HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

2015

Preliminary HSC

Assessment Task 3

Examiners ~ S. Gutesa, P. Biczo, D. Crancher, J. Dillon, G. Huxley, G. Rawson

General Instructions

- Reading time – 5 minutes.
- Working time – $1\frac{1}{2}$ hours
- Attempt **all** questions.
- Your ***NAME*** is to be written on the multiple choice answer sheet and **each** answer booklet for questions 8 – 14.
- Board approved calculators and Math Aids may be used.
- This examination must **NOT** be removed from the examination room
- **Section A** consists of seven (7) multiple choice questions worth 1 mark each. Fill in your answer on the multiple choice answer sheet provided.
- **Section B** requires all necessary working to be shown in every question. This section consists of seven (7) questions worth 8 marks each. Marks may not be awarded for careless or badly arranged work.
  Each question is to be started in a new answer booklet. Additional booklets are available if required.

Name : _____

Teacher : _____

SECTION A – 7 multiple choice questions (1 mark each)

Question 1

Which of the following statements is **not true** for all values of x?

A If $x < 0$ **and** $x < -4$ then $x < -4$.

B If $x < 0$ **and** $x > -4$ then $-4 < x < 0$.

C If $x > 0$ **and** $x > -4$ then $x > -4$.

D If $x > 0$ **and** $x < -4$ then x has no solution.

Question 2

In how many ways can 10 boys be arranged in a line if the first boy in the line is Peter and the last boy in the line is Ben?

A 80 640

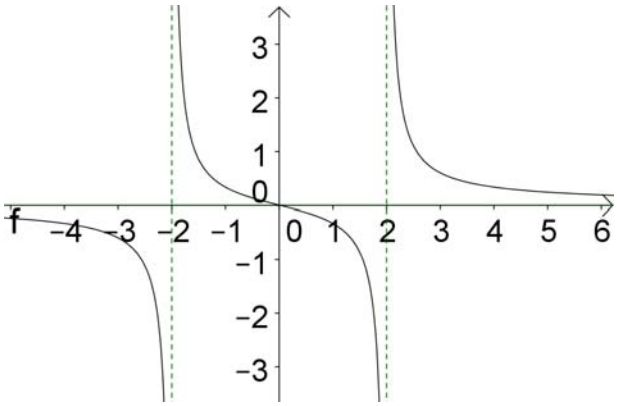
B 40 320

C 3 628 800

D 1 814 400

Question 3

Which equation best describes this graph?



A $y = \dfrac{x}{x^2 - 4}$

B $y = \dfrac{x - 2}{x^2 - 4}$

C $y = \dfrac{x + 2}{x^2 - 4}$

D $y = \dfrac{-x}{x^2 - 4}$

Question 4

Simplify $2\sin 3\theta \cos 3\theta$

A $\sin 2\theta$ **B** $\sin 6\theta$

C $\sin 3\theta$ **D** $2\sin 3\theta$

Question 5

The value of $\dfrac{2\tan 25°}{1+\tan^2 25°} =$

A $\cos 12\cdot 5°$ **B** $\sin 12\cdot 5°$

C $\cos 50°$ **D** $\sin 50°$

Question 6

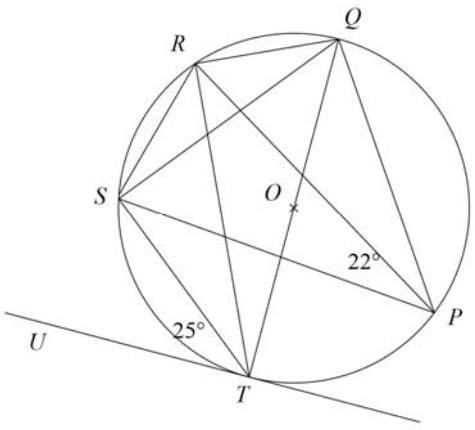
What is the gradient of the line which makes an angle of $45°$ with the line $y = 3x - 1$?

A $\dfrac{1}{3}$ **B** $\dfrac{1}{2}$

C 2 **D** 3

Question 7

A circle with centre O has a tangent TU, diameter QT, $\angle STU = 25^o$ and $\angle RPS = 22^o$.



What is the size of $\angle RTQ$?

A 22° **B** 25°

C 43° **D** 47°

SECTION B – 7 questions (8 marks each)

Question 8 (8 marks) Use a SEPARATE writing booklet **Marks**

(a) If each distinct arrangement of the letters of DELETED is called a word:

(i) How many words are possible? **1**

(ii) In how many of these words will the D's be separated? **1**

(b) In how many ways can a committee of 5 be selected from 10 people, consisting of 5 men and 5 women, if one man, Ken, refuses to work in the same committee with Sue? **2**

(c) Kent wants to celebrate his 18^{th} birthday by having a dinner party for himself and nine of his friends (five girls and four boys).

In how many ways can the people be seated at a round table if:

(i) the boys and girls are to alternate? **1**

(ii) Kent is to be seated between two particular girls? **1**

(d) In how many ways can 8 different gifts be distributed to two children so that each child receives an odd number of gifts? **2**

Question 9 (8 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve for x : $|x+1| \leq |x-3|$. **2**

(b) Solve for x : $\dfrac{2}{x-1} \geq 3$. **3**

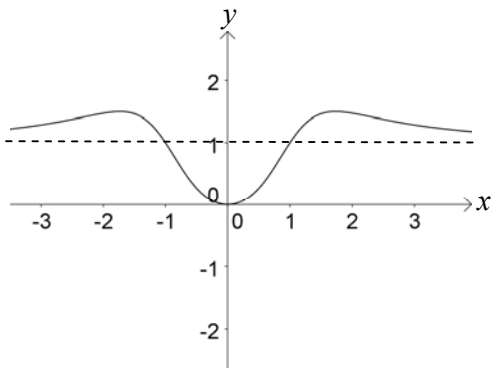
(c) If $a > 0$ and $b > 0$, show that $a^4 + b^4 \geq a^3b + ab^3$. **3**

Question 10 (8 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Find the equations of the vertical and horizontal asymptotes of $y = \dfrac{x-2}{x-4}$ and hence sketch the graph of $y = \dfrac{x-2}{x-4}$. **3**

(ii) Using the graph from part (i) and further sketching, find the values of x for which $\dfrac{x-2}{x-4} \leq 3$. **2**

(b) The graph of the function $f(x) = \dfrac{x^4 + 3x^2}{x^4 + 3}$ is given below.



Sketch the following graphs, clearly showing any x or y intercepts.

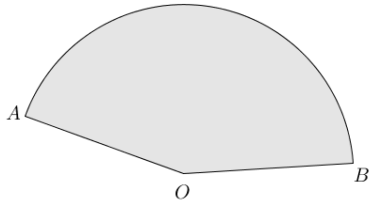
(i) $y = f(x) - 1$ **1**

(ii) $y = |f(x) - 1|$ **1**

(iii) $y = f(x-2)$, for $1 \leq x \leq 3$ **1**

Question 11 (8 marks) Use a SEPARATE writing booklet **Marks**

(a) The circle sector shown below has radius 12 cm and $\angle AOB = \frac{2\pi}{3}$.



(i) Calculate the exact area of the sector. **1**

(ii) What is the length of the arc AB, in exact form? **1**

(b) Prove that $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$ for all values of θ. **2**

(c) Two boats P and Q are observed from the top of a vertical tower CT of height 120 m. The base of the tower, C, is at sea level.

P is on a bearing of $305°$ from the tower and its angle of depression from T is $22°$.
Q is on a bearing of $025°$ from the tower and its angle of depression from T is $27°$.

(i) Show that $\angle PCQ = 80°$. **1**

(ii) Calculate the distance between the two boats. **3**

Question 12 (8 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve $\sin\left(\theta+\frac{\pi}{6}\right)=2\sin\left(\theta-\frac{\pi}{6}\right)$ for $0\le\theta\le 2\pi$ **3**

(b) (i) Let $\cos x-\sin x=R\cos\left(x+\alpha\right)$.

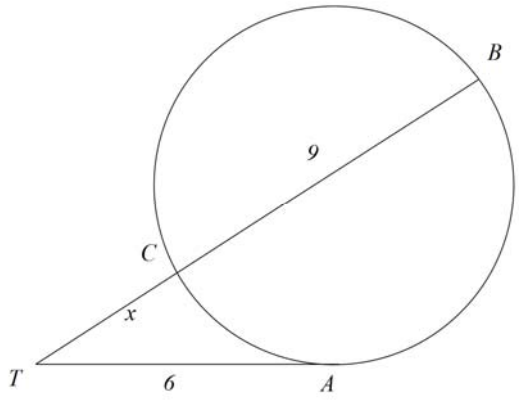
Find positive values for R and for α. **2**

(ii) Hence solve the equation $\cos x-\sin x=1$, for $0°\le\theta\le 360°$
(Give the solutions correct to the nearest minute where necessary.) **2**

(iii) Find the maximum value of $\cos x-\sin x$. **1**

Question 13 (8 marks) Use a SEPARATE writing booklet **Marks**

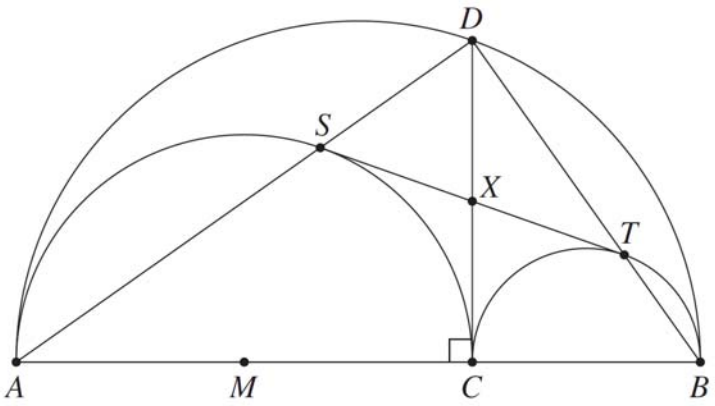
(a) Line TA is a tangent to the circle at A and TB is a secant meeting the circle at B and C. Given that $TA = 6$, $CB = 9$ and $TC = x$, what is the value of x?



Find the value of x, giving reasons. **2**

(b) The diagram shows a large semicircle with diameter AB and two smaller semicircles with diameters AC and BC, respectively, where C is a point on the diameter AB. The point M is the centre of the semicircle with diameter AC.

The line perpendicular to AB through C meets the largest semicircle at the point D. The points S and T are the intersections of the lines AD and BD with the smaller semicircles. The point X is the intersection of the lines CD and ST.



(i) Explain why $CTDS$ is a rectangle. **2**

(ii) Show that $\triangle MXS$ and $\triangle MXC$ are congruent. **3**

(iii) Show that the line ST is a tangent to the semicircle with diameter AC. **1**

Question 14 (8 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the coordinates of the point P that divides the interval joining A (1, 6) and B (5, -2) internally in the ratio 3:1. **2**

(b) (i) Show that the curves $y = 3x^2$ and $y = 4x - x^2$ intersect at both the origin and the point (1, 3). **2**

(iii) Find the acute angle, to the nearest minute, between the two curves at (1, 3). **2**

(c) Determine the ratio in which the point P (-3, 8) divides the interval joining A (6, -4) and B (0, 4). **2**

END OF EXAMINATION.

SECTION A

Multiple choice answer sheet.

- Detach this sheet and use it to mark the answers to the questions in Section A
- Mark the answer by shading the letter that matches with the correct answer
- If you make a mistake, draw a cross through the incorrect answer

Name: _____

Teacher: _____

1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D

Year 11	Mathematics Extension 1	Yearly Examination 2015
Question 8	Solutions and Marking Guidelines	

Outcome Addressed in this Question

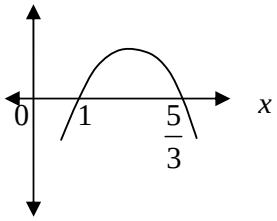
PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Part	Solutions	Marking Guidelines
(a) (i)	Number of ways $= \dfrac{7!}{2!3!} = 420$	**Award 1** ~ correct answer
(ii)	Put the *D*'s together ~ this can be done in $\dfrac{6!}{3!} = 120$ ways. $\therefore$ Number of ways with the *D*'s separated is $420 - 120 = 300$ ways.	**Award 1** ~ correct answer
(b)	Ken in, Sue out: $\binom{8}{4}$ ways Ken out, Sue in: $\binom{8}{4}$ ways Ken out, Sue out: $\binom{8}{5}$ ways $\therefore$ Number of ways $= 196$	**Award 2** for correct solution. **Award 1** for substantial progress towards solution.
(c) (i)	Place one boy first, the rest can be arranged in $4! \times 5! = 2880$ ways	**Award 1** ~ correct answer
(ii)	Place Kent and the two girls first. This can be done in $2!$ ways. The remainder can be arranged in $7!$ ways. $\therefore$ Number of ways $= 2! \times 7! = 10080$	**Award 1** ~ correct answer
(d)	**A** 1 3 5 7 **B** 7 5 3 1 Number of ways $= \binom{8}{1}\binom{7}{7} + \binom{8}{3}\binom{5}{5} + \binom{8}{5}\binom{3}{3} + \binom{8}{7}\binom{1}{1}$ $= 128$	**Award 2** for correct solution **Award 1** for substantial progress towards solution

Year 11 Mathematics Extension 1 Examination 2015	
Question No. 9 Solutions and Marking Guidelines	

Outcomes Addressed in this Question

P4 Chooses and applies appropriate algebraic techniques
P5 Understands the concept of a function and the relationship between a function and its graph.
PE3 Solves problems involving inequalities

Outcome	Solutions	Marking Guidelines
PE3, P4	(a) $\lvert x+1\rvert \le \lvert x-3\rvert$ $\therefore (x+1)^2 \le (x-3)^2$ (as both sides positive / zero, inequality remains true when squaring) $\therefore x^2+2x+1 \le x^2-6x+9$ $\therefore 8x \le 8$ $\therefore x \le 1.$	2 marks : correct solution 1 mark : substantial progress towards correct solution
PE3, P5	(b) $\dfrac{2}{x-1} \ge 3$ $2(x-1) \ge 3(x-1)^2$ (multiplying both sides by $(x-1)^2$) $2(x-1)-3(x-1)^2 \ge 0$ $(x-1)(2-3(x-1)) \ge 0$ $(x-1)(5-3x) \ge 0$  From the graph, $1 \le x \le \dfrac{5}{3}$. But $x \ne 1$, as denominator cannot be zero, so $1 < x \le \dfrac{5}{3}.$	3 marks : correct solution 2 mark : partially correct solution. 1 mark : substantial progress towards correct solution
PE3	(c) Consider the difference $a^4+b^4-(a^3b+ab^3) = a^4-a^3b+b^4-ab^3$ $= a^3(a-b)+b^3(b-a)$ $= a^3(a-b)-b^3(a-b)$ $= (a-b)(a^3-b^3)$ $= (a-b)(a-b)(a^2+ab+b^2)$ $= (a-b)^2(a^2+ab+b^2)$ Since $a>0$ and $b>0$, then $a^2>0$, $ab>0$ and $b^2>0$. $\therefore a^2+ab+b^2>0.$ Also, $(a-b)^2 \ge 0.$ Hence, $(a-b)^2(a^2+ab+b^2) \ge 0,$ $a^4+b^4-(a^3b+ab^3) \ge 0,$ Hence $a^4+b^4 \ge a^3b+ab^3.$	3 marks : correct solution 2 mark : partially correct solution. 1 mark : substantial progress towards correct solution

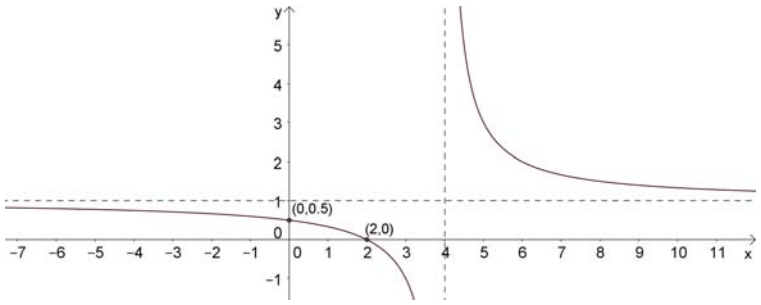
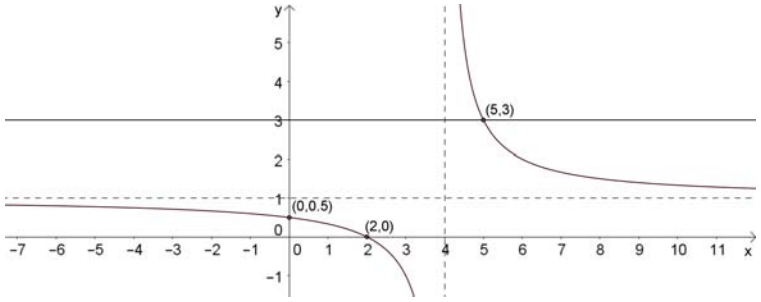
Year 11 Task 3 : Yearly	Mathematics Extension 1	Examination 2015
Question No. 10	Solutions and Marking Guidelines	

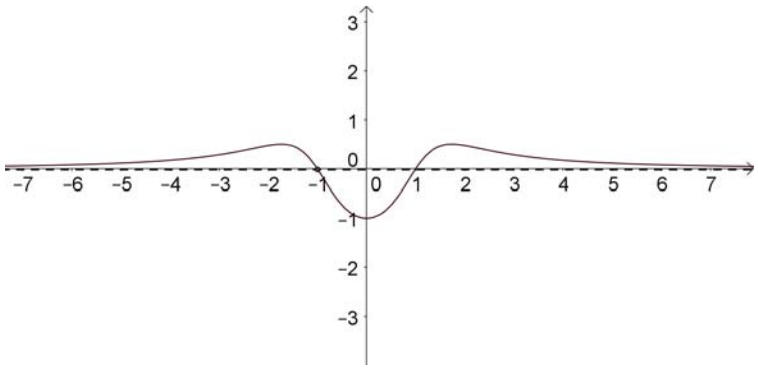
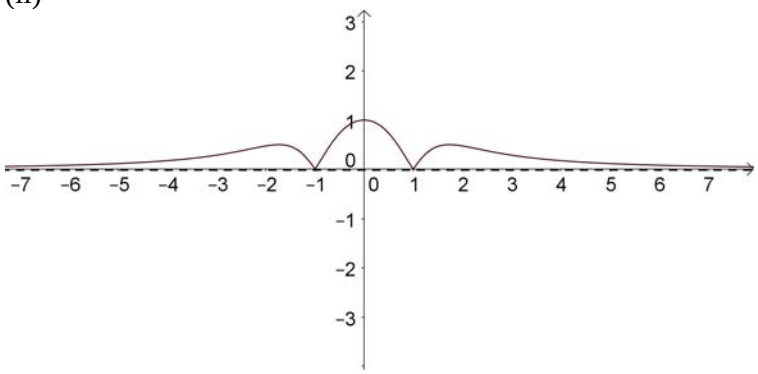
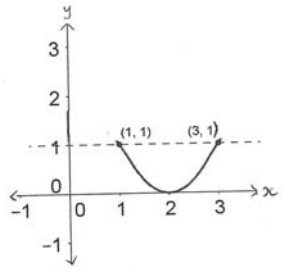
Outcomes Addressed in this Question

PE3 solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

Outcome	Solutions	Marking Guidelines
	Question 10 a)	
PE6,P4	(i) Vertical asymptote at $x = 4$	1 mark for correct vertical asymptote
	Horizontal asymptote at $y = 1$	1 mark for correct horizontal asymptote
		1 mark for correct graph
PE3,P4	(ii)	
		2 marks for complete correct solution 1 mark for partial correct solution
	Solution is $x < 4,\ x \geq 5$.	

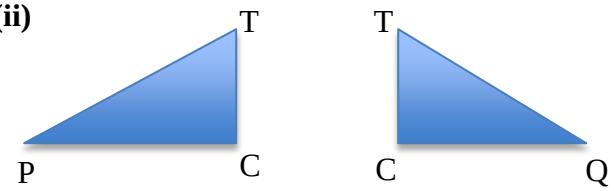
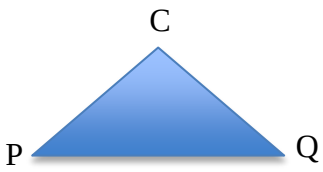
PE6,P4	b) (i)  	1 mark for correct graph
PE6,P4	(ii)  	1 mark for correct graph
PE6,P4	(iii)  	1 mark for correct graph

Year 11 Mathematics Extension 1 Task 3, 2015

Question No. 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

P4 - chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

Outcome	Solutions	Marking Guidelines
P4	**(a)** **(i)** $A=\frac{1}{2}\left(12^2\right)\left(\frac{2\pi}{3}\right)=48\pi \text{ cm}^2$ **(ii)** $l=12\left(\frac{2\pi}{3}\right)=8\pi \text{ cm}$ **(b)** $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1}{\sec^2\theta}-\frac{\tan^2\theta}{\sec^2\theta}$ $=\cos^2\theta-\frac{\sin^2\theta}{\cos^2\theta}\times\cos^2\theta$ $=\cos^2\theta-\sin^2\theta$ $=\cos 2\theta$ **(c) (i)** $\angle PCQ=(360-305)^\circ+25^\circ=80^\circ$ **(ii)**   In ΔPCT, $PC=\frac{120}{\tan 22^\circ}$ In ΔQCT, $QC=\frac{120}{\tan 27^\circ}$ In ΔPCQ, $PQ^2=PC^2+QC^2-2(PC)(QC)\cos 80^\circ$ $\Rightarrow PQ=346$ m to the nearest m.	**(a)(i)** **1 mark:** Correct, exact answer. **(ii)** **1 mark:** Correct, exact answer. **(b)** **2 marks:** Correct solution. **1 mark:** Relevant progress involving trig identities. **(c) (i)** **1 mark**: Correct expression. **(ii)** **3 marks:** Correct solution, including all steps. **2 marks:** Significant progress. **1 mark:** Some progress using 3D trigonometry. *Lots of diagrams had T and C in the same place, which made it impossible to see the 3D involved.*

Year 11 Mathematics Extension 1 Task 3 2015		
Question No. 12 Solutions and Marking Guidelines		
Outcomes Addressed in this Question		
P4 - chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques		
Outcome	**Solutions**	**Marking Guidelines**
P4	**(a)** $\sin\left(\theta+\frac{\pi}{6}\right)=2\sin\left(\theta-\frac{\pi}{6}\right)$ $\sin\theta\cos\frac{\pi}{6}+\cos\theta\sin\frac{\pi}{6}=2\sin\theta\cos\frac{\pi}{6}-2\cos\theta\sin\frac{\pi}{6}$ $\frac{3}{2}\cos\theta=\frac{\sqrt{3}}{2}\sin\theta$ $\tan\theta=\sqrt{3}\quad(\cos\theta\neq0)$ $\theta=\frac{\pi}{3},\frac{4\pi}{3}$ Check that $\cos\theta=0$ is not a possible solution. **(b) (i)** $\cos x-\sin x=R\cos x\cos\alpha-R\sin x\sin\alpha$ $\rightarrow R=\sqrt{2}\ ;\quad \alpha=45°$ **(ii)** $\sqrt{2}\cos(x+45°)=1 \qquad 45°\le x+45°\le 405°$ $\cos(x+45°)=\frac{1}{\sqrt{2}}$ $x+45°=45°,315°,405°$ $x=0°,270°,360°$ **(iii)** $\cos(x+45°)$ has a maximum value of 1. Therefore $\sqrt{2}\cos(x+45°)$ has a maximum = $\sqrt{2}$	**(a)** **3 marks:** Correct solution, that should include: correct expansion of trigonometric sum; correct value for tanθ; and both solutions in radians.. (Or equivalent) **2 marks** 2 components of the above correct. **1 mark** 1 component of above correct. **(b)** **(i) 2 marks:** both answers correct according to the sum format that is given in the question. **1 mark:** 1 answer correct. **(ii) 2 marks:** Correct working and achieving all possible solutions. **1 mark:** Considerable progress. **(iii) 1 mark:** Correct answer, dependent upon the value for *R* given in part (i). *In (b) (i), many people had memorised a format that wasn't relevant to the cos of a sum. You need to be able to adapt to what the question is seeking.*

Year 11 Ext 1 Mathematics		Task 3 2015
Question No. 13	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

PE2 - uses multi-step deductive reasoning in a variety of contexts

PE3 - solves problems involving permutations and combinations, inequalities, polynomials, circle geometry and parametric representations

Outcome	Solutions	Marking Guidelines
PE3	(a) $BT.TC = AT^2$ (product of intercepts of secant is equal to the square of intercept on tangent) $(x+9)x = 6^2$ $x^2 + 9x - 36 = 0$ $(x-3)(x+12) = 0$ $x = 3$ or -12 $x = 3$ (x is positive)	**2 marks:** correct solution, **1 mark:** substantially correct solution
PE2	(b)(i) $\angle ADB = 90°$ (angle in semicircle) $\angle ASC = 90°$ (" ") $\angle CTB = 90°$ (" ") $\therefore CTDS$ is a rectangle (quadrilateral with 3 right angles)	**2 marks:** correct solution, **1 mark:** substantially correct solution
PE2	(b)(ii) In ΔMXS & ΔMXC MX is common $XS = XC$ (equal diagonals of rectangle bisect each other) $MS = MC$ (radii) $\therefore \Delta MXS$ & ΔMXC (SSS)	**3 marks:** correct solution, **2 marks:** substantially correct solution **1 mark:** limited progress *towards* correct solution
PE2	(b)(iii) $\angle MSX = \angle MCS$ (matching angles in congruent triangles) $= 90°$ $\therefore$ ST is tangent (perpendicular to radius)	**1 mark:** correct solution,

Year 11 Mathematics Extension 1 Task 3 2015		
Question No. 14 Solutions and Marking Guidelines		
Outcomes Addressed in this Question		
P4 - chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques		
Outcome	**Solutions**	**Marking Guidelines**
	(a) $A(1,6)\ B(5,-2)\ m:n=3:1\ P(x,y)$ $x=\frac{mx_2+nx_1}{m+n}$ $\quad y=\frac{my_2+ny_1}{m+n}$ $x=\frac{(3)(5)+(1)(1)}{(3)+(1)}$ $\quad y=\frac{(3)(-2)+(1)(6)}{(3)+(1)}$ $x=\frac{16}{4}$ $\quad y=\frac{0}{4}$ $x=4$ $\quad y=0$ $\therefore P$ has coordinates $(4,0)$.	**2 marks** Correct solution. **1 mark** Some progress towards correct solution.
	(b)(i) Point of intersection: (solve simultaneously or show full substitution of coordinates into both equations) $y=3x^2....(1)$ $y=4x-x^2...(2)$ $3x^2=4x-x^2$ $4x^2-4x=0$ $4x(x-1)=0$ $\therefore$ Points of intersection at $x=0,\ x=1$. When $x=0,\ y=3(0)^2=0$ When $x=1,\ y=3(1)^2=3$ $\therefore$ The two curves intersect at the origin and $(1,3)$.	**2 marks** Correct solution. **1 mark** Some progress towards correct solution.
	(b)(ii) For $y=3x^2,\ y'=6x$ When $x=1,\ y'=6(1)=6$ $\therefore m_1=6$ For $y=4x-x^2,\ y'=4-2x$ When $x=1,\ y'=4-2(1)=2$ $\therefore m_2=2$ $\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$ $\tan\theta=\frac{6-2}{1+(6)(2)}$ $\tan\theta=\frac{4}{13}$ $\therefore\theta=17°6'$ (to the nearest minute).	**2 marks** Correct solution. **1 mark** Some progress towards correct solution.

	(c) $A(6,-4)\ B(0,4)\ P(-3,8)$ Let $m:n=k:1$ $x=\dfrac{mx_2+nx_1}{m+n}$ $-3=\dfrac{(k)(0)+(1)(6)}{(k)+(1)}$ $-3=\dfrac{6}{k+1}$ $-3(k+1)=6$ $-3k-3=6$ $-3k=9$ $\therefore k=-3$ $\therefore$ The ratio is $-3:1$ or external division in the ratio $3:1$.	**2 marks** Correct solution. **1 mark** Some progress towards correct solution.

Multiple choice:
1. C
2. B
3. A
4. B
5. D
6. B
7. C