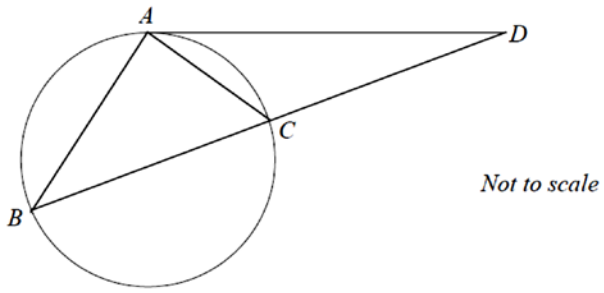
Answer question 1 to 5 on the Multiple Choice answer sheet

Question 1

The point A has coordinates (-6,4) and the point B has coordinates (5,1). Find the coordinates of the point which divides AB internally in the ratio 3:4

(A) (-39,13) (B) $\left(-\frac{12}{7},\frac{23}{7}\right)$ (C) $\left(-\frac{9}{7},\frac{19}{7}\right)$ (D) $\left(\frac{2}{7},\frac{16}{7}\right)$

Question 2



BC is the diameter of the circle. A is a point on the circle. The tangent at A meets BC produced at D. $\angle DAC = 35^o$. What is the size of of $\angle BDA$

(A) 10^o (B) 15^o (C) 20^o (D) 25^o

Question 3

P(x) is an odd polynomial. When P(x) is divided by (x-2) the remainder is 5.

What is the remainder when P(x) is divided by (x+2)

(A) -5 (B) -5x (C) 5x (D) 5

Question 4

Which of the following is an expression for $\frac{1}{1-\tan x} - \frac{1}{1+\tan x}$?

(A) $\frac{2\tan x}{\sec^2 x}$ (B) tan2x (C) $\frac{\tan 2x}{\tan x}$ (D) tanxtan2x

Question 5

At Euclid High School the Year 12 grade consists of n boys and n girls

A committee of 4 is chosen from Year 12 students

How many different committees can be formed containing 2 boys and 2 girls?

(A) $n^2(n^2-2n+1)$ (B) $\frac{n^2(n^2-2n+1)}{4}$ (C) n^2-n (D) $\frac{n^2-n}{2}$

Question 6 Start a New Sheet of Paper (20 marks)

(a) Six students are to be seated in a row on the stage for an assembly.
How many ways can they be placed if:

(i) There are no restrictions on where they sit? 1

(ii) Two particular students insist on sitting next to each other? 1

(iii) Two particular students refuse to sit next to each other? 1

(b) A serial number is made up of 4 letters followed by 2 numbers if zero isn't allowed how many serial numbers are there. 1

(c) Let P(x) = $2x^3 - 3x^2 - 3x + 2$

Express P(x) as a product of its 3 linear factors 3

(d) What is the general solution to the equation $2\sin^2\theta + 5\cos\theta + 1 = 0$
Give answer in radians 3

(e) (i) Show that $\tan 75^\circ = 2 + \sqrt{3}$ 2

(ii) The lines y = mx and $x = y\sqrt{3}$ meet at an angle of 75°. Find the value(s) of m 3

(f) (i) Change $\frac{3\pi}{8}$ radians to degrees 1

(ii) Convert 109° to radians to 1 decimal place 1

(g) Solve the inequality $\frac{2x-5}{x-4} \geq x$ 3

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Question 7 Start a New Sheet of Paper (20 marks)

(a) How many different arrangements are possible of COONABARABRAN if all the letters are used. 2

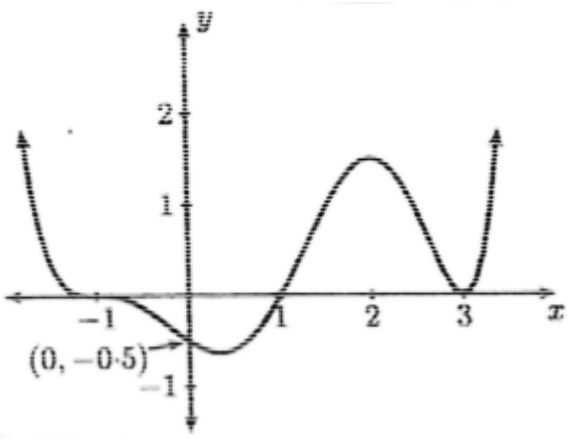
(b) A committee of 5 is to be chosen from 5 boys and 7 girls.
Find how many committees are possible

(i) If there are no restrictions? 1

(ii) A particular boy is to be on the committee? 1

(iii) There is a majority of boys? 3

(c)



Write down a possible function for the polynomial function sketched above. (Do NOT use calculus) 2

(d) Given that α, β, γ are the roots of $2x^3 + 3x^2 + 4x + 5$ find the value of $\alpha^2 + \beta^2 + \gamma^2$ 3

(e) Solve the equation $3\sin\theta - \cos\theta = 1$ for $0° \leq \theta \leq 360°$
Give the answer to the nearest minute. 4

(f) Solve the equation cos2x+sin2x+1=0 for $0 < x < 2\pi$ 4

Question 8 Start a New Sheet of Paper (20 marks)

(a) A group consisting of two adults, two boys and two girls is to be seated at a round table. The adults are to be seated together. The girls and boys are to sit in alternating positions. How many different seating arrangements are possible? 2

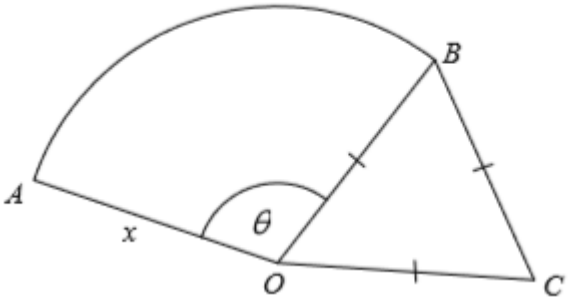
(b) How many different arrangements of the word MAMMOTH can be made if only five letters are used? 3

(c) Prove ($p^2 - q^2)(p^4 - q^4) \leq (p^3 - q^3)^2$ for all real p and q 2

(d) (i) Graph y = cos2x and y = x on the same axes between $-\pi \leq x \leq \pi$ 3

(ii) How many solutions to cos2x = x are there if $-\pi \leq x \leq \pi$ 1

(e) The diagram shows a sector OAB of a circle, centre O and radius x metres. Arc AB subtends an angle of θ radians at O. An equilateral triangle BCO adjoins the sector



(i) Write an expression in terms of θ and x for

(α) the perimeter of OABC. 1

(β) the area of OABC. 2

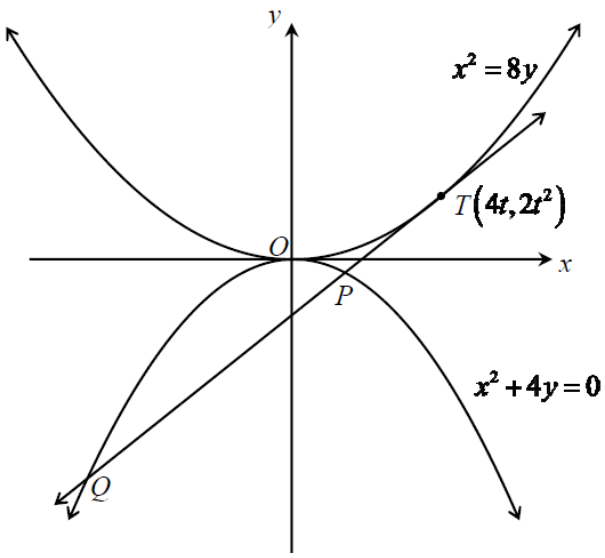
(ii) Given that the perimeter has the value $(12 - 2\sqrt{3})$ metres, show that the area A is given by 2

$$A=\frac{(6-\sqrt{3})^2(2\theta+\sqrt{3})}{(\theta+3)^2}$$

(iii) For what value of θ is the area a maximum? Justify your answer. 4

Question 9 Start a New Sheet of Paper (20 marks)

(a) Consider the point T(4t,$2t^2$) on the parabola x^2=8y



(i) Show that the equation of the tangent at T has the equation y-tx+$2t^2$=0 2

(ii) The tangent meets the parabola $x^2 + 4y = 0$ at two points P (x_1, y_1) and $Q(x_2, y_2)$ as shown. Show that x_1 and x_2 are the roots of the quadratic equation $x^2 + 4tx - 8t^2 = 0$ 2

(iii) Write an expression for $\frac{x_1+x_2}{2}$ 1

(iv) If M(x,y) is the midpoint of PQ ,find the coordinates of M in terms of t. 2

(v) Find the locus of M as T varies 1

Question 9 continued on next page

Question 9 continued

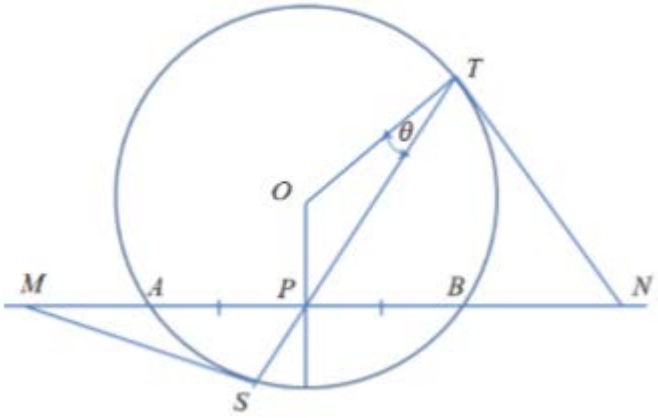
(b) The elevation of a tower T at a place P due east of it is 42°. At a place Q due south of P the elevation is 33°. The distance from P to Q is 450 metres.

(i) Draw a diagram labelling all the information given 1

(ii) Prove $h = \frac{450}{\sqrt{cot^2 33^\circ - cot^2 42^\circ}}$ where h =height of the tower 2

(iii) Find the height of the tower to 2 decimal places 1

(c)



In the diagram above, P is the midpoint of the chord AB in the circle with centre O. A second chord ST passes through P and the tangents at the endpoints meet AB produced at M and N respectively. Join OS.

(i) Explain why OPNT is a cyclic quadrilateral.

(ii) Explain why OPSM is also cyclic. 3

(iii) Let θ =∠ OTS . Show that θ =∠OMP=∠ ONP 2

(iv) Hence, prove that AM = BN. 2

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END OF EXAMINATION