Year 11 Preliminary Extension I Exam 2008

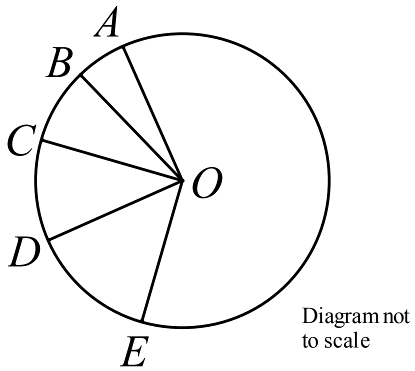
Question 1. **Marks**

(a) Find $\lim_{x\to 0}\left(\dfrac{\sin 2x}{\tan 5x}\right)$ **2**

(b) If $x=\alpha$, $x=\beta$ and $x=\gamma$ are the roots of the equation $3x^3+4x^2-2x+1=0$, find the value of $\alpha^2+\beta^2+\gamma^2$. **2**

(c) Find the angle to the nearest minute between the lines $4x-5y+3=0$ and $y=3x-2$. **2**

(d) Circle with centre *O* and radii *AO,BO,CO,DO* and *EO* is shown below with $\angle AOB=15°, \angle BOC=25°, \angle COD=35°$ and $\angle DOE=40°$. Prove that *AD*=*CE* **3**



(e) (i) Show that $x-3$ is a factor of $x^3-12x^2+47x-60$. **1**
(ii) Hence express $x^3-12x^2+47x-60$ as a product of linear factors. **2**

(f) Find an expression between *x* and *y* that is independent of *t* when $x=e^t+e^{-t}$ and $y=e^t-e^{-t}$. **2**

(g) Find the number of ways all the letters from the word *choice* can be arranged around a circle. **1**

Question 2.

(a) If $\sin A=\dfrac{1}{4}$ and angle *A* is obtuse, find the exact values of:

(i) $\sin 2A$ **2**

(ii) $\cos\dfrac{A}{2}$ **2**

(b) A polynomial *P(x)* has a remainder of 2 when divided by $x-1$ and a remainder of 3 when divided by *x+2.* **3**
Find the remainder when *P(x)* is divided by x^2+x-2.

(c) How many ways can a group of 2 men and 3 women be chosen from 6 men and 5 women? **2**

(d) (i) Write $\sqrt{2}\cos x+\sqrt{7}\sin x$ in the form $R\cos(x-\alpha)$, where *R>0* and α is to the nearest minute. **2**

(ii) Graph $y=\sqrt{2}\cos x+\sqrt{7}\sin x$ for $0°\le x\le 360°$, showing all points where the curve crosses the *x* and *y* axes. **2**

(iii) Solve $\sqrt{2}\cos x+\sqrt{7}\sin x=1$ for *x* in the domain $0°\le x\le 360°$ **2**

Question 3.

(a) Find the number of words that can be formed using all the letters from{ *a, b, c, d, e , f, g}* if a vowel is at each end and the letters *f* and *g* are together. **2**

(b) Find the equation in expanded form of a monic cubic polynomial with real roots : $1, 2+\sqrt{3}$ and $2-\sqrt{3}$. **2**

(c) Differentiate with respect to x in simplest terms:

(i) $y = \dfrac{\sin(x+7)}{\cos(x-9)}$ **3**

(ii) $y = \log_x 3$ **2**

(d) The points $P(2p, p^2)$ and $Q(2q, q^2)$ lie on the parabola with equation $x^2 = 4y$, such that $pq = 4$. The normals to the parabola at P and Q intersect at N.

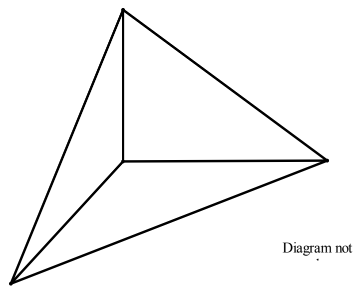
(i) Derive the equation of the normal at the point P. **2**

(ii) Show that the intersection point N of the normals is $(-4(p+q), p^2+q^2+6)$. **2**

(iii) Find the Cartesian equation of the locus of N, stating any restrictions. **2**

Question 4.

(a) A vertical post BD of length h metres is held in position by ropes AD and DC. The distance AC is 30 metres, $\angle ABC = 100°$ and the angles of elevation of D from A and C are $10°$ and $8°$ respectively.

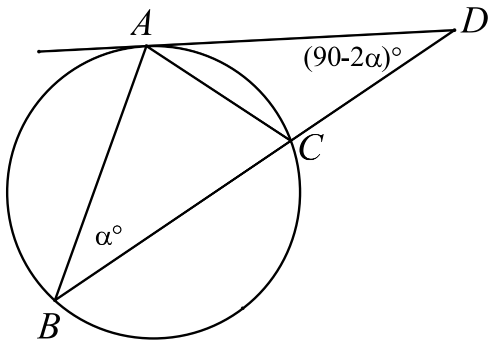


(i) Show that **2**

$$h = \frac{30\tan 80°\tan 10°}{\sqrt{\tan^2 8° + \tan^2 10° - 2\tan 8°\tan 10°\cos 100°}}$$

(ii) Find the height of the post to the nearest centimetre. **1**

(b) Triangle ABC is inscribed in a circle. The tangent to the circle at A meets BC produced at D as shown with $\angle ABC = \alpha°$ and $\angle ADC = (90-2\alpha)°$, where $0° < \alpha° < 45°$. Prove BC is the diameter of the circle. **4**



(c) (i) Show $\dfrac{d}{dx}\left(\dfrac{x^2-3x+2}{x^2+4x+3}\right) = \dfrac{7x^2+2x-17}{(x^2+4x+3)^2}$ **2**

(ii) Find the location of the turning points of $y = \dfrac{x^2-3x+2}{x^2+4x+3}$, and test their nature. **3**

(iii) Graph $y = \dfrac{x^2-3x+2}{x^2+4x+3}$, showing all asymptotes, turning points, and points where the curve crosses the coordinate axes. **3**

End of Exam

YR11 PRELIMINARY EXT 1 2008

1(a) $\lim_{x\to 0} \frac{\sin 2x}{\tan 5x} = \lim_{x\to 0}\left[\frac{\sin 2x}{2x}\cdot\frac{5x}{\sin 5x}\cdot\cos 5x\cdot\frac{2}{5}\right]$

$= 1\cdot 1\cdot 1\cdot \frac{2}{5}$

$= \frac{2}{5}$

(b) $\alpha+\beta+\gamma = -\frac{4}{3}$

$\alpha\beta+\alpha\gamma+\beta\gamma = -\frac{2}{3}$

$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2[\alpha\beta+\alpha\gamma+\beta\gamma]$

$= \left(-\frac{4}{3}\right)^2 - 2\cdot -\frac{2}{3}$

$= \frac{16}{9} + \frac{4}{3}$

$= \frac{28}{9}$

$= 3\frac{1}{9}$

(c) $m_1 = \frac{4}{5}$ $m_2 = 3$

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= \left|\frac{\frac{4}{5}-3}{1+\frac{4}{5}\cdot 3}\right|$

$= \left|\frac{-11}{17}\right|$

$\theta = 32^\circ 54'$

(d) $\angle AOD = 15+25+35$

$= 75^\circ$

$\angle COE = 35+40$

$= 75^\circ$

$\therefore AD = CE$ (Equal angles subtend equal chords)

(e) (i) $P(3) = 3^3 - 12\times 3^2 + 47\times 3 - 60$

$= 27 - 108 + 141 - 60$

$= 168 - 168$

$= 0$

$\therefore x-3$ is a factor

(ii) $x^3 - 12x^2 + 47x - 60 = (x-3)(x^2-9x+20)$

$= (x-3)(x-4)(x-5)$

(f) $x^2 = e^{2t} + 2 + e^{-2t}$

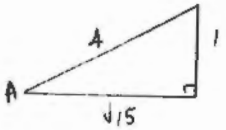
$y^2 = e^{2t} - 2 + e^{-2t}$

$\therefore x^2 - y^2 = 4$

(g) c e h c o ways $= \frac{5!}{2!}$

$= 60.$

2(a)



(i) $\sin 2A = 2\sin A\cos A$

$= 2\cdot\frac{1}{4}\cdot -\frac{\sqrt{15}}{4}$

$= -\frac{\sqrt{15}}{8}$

$\cos A < 0$

A obtuse

(ii) $\cos A = 2\cos^2\frac{A}{2} - 1$

$2\cos^2\frac{A}{2} = 1 + \left(-\frac{\sqrt{15}}{8}\right)$

$\cos^2\frac{A}{2} = \frac{8-\sqrt{15}}{16}$

$\cos\frac{A}{2} = \frac{\sqrt{8-\sqrt{15}}}{4}$

Halve obtuse → acute angle

$\cos\frac{A}{2} > 0.$

(b) $P(x) = (x-1)(x+2)Q(x) + ax + b$

$x=1$ $2 = a+b$

$x=-2$ $3 = -2a+b$

$a = -\frac{1}{3}$ $b = 2\frac{1}{3}$

Remainder $R = -\frac{x}{3} + 2\frac{1}{3}$

(c) No ways $= \binom{6}{2}\binom{5}{3}$

$= 15\times 10$

$= 150$

d(i) $R\cos(x-\alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$

$R\sin\alpha = \sqrt{7}$ $R>0$ $\left.\begin{array}{l}\sin\alpha>0\\ \cos\alpha>0\end{array}\right\}$ $0<\alpha<90^\circ$

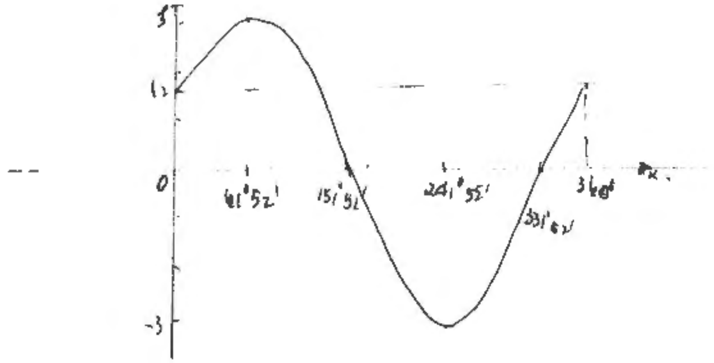
$R\cos\alpha = \sqrt{2}$

$\tan\alpha = \sqrt{\frac{7}{2}}$

$\alpha = 61^\circ 52'$

$R = 3$

$\therefore \sqrt{2}\cos x + \sqrt{7}\sin x = 3\cos(x - 61^\circ 52')$



(iii) $\sqrt{2}\cos x + \sqrt{7}\sin x = 1$

$3\cos(x - 61°52') = 1$

$\cos(x - 61°52') = \frac{1}{3}$

$x - 61°52' = 70°32'$ or $289°28'$

$x = 132°24'$ or $351°20'$

3(a) ways $\{a, e\} = 2$

ways $\{fg\} = 2$

ways $b\,c\,d\,(fg) = 4!\ 2$

Total ways $= 4! \times 2 \times 2 = 96$

(b) $(x-1)(x-2+\sqrt{3})(x-2-\sqrt{3}) = 0$

$(x-1)((x-2)^2 - 3) = 0$

$(x-1)(x^2 - 4x + 1) = 0$

$x^3 - 5x^2 + 5x - 1 = 0$

(c) (i) $y = \dfrac{\sin(x+7)}{\cos(x-9)}$

$\dfrac{dy}{dx} = \dfrac{\cos(x-9)\cos(x+7) - \sin(x+7)\cdot -\sin(x-9)}{\cos^2(x-9)}$

$= \dfrac{\cos(x-9-(x+7))}{\cos^2(x-9)}$

$= \dfrac{\cos 16}{\cos^2(x-9)}$

(ii) $y = \log_x 3$

$= \dfrac{\ln 3}{\ln x}$

$\dfrac{dy}{dx} = \dfrac{-1}{x}\ \dfrac{\ln 3}{\ln^2 x}$

$= \dfrac{-1}{x} \cdot \dfrac{\ln 3}{\ln x} \cdot \dfrac{1}{\ln x}$

$= \dfrac{-\log_x 3}{x \ln x}$

3(d) $x = 2p \quad y = p^2$

$\frac{dx}{dp} = 2 \quad \frac{dy}{dp} = 2p$

$\frac{dy}{dx} = \frac{\frac{dy}{dp}}{\frac{dx}{dp}}$

$= \frac{2p}{2}$

$= p$

$m_N = -\frac{1}{p}$

Eqn normal

$y - p^2 = \frac{-1}{p}(x - 2p)$

$py - p^3 = -x + 2p$

$x + py = 2p + p^3$ ①

$x + qy = 2q + q^3$ (2)

$\therefore \cancel{pq}\ y(p-q) = 2(p-q) + p^3 - q^3$

$= 2(p-q) + (p-q)(p^2 + pq + q^2)$

$= (p-q)\left[2 + p^2 + 4 + q^2\right]$

$y = p^2 + q^2 + 6 \qquad p \neq q$

$\therefore \quad x = 2p + p^3 - p^3 - pq^2 - 6p$

$= -\cancel{4q} - 4p - pq \cdot q$

$= -4p - 4q$

$= -4(p+q)$

$N \equiv \left(-4(p+q),\ p^2 + q^2 + 6\right)$ ~2

(iii) $x = -4(p+q)$

$x^2 = 16\left[p^2 + q^2 + 2pq\right]$

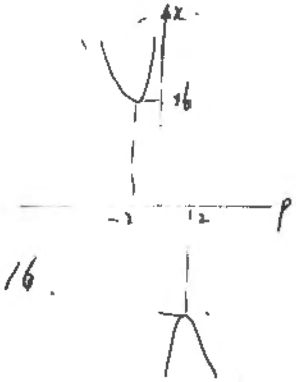
$= 16\left[p^2 + q^2 + 8\right]$

$= 16\left[y - 6 + 8\right]$

$x^2 = 16(y + 2)$

Restriction $x = -4\left(p + \frac{4}{p}\right)$

Restriction $x \geq 16$ or $x \leq -16$.



4(i) $AB = \frac{h}{\tan 10^\circ} \qquad BC = \frac{h}{\tan 8^\circ}$

$AC^2 = AB^2 + BC^2 - 2AB.BC \cos\angle ABC$

$30^2 = \frac{h^2}{\tan^2 10^\circ} + \frac{h^2}{\tan^2 8^\circ} - \frac{2h^2 \cos 100^\circ}{\tan 10^\circ \tan 8^\circ}$

$= h^2\left[\frac{\tan^2 8^\circ + \tan^2 10^\circ - 2\tan 8^\circ \tan 10^\circ \cos 100^\circ}{\tan^2 10^\circ \tan^2 8^\circ}\right]$

$h = \frac{30 \tan 8^\circ \tan 10^\circ}{\sqrt{\tan^2 8 + \tan^2 10^\circ - 2\tan 8^\circ \tan 10^\circ \cos 100^\circ}}$

$= 3.05$ m.

(7)

(a) $\angle DAC = \angle ABC$ (Angle between chord & Tangent)

$= \alpha$

$\therefore \angle DCA = 180^\circ - \alpha - (90^\circ - 2\alpha)$ (Angle sum ΔADC)

$= 90 + \alpha$.

$\therefore \angle BAC = 90^\circ$ (Exterior angle ΔABC)

$\therefore$ BC is diameter (angle semi circle)

(b)(i) $y = \dfrac{x^2 - 3x + 2}{x^2 + 4x + 3}$

$$y' = \frac{(x^2+4x+3)(2x-3) - (x^2-3x+2)(2x+4)}{(x^2+4x+3)^2}$$

$$= \frac{2x^3 - 3x^2 + 8x^2 - 12x + 6x - 9 - [2x^3 + 4x^2 - 6x^2 - 12x + 4x + 8]}{(x^2+4x+3)^2}$$

$$= \frac{+5x^2 - 6x - 9 - [-2x^2 - 8x + 8]}{(x^2+4x+3)^2}$$

$$= \frac{7x^2 + 2x - 17}{(x^2+4x+3)^2}$$

Turning points $y' = 0$.

$7x^2 + 2x - 17 = 0$

$$x = \frac{-2 \pm \sqrt{4 + 4 \times 7 \times 17}}{14}$$

$$= \frac{-1 \pm \sqrt{120}}{7}$$ ①.

$$= \frac{-1 \pm 2\sqrt{30}}{7}$$ $x = -1.71$ or 1.42

$y = -10.98$ $\quad -0.023$

(8)

For nature turning points test gradient since $\frac{dy}{dx}$ is continuous except $x=-1$ & $x=-3$

Test $x = -1.71$

x	-1.8	-1.71	-1.6
$\frac{dy}{dx}$	2.26	0	-3.23

①

relative max at $(-1.71, -10.98)$

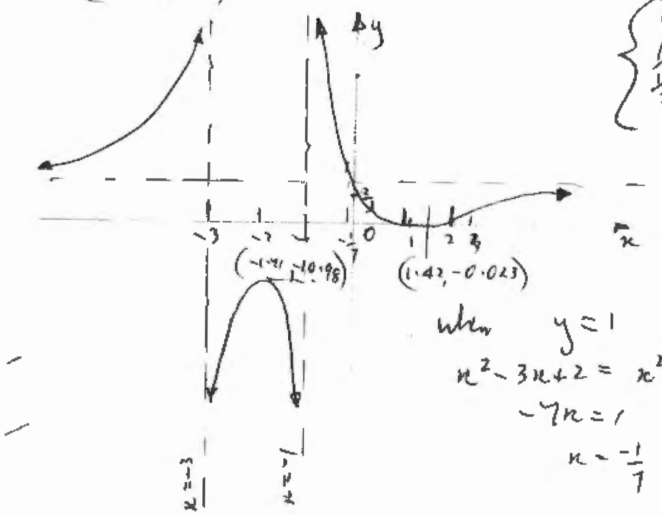
Test $x = +1.42$.

x	1.3	1.42	1.5
$\frac{dy}{dx}$	-0.03	0	0.01

①

relative min at $(1.42, -0.023)$

(ii) $y = \dfrac{(x-1)(x-2)}{(x+3)(x+1)}$



1 - Asymptotes
1/2 Roots
1/2 y intercept
1 shape

when $y = 1$

$x^2 - 3x + 2 = x^2 + 4x + 3$

$-7x = 1$

$x = -\frac{1}{7}$