YEAR 11 Extension 1 – Preliminary Examination 2006

Question 1 (15 Marks) **Marks**

(a) Factorise completely: $3x^3 + 375$ **2**

(b) Differentiate with respect to x:

(i) $y = 1 - \dfrac{3}{\pi}$ **1**

(ii) $y = \dfrac{e^{2x} - e^{x}}{e^{x}}$ **2**

(iii) $y = \log_e(\cos 2x)$ **2**

(c) Find the coordinates of the point dividing the interval from $(-7, 0)$ to $(3, 5)$ in the ratio 3:2, externally. **2**

(d) Find the size of the acute angled formed by the straight lines: $2x - y + 3 = 0$ and $x - 3y - 2 = 0$. **3**

(e) Solve $\tan\theta = \sin 2\theta$ for $0 \le \theta \le 360^0$. **3**

Question 2 (15 Marks) **Start a new page** **Marks**

(a) The equation $(x - 3y + 5) + k(x + 2y) = 0$ represents the family of straight lines passing through the point of intersection of point P.

(i) For what values of k is one of the lines in the family parallel to the straight line $x + y - 2 = 0$? **2**

(ii) For what values of k does one of the lines in the family pass through the centre of the circle $x^2 + y^2 - 10y + 21 = 0$? **2**

(iii) Find the coordinates of P. **2**

(b) The polynomial $P(x) = x^4 - 3x^3 + ax^2 + bx - 6$ has a remainder of 8 when divided by $(x + 1)$. If $(x - 3)$ is a factor of $P(x)$, find the values of a and b. **3**

(c) A$(-4, -6)$ and $B(2, 3)$ are two given points. P(x, y) is a point such that the ratio AP: PB = 2: 1. Find the locus of P and describe it geometrically. **3**

(d) The polynomial $x^3 - 2x^2 - mx - 16 = 0$ has roots α, β and $\alpha\beta$. Find all possible values of m. **3**

Question 3 (15 Marks) **Start a new page** **Marks**

(a) Give the general solution of $\cos x + \sin^2 x = \frac{5}{4}$. **3**

(b)

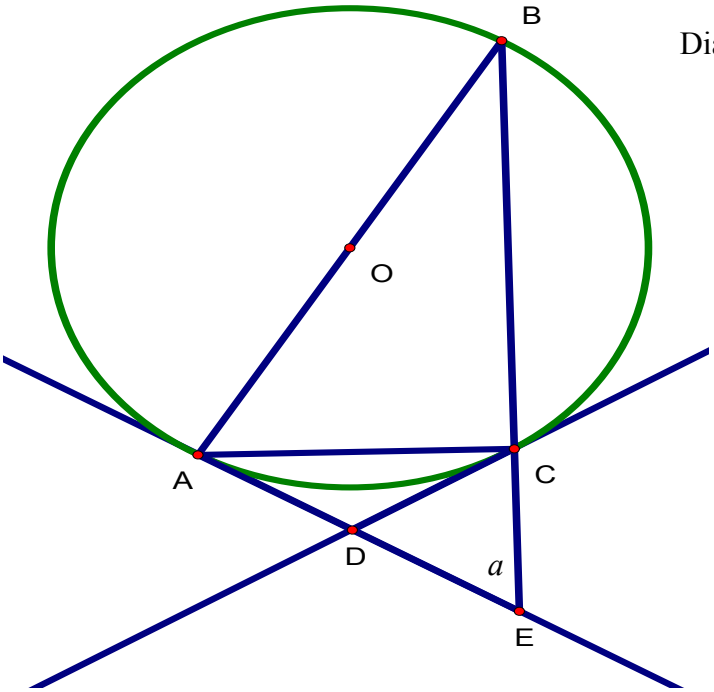


Diagram not to scale

DA, DC are tangents to the circle above. AB is the diameter and $\angle CED = a$. **3**
AE and DC are tangents to the circle.
Copy the diagram neatly onto your answer sheet and prove that $\angle ADC = 2a$.

(b) 2 Koreans, 3 Germans and 4 Americans stand in a queue to board a plane heading for Sydney.

(i) How many ways can they stand in this queue without restrictions? **1**

(ii) How many ways can they stand in this queue, if all the Americans must be together? **2**

(c) There are 5 men and 5 women in a sports team. In how many ways can they be formed into

(i) one circle with no two girls next to one another? **2**

(ii) two concentric circles, with a girl behind each boy? **2**

(iii) Show that 10 people may be seated at two round tables, 5 persons at each, in $\frac{10!}{5^2}$ ways. **2**

Question 4 (15 Marks) **Start a new page** **Marks**

(a) $A(h, k + a)$ is a fixed point on the Cartesian Number plane and a, h, k are constants. $P(x, y)$ is a variable point on the same plane and L is a fixed line on the same plane with equation $y = k - a$.

(i) Find the perpendicular distance from the point P to the line L. **1**

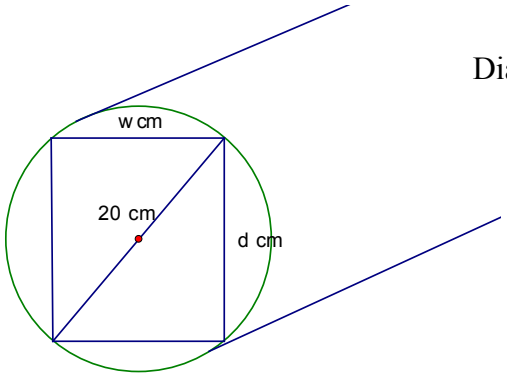
(ii) Prove that the locus of P is given by $(x - h)^2 = 4a(y - k)$ **3**

if distance PA is equal to the perpendicular distance found in part (i).

(iii) If $h = 1$, $k = -2$ and $|a| = 3$; draw a neat sketch of the locus, clearly showing the vertex and focus. **3**

(b) A rectangular beam of width w cm and depth d cm is cut from a cylindrical wooden log as shown.

Diagram not to scale



The diameter of the cross – section of the log is 20 cm. The strength S of the beam is proportional to the product of its width and the square of its depth.

(i) Show that $S = k\,w(400 - w^2)$. **2**

(ii) What numerical dimensions will give a beam of maximum strength? Justify your answer. **3**

(iii) A square beam with diagonal 20 cm could have been cut from the Same log. Show that the rectangular beam of maximum strength is more than 60% stronger than this square beam. **3**

~End of exam ~