) For the function  $y = x^2 + 4x$ :
- (i) Find the equation of the axis of symmetry of the parabola. **1**
- (ii) Find the co-ordinates of the vertex of the parabola and hence, state the range of the function. **2**
- (c) (i) Sketch the graph of  $y = |x + 2|$ , showing all important features. **2**
- (ii) Hence, solve  $|x + 2| > 4 - |x|$ . **2**
- (d) Copy or trace the following curves onto your answer sheet. **1**

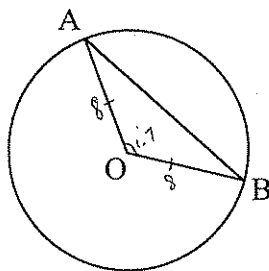
Using addition of ordinates, sketch the curve  $y = 2^x + x$ .

**QUESTION 5****10 marks**

Start a NEW answer booklet

**Marks**

- (a) (i) Convert  $36^\circ$  to radians, expressing your answer in terms of  $\pi$ . 1
- (ii) In the diagram below, the circle has a radius of 8cm and  $\angle AOB = 1.7$  radians. 2



Find the area of the minor segment enclosed by the chord AB.

- (b) Differentiate the following with respect to  $x$ :

(i)  $5x^4 - \frac{4}{x}$  2

(ii)  $\sqrt{2x+7}$  2

- (c) Prove, by the process of mathematical induction, that: 3

$$\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

for all positive integers,  $n$ .

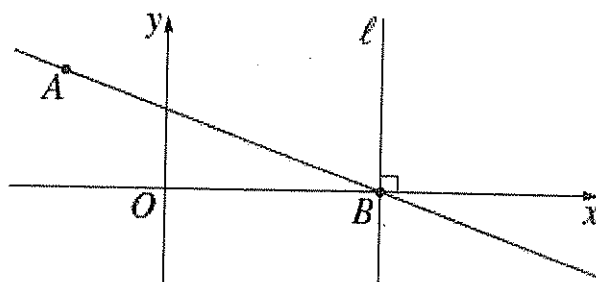
**QUESTION 6**

**10 marks**

Start a NEW answer booklet

**Marks**

- a) The diagram shows the points  $A(-1,3)$  and  $B(2,0)$ .  
The line  $l$  is drawn perpendicular to the  $x$ -axis through the point  $B$ .



**NOT TO SCALE**

- |       |  |   |
|-------|--|---|
| (i)   | Calculate the length of the interval $AB$ .  | 1 |
| (ii)  | Find the gradient of the line $AB$ .   | 1 |
| (iii) | What is the size of the acute angle between the line $AB$ and the line $l$ ?                             | 1 |
| (iv)  | Show that the equation of the line $AB$ is $x + y - 2 = 0$ .   | 1 |
| (v)   | Copy the diagram into your writing booklet and shade the region defined by $x + y - 2 < 0$ .             | 1 |
| (vi)  | Write down the equation of the line $l$ .  | 1 |
| (vii) | The point $C$ is on the line $l$ such that $AC$ is perpendicular to $AB$ . Find the coordinates of $C$ . | 2 |
| (b)   | Let $A$ be the point $(3, -1)$ and $B$ be the point $(9, 2)$ .   | 2 |

Find the coordinates of the point  $P$  which divides the interval  $AB$  externally in the ratio  $5:2$ .

**QUESTION 7****10 marks**

Start a NEW answer booklet

**Marks**

(a) Evaluate  $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1}$ . 2

(b) Given that  $\frac{d}{dx}(\sin x) = \cos x$ , find the derivative of  $y = (x^2 - 5)\sin x$ . 2

(c) Given that  $f(x) = x^3 - 3x$  find the equation of the tangent to the curve  $y = f(x)$  at the point on the curve where  $x = -1$ . 3

(d) Differentiate: 3

$$y = \frac{x^2}{(2x+1)^3}, x \neq -\frac{1}{2}$$



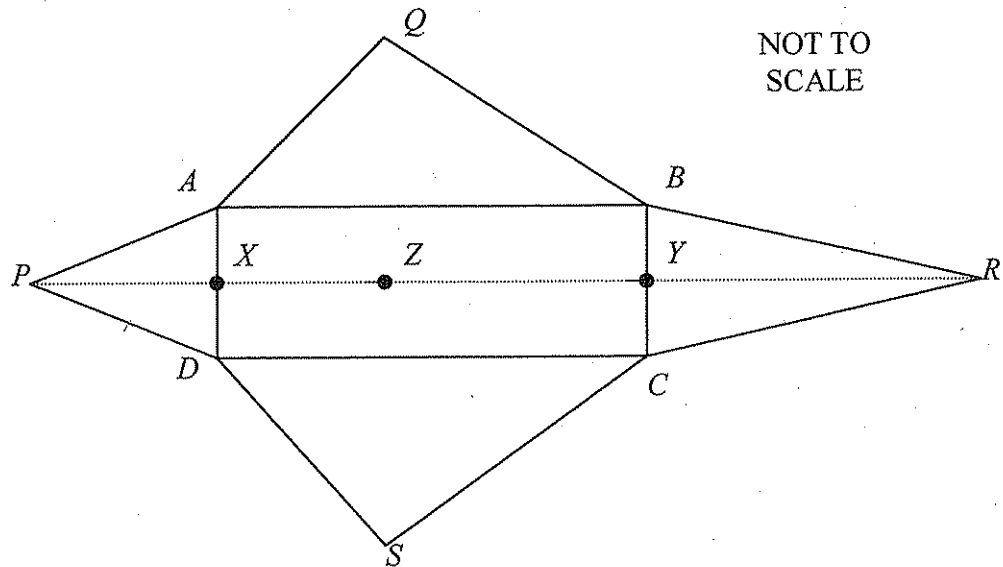
**QUESTION 8**

**10 marks**

Start a NEW answer booklet.

**Marks**

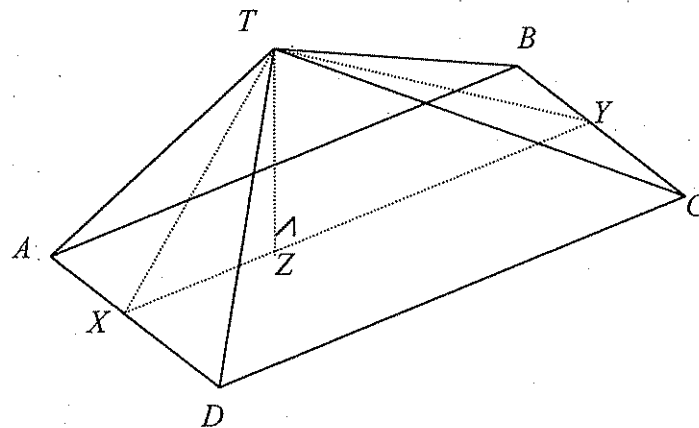
(a)



The figure above shows the net of an oblique pyramid with a rectangular base.

In this figure,  $PXZYR$  is a straight line,  $PX = 15$  cm,  $RY = 20$  cm,  $AB = 25$  cm and  $BC = 10$  cm. Further,  $AP = PD$  and  $BR = RC$ .

When the net is folded, points  $P$ ,  $Q$ ,  $R$ , and  $S$  all meet at the apex  $T$ , which lies vertically above the point  $Z$  in the horizontal base, as shown below.



- (i) Show that  $\triangle TXY$  is right-angled. 1
- (ii) Hence, show that  $T$  is 12 cm above the base. 2
- (iii) Hence, find the angle that the face  $DCT$  makes with the base. 2

Question 8 continues on next page.

**QUESTION 8 (continued)**

**Marks**

- (b) Prove the identity:

**2**

$$\operatorname{cosec}2A + \cot2A = \cot A.$$

- (c) Solve the equation below for  $0^\circ \leq \theta \leq 360^\circ$ :

**3**

$$5\cos^2\theta + 2\sin\theta - 2 = 0$$

Year 11 Mathematics Extension 1		Yearly Examination 2011
Question No. 1 Solutions and Marking Guidelines		
Outcomes Addressed in this Question		
<b>P3</b>	performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities	
<b>P4</b>	chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques	
<b>PE3</b>	solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations	
Outcome	Solutions	Marking Guidelines
(a) P4	$\log_2 5 = \frac{\log 5}{\log 2}$ $= 2.321928095$ $\approx 2.3219$	Award 2 for correct answer Award 1 attempts to use "change of base" and round to 4 d.p.
(b) P4	$3 \times 2^{2.5} = 16.97056275$ $\approx 17.0$	Award 1 for correct answer
(c) P3	$\frac{1+\sqrt{3}}{2-\sqrt{3}} = \frac{1+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ $= \frac{5+3\sqrt{3}}{4-3}$ $= 5+3\sqrt{3}$	Award 2 for correct solution Award 1 multiplies by the conjugate
(d) PE3	$\frac{3-2x}{x-3} < 4 \quad (x \neq 3)$ $(3-2x)(x-3) < 4(x-3)^2$ $4(x-3)^2 - (3-2x)(x-3) > 0$ $(x-3)(4(x-3) - (3-2x)) > 0$ $(x-3)(6x-15) > 0$ $\therefore x < 2\frac{1}{2} \text{ or } x > 3$	Award 3 for correct solution Award 2 multiplies both sides by $(x-3)^2$ and attempts to solve the resulting quadratic inequality. Award 1 multiplies both sides by $(x-3)$ and attempts to solve the resulting linear inequality.
(e) PE3	<p>Consider <math>a + \frac{1}{a} - 2</math></p> $a + \frac{1}{a} - 2 = \frac{a^2 + 1 - 2a}{a}$ $= \frac{(a-1)^2}{a}$ <p>If <math>a &gt; 0</math>, <math>(a-1)^2 \geq 0</math> and <math>\frac{(a-1)^2}{a} \geq 0</math></p> $\therefore a + \frac{1}{a} - 2 \geq 0$ $\therefore a + \frac{1}{a} \geq 2$	Award 2 for correct solution Award 1 for attempting to prove the result by a valid means

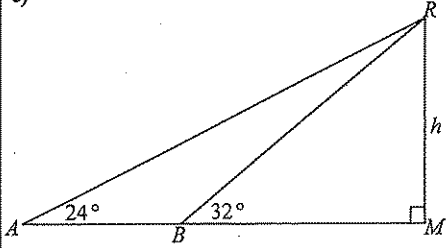
P4 Chooses and applies appropriate geometric techniques.

Outcome	Solutions	Marking Guidelines
P4	(a) $\angle BCE = 93^\circ + 61^\circ$ (Exterior angle of $\triangle ABC$ equals the sum of the two interior opposite angles.) $\therefore \angle BCE = 154^\circ$ $\angle BCD = \frac{154^\circ}{2}$ ( $CD$ bisects $\angle BCE$ ) $\therefore \angle BCD = 77^\circ$	2 marks Correct answer with full reasons. 1 mark Correct answer with incomplete reasons OR incorrect answer with relevant and correct reasoning.
P4	(b)(i) 	1 mark Correctly drawn diagram showing all given information
P4	(ii) $AX = XC$ (given) $\therefore \triangle AXC$ is isosceles (two equal sides) Now, $\angle ACX = \alpha^\circ$ (angles opposite equal sides in isosceles $\triangle AXC$ ) In $\triangle ABC$ , $2\alpha + \alpha + \alpha + 90^\circ = 180^\circ$ (angle sum of triangle) $4\alpha = 90^\circ$ $\therefore \alpha = 22.5^\circ$	2 marks Correct answer with full reasons. 1 mark Correct answer with incomplete reasons OR incorrect answer with relevant and correct reasoning.
P4	(c)(i) $BX \parallel CZ$ ( $BXZC$ is a parallelogram) $AX \parallel CZ$ ( $BX$ produced to $A$ ) In $\triangle AXY$ and $\triangle CZY$ $\angle AXY = \angle CYZ$ (vertically opposite angles are equal) $AY = CY$ ( $Y$ is the midpoint of $AC$ ) $\angle XAY = \angle ZCY$ (alternate angles equal $AX \parallel ZC$ ) $\therefore \triangle AXY \cong \triangle CZY$ (AAS)	3 marks Correct solution 2 marks Substantial progress towards correct solution 1 mark Limited progress towards correct solution
P4	ii) $AX = CZ$ (matching sides of congruent triangles are equal) $BX = CZ$ (opposite sides are equal, $BXZC$ is a parallelogram) $\therefore AX = BX$ $\therefore X$ is the midpoint of $AB$ .	2 marks Correct solution 1 mark Substantial progress towards correct solution

**Outcomes Addressed in this Question**

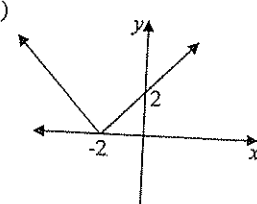
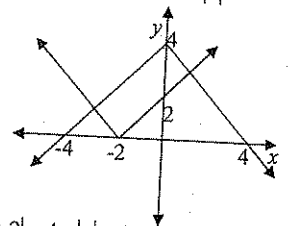
PE2 - uses multi-step deductive reasoning in a variety of contexts  
 P3 - performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities  
 P4 - chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques  
 P5 - understands the concept of a function and the relationship between a function and its graph  
 PE5 - determines derivatives which require the application of more than one rule of differentiation  
 P6 - relates the derivative of a function to the slope of its graph  
 PE6 - makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations  
 P8 - understands and uses the language and notation of calculus

	Sample Solution	Marking Guidelines
P4	<p>3. a)</p> $LHS = \frac{2 \tan \frac{\pi}{6}}{1 - \tan^2 \frac{\pi}{6}}$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px 0;"> <p>Since, double angle result for tan is: <math>\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}</math></p> </div> $= \tan 2\left(\frac{\pi}{6}\right)$ $= \tan \frac{\pi}{3}$ <p>Therefore,</p> $\frac{2 \tan \frac{\pi}{6}}{1 - \tan^2 \frac{\pi}{6}} = \tan \frac{\pi}{3}$	<p><b>2 marks for complete correct solution</b></p> <p><b>1 marks for partially correct solution</b></p>
P4	<p>b)</p> $\sin 75^\circ$ $= \sin(30^\circ + 45^\circ)$ $= \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$ $= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$ $= \frac{1 + \sqrt{3}}{2\sqrt{2}}$	<p><b>2 marks for complete correct solution</b></p> <p><b>1 marks for partially correct solution</b></p>

	<p>e)</p>  <p>i)</p> $\frac{BR}{\sin 24^\circ} = \frac{1}{\sin 8^\circ}$ $BR = \frac{\sin 24^\circ}{\sin 8^\circ}$ <p>ii)</p> $\cos 32^\circ = \frac{BM}{BR}$ $\therefore BM = BR \cos 32^\circ$ $= \frac{\sin 24^\circ}{\sin 8^\circ} \cos 32^\circ$ $= 2.478440398$ $= 2.478 \text{ km or } 2478 \text{ m}$ <p>iii)</p> $\frac{h}{BM} = \tan 32^\circ$ $h = BM \tan 32^\circ$ $h = \left( \frac{\sin 24^\circ}{\sin 8^\circ} \cos 32^\circ \right) \tan 32^\circ$ $= 1.548701446$ $= 1.549 \text{ km or } 1549 \text{ m}$ <p>d)</p> $LHS = \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta}$ $= \frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta \cos^2 \theta}$ $= \frac{1}{\sin^2 \theta \cos^2 \theta}$ $= \frac{1}{\sin^2 \theta} \times \frac{1}{\cos^2 \theta}$ $= \operatorname{cosec}^2 \theta \sec^2 \theta$ $= RHS$ $\therefore \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} = \sec^2 \theta \operatorname{cosec}^2 \theta$	<p><b>2 marks for complete correct solution</b></p> <p><b>1 marks for partially correct solution</b></p> <p><b>1 mark for correct solution</b></p> <p><b>3 marks for complete correct solution</b></p> <p><b>2 marks for partially correct solution no reasoning</b></p> <p><b>1 mark for any correct statement</b></p>
P4		
P4		
P3		

**Outcomes Addressed in this Question**

- P4** Chooses and applies appropriate algebraic, trigonometric and geometric techniques  
**P5** Understands the concept of a function and the relationship between a function and its graph  
**PE3** Solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations.  
**PE6** Makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations.

Outcome	Solutions	Marking Guidelines
P4	<p>(a)(i) A function is odd if <math>f(-x) = -f(x)</math></p> $f(x) = 2x^3$ $f(-x) = 2(-x)^3$ $= -2x^3$ $= -(2x^3)$ $= -f(x)$	<p>2 marks : correct definition and correct solution                      1 mark : one of above</p>
P5	<p>(b)(i) Axis of symmetry of <math>y = ax^2 + bx + c</math> is given by</p> $x = \frac{-b}{2a}$ $= \frac{-4}{2}$ <p><math>\therefore</math> axis of symmetry is <math>x = -2</math></p> <p>(ii) Vertex is on the axis of symmetry.                      Substituting <math>x = -2</math> into <math>y = x^2 + 4x</math>,  <math>y = 4 - 8 = -4</math>  <math>\therefore</math> vertex is <math>(-2, -4)</math>                      As this is a concave up parabola, range is <math>y \geq -4</math>.</p>	<p>1 mark : correct answer</p> <p>2 marks : correct vertex &amp; range                      1 mark : one of above</p>
P5	<p>(c)(i)</p> 	<p>2 mark : correct graph                      1 mark: incomplete graph</p>
PE3	<p>(ii) Sketching <math>y = 4 -  x </math> on the same diagram,</p>  <p><math> x + 2  &gt; 4 -  x </math> when the graph of <math>y =  x + 2 </math> is above the graph of <math>y = 4 -  x </math>.</p>	<p>2 marks : correctly graphs <math>y = 4 -  x </math> and correct solution to inequality                      1 mark : substantial progress towards correct solution, by using the given graph</p>

The graphs meet when  $y = x + 2$  and  $y = 4 - x$  intersect and when  $y = -(x + 2)$  and  $y = 4 - (-x)$  intersect.

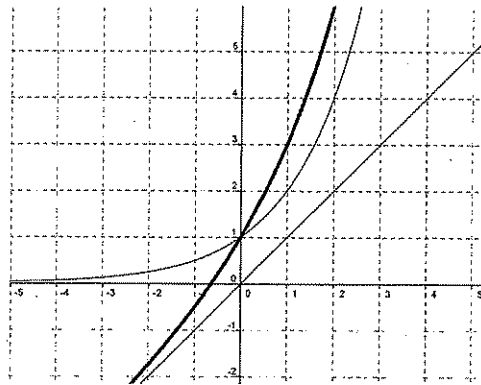
$$\begin{aligned} \text{Solving } x + 2 &= 4 - x, \\ 2x &= 2, \\ \therefore x &= 1. \end{aligned}$$

$$\begin{aligned} \text{Solving } -x - 2 &= 4 + x, \\ 2x &= -6, \\ \therefore x &= -3. \end{aligned}$$

From the graph, it can be seen that the solution to the inequality is given by  $x < -3$  and  $x > 1$ .

(d) Add the y values at particular values of x.

PE6



1 mark : correct solution



**Outcomes Addressed in this Question**

P4 chooses and applies appropriate arithmetic, algebraic, and trigonometric techniques  
 P7 determines the derivative of a function through routine application of the rules of differentiation  
 HE2 uses inductive reasoning in the construction of proofs

Outcome	Solutions	Marking Guidelines
P4	(a) (i) $36^\circ = \frac{36\pi}{180}$ $= \frac{\pi}{5}$	<b>1 mark:</b> correct answer
P4	(ii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$ $= \frac{1}{2} \times 8^2(1.7 - \sin 1.7)$ $= 22.7 \text{ cm}^2$	<b>2 marks:</b> correct solution <b>1 mark:</b> substantially correct solution
P7	(b) (i) $\frac{d}{dx}\left(5x^4 - \frac{4}{x}\right) = 20x^3 + \frac{4}{x^2}$	<b>2 marks:</b> correct solution <b>1 mark:</b> substantially correct solution
P7	(ii) $\frac{d}{dx}\sqrt{2x+7} = \frac{d}{dx}(2x+7)^{\frac{1}{2}}$ $= \frac{1}{2}(2x+7)^{-\frac{1}{2}} \times 2$ $= \frac{1}{\sqrt{2x+7}}$	<b>2 marks:</b> correct solution <b>1 mark:</b> substantially correct solution
HE2	(c) Show true for $n = 1$ $\text{LHS} = \frac{1}{(3 \times 1 - 1)(3 \times 1 + 1)} \quad \text{RHS} = \frac{1}{3(1) + 1}$ $= \frac{1}{4} \quad = \frac{1}{4}$ <p>Assume true for <math>n = k</math>.</p> $\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1}$ <p>Prove true for <math>n = k + 1</math></p> $\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3[k+1]-2)(3[k+1]+1)}$ $= \frac{k}{3k+1} + \frac{k+1}{3(k+1)+1}$ <p>i.e.</p> $\frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)} = \frac{k+1}{3k+4}$	<b>3 marks:</b> correct solution  <b>2 mark:</b> substantially correct solution  <b>1 mark:</b> partially correct solution

$$\begin{aligned}\text{LHS} &= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)} \\ &= \frac{k(3k+4)}{(3k+1)(3k+4)} + \frac{1}{(3k+1)(3k+4)} \\ &= \frac{3k^2 + 4k + 1}{(3k+1)(3k+4)} \\ &= \frac{(3k+1)(k+1)}{(3k+1)(3k+4)} \\ &= \frac{k+1}{3k+4} \\ &= \text{RHS}\end{aligned}$$

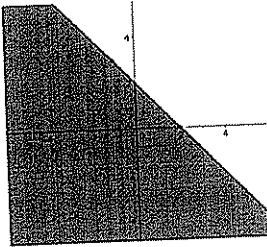
∴ Proven by mathematical induction for  $n > 0$ .

## Year 11 Yearly 2011 – Extension 1 Mathematics

### Solutions and Marking Guidelines

Question No: 6

Outcome Addressed in this Question: Chooses and applies appropriate coordinate techniques to solve problems.

Sample Solution		Marking Guidelines
a)	i) $\sqrt{18}$	<b>1 mark</b> – correct solution
	ii) -1	<b>1 mark</b> – correct solution
	iii) $45^\circ$	<b>1 mark</b> – correct solution
	iv) $y-0 = -1(x-2)$ $y-0 = -x+2$ $\therefore x+y-2=0$	<b>1 mark</b> – correct equation derived with all steps clearly shown
	v) 	<b>1 marks</b> – correct region shaded with dotted line clearly shown.
	vi) $x=2$	<b>1 mark</b> – correct solution
	vii) <p style="text-align: center;">gradient of AC=1  <math>\therefore</math> Equation of AC is <math>y-3=1(x+1)</math>  <math>y-3=x+1</math>  <math>y=x+4</math></p> <p style="text-align: center;">When <math>x=2</math>, <math>y=2+4=6</math>  <math>\therefore C(2,6)</math></p>	<b>2 marks</b> – correct solution  <b>1 mark</b> – substantial progress towards correct solution
b)	$(3,-1) \quad (9,2)$ $5 : -2$ $P\left(\frac{5 \times 9 + -2 \times 3}{5 + (-2)}, \frac{5 \times 2 + -2 \times -1}{5 + (-2)}\right)$ $P\left(\frac{45-6}{3}, \frac{10+2}{3}\right)$ $P(13,4)$	<b>2 marks</b> – correct solution  <b>1 mark</b> – substantial progress towards correct solution

**Year 11 Yearly 2011 – Extension 1 Mathematics**  
**Solutions and Marking Guidelines**

Question No: 7

Outcome Addressed in this Question: Determines derivatives which require various rules of differentiation

Sample Solution		Marking Guidelines
a)	$\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1}$ $= \lim_{x \rightarrow 1} \frac{(x-1)(x-2)}{x-1}$ $= \lim_{x \rightarrow 1} (x-2)$ $= 1 - 2$ $= -1$	<p><b>2 marks</b> – correct solution</p> <p><b>1 mark</b> – substantial progress towards correct solution</p>
b)	$y' = 2x \sin x + (x^2 - 5) \cos x$	<p><b>2 marks</b> – correct solution</p> <p><b>1 mark</b> – substantial progress towards correct solution</p>
c)	$f'(x) = 3x^2 - 3$ <p>at <math>x = -1</math>      <math>f'(x) = 0</math></p> $f(x) = 2$ <p><math>\therefore</math> Equation of tangent is <math>y = 2</math></p>	<p><b>3 marks</b> – correct solution</p> <p><b>1 mark</b> – substantial progress towards correct solution</p> <p><b>3 marks</b> – correct solution</p> <p><b>2 marks</b> – correctly calculates <math>f(x)</math> and <math>f'(x)</math></p> <p><b>1 mark</b> – correctly calculates <math>f(x)</math> or <math>f'(x)</math></p>
d)	$y' = \frac{(2x+1)^3 \cdot 2x - x^3 \cdot 3(2x+1)^2 \cdot 2}{((2x+1)^3)^2}$ $= \frac{2x \cdot (2x+1)^3 - 6x^3 \cdot (2x+1)^2}{(2x+1)^6}$ $= \frac{2x \cdot (2x+1)^2 (2x+1 - 3x)}{(2x+1)^6}$ $= \frac{2x \cdot (1-x)}{(2x+1)^4}$	<p><b>3 marks</b> – correct solution</p> <p><b>2 mark</b> – correctly differentiates <math>y</math>, but does not simplify correctly</p> <p><b>1 mark</b> – substantial progress towards correct derivative</p>

Outcomes Addressed in this Question

P4 chooses and applies appropriate arithmetic, algebraic, and trigonometric techniques

Outcome	Solutions	Marking Guidelines
	<p>(a) (i) <math>TX^2 + TY^2 = PX^2 + RY^2</math>  <math>= 15^2 + 20^2</math>  <math>= 625</math>  <math>= XY^2</math>  <math>\therefore \triangle TXY</math> is right angled.</p> <p>(ii) <math>A = \frac{1}{2} \times 15 \times 20</math>  <math>= 150</math>  <math>A = \frac{1}{2} \times XY \times TZ</math>  <math>150 = \frac{1}{2} \times 25 \times TZ</math>  <math>TZ = 12</math></p> <p>(iii) <math>\tan \theta = \frac{12}{5}</math>  <math>\theta = 67^\circ 23'</math></p> <p>(b) LHS = <math>\operatorname{cosec} 2A + \cot 2A</math>  <math>= \frac{1}{\sin 2A} + \frac{\cos 2A}{\sin 2A}</math>  <math>= \frac{1 + \cos 2A}{\sin 2A}</math>  <math>= \frac{2 \cos^2 A}{2 \sin A \cos A}</math>  <math>= \frac{\cos A}{\sin A} = \cot A</math>  <math>= \text{RHS}</math></p> <p>(c) <math>5 \cos^2 \theta + 2 \sin \theta - 2 = 0</math>  <math>5(1 - \sin^2 \theta) + 2 \sin \theta - 2 = 0</math>  <math>5 \sin^2 \theta - 2 \sin \theta - 3 = 0</math>  <math>(5 \sin \theta + 3)(\sin \theta - 1) = 0</math>  <math>\sin \theta = -\frac{3}{5}</math> or 1  <math>\theta = 216^\circ 52', 323^\circ 8', 90^\circ</math></p>	<p><b>1 mark:</b> correct solution</p> <p><b>2 marks:</b> correct solution  <b>1 mark:</b> substantially correct solution</p> <p><b>2 marks:</b> correct solution  <b>1 mark:</b> substantially correct solution</p> <p><b>2 marks:</b> correct solution  <b>1 mark:</b> substantially correct solution</p> <p><b>3 marks:</b> correct solution  <b>2 mark:</b> substantially correct solution  <b>1 mark:</b> partially correct solution</p>