Name ______________________________

Class ____________

YEAR 11

ASSESSMENT TASK 3

2019

MATHEMATICS EXTENSION 1

General Instructions	• **Reading time – 5 minutes** • **Working time – 2 hours** • **Write using black pen** • **Calculators approved by NESA may be used** • **A separate reference sheet is provided** • **For questions in Section II, show relevant mathematical reasoning and/or calculations**
Total marks: 85	**Section I – 5 marks** • **Attempt Questions 1–5** • **Allow about 7 minutes for this section** **Section II – 80 marks** • **Attempt Questions 6-9** • **Allow about 1 hours and 53 minutes for this section**

The answers to all questions are to be returned in separate *stapled* bundles, clearly marked with Question 6, Question 7, etc., with your student number.

Question 1

Which set provides a solution to the inequality $|3x-2| \geq |x+4|$?

A. $\{x\colon -\frac{1}{2} \leq x \leq 3\}$
B. $\{x\colon\ x \leq -\frac{1}{2}$ or $x \geq 3\}$
C. $\{x\colon\ x \leq -\frac{1}{2}\}$
D. $\{x\colon\ x \leq 3\}$

Question 2

Let $P(x) = x^3 + 3x^2 + ax + b$, where a and b are integers. When $P(x)$ is divided by $(x+1)$, the remainder is 4. When $P(x)$ is divided by $(x-1)$, the remainder is -2.

What is the remainder when $P(x)$ is divided by (x^2-1)?

A. $4x+2$
B. $-4x-2$
C. $3x-1$
D. $-3x+1$

Question 3

The cubic equation $x^3 + px^2 + qx + r = 0$, where p, q and r are integers, has roots α, β and γ, such that $\alpha + \beta + \gamma = 15$ and $\alpha^2 + \beta^2 + \gamma^2 = 83$.

What is the value of $p+q$?

A. 56 B. -56 C. -86 D. 86

Question 4

A team of six students is to be formed from a class of ten students. How many different teams can be formed if two particular students cannot both be selected for the team?

A. 252 B. 210 C. 168 D. 140

Question 5

What does the expression $\dfrac{\cos 3x - \cos 5x}{\sin 3x + \sin 5x}$ simplify to?

A. $\cot x$
B. $-\tan x$
C. $\tan x$
D. $-\cot x$

Question 6 (20 Marks) **START A NEW PAGE** **Marks**

(a) Find the exact value of: $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{5}{13}\right)$ **2**

(b) If $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{\sin\theta}{1-\cos\theta}=4,$ find the exact value of $\cot 2\theta$ for $0 \le \theta \le \frac{\pi}{2}$. **4**

(c) Show that $\cos 37\frac{1}{2}^{\circ} \sin 7\frac{1}{2}^{\circ} = \frac{\sqrt{2}-1}{4}$, without using a calculator. **2**

(d) Sketch the graph of $y = \frac{1}{2}\cos^{-1} 2x$. **2**

(e) Consider the function $f(x) = \frac{x-1}{x-2}$

(i) Show that $f^{-1}(x) = \frac{2x-1}{x-1}$ **1**

(ii) Sketch f and f^{-1} on the same system of axes, showing any asymptotes and intercepts. **3**
Clearly label your graphs.

(f) The speed, $V\ m\ s^{-1}$, of a parachute, t seconds after jumping from an aeroplane is modelled by the equation

$$V = 42\left(1 - \frac{1}{e^{\frac{t}{\pi}}}\right).$$

(i) Find the acceleration of the parachutist after $\boldsymbol{\pi}$ seconds, to two decimal places. **2**

(ii) The parachute opens when it reaches a speed of $21\ m\ s^{-1}$. **2**

Find the exact time of falling before the parachute opens?

(g) Show that among the 900 students at James Ruse, at least 3 students share a birthday. **2**

Question 7 (20 Marks) **START A NEW PAGE** **Marks**

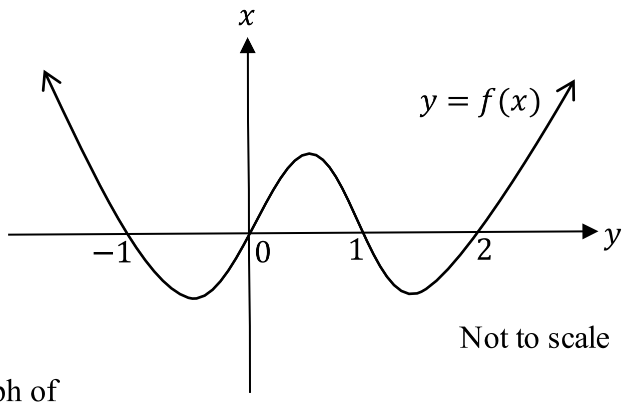
(a) For what values of x is the inequality $\frac{x}{x+1} \geq \frac{2}{x+3}$ satisfied? **4**

(b) Find the values of a and b if $2x^3 - (2a+1)x^2 + (2+b)x - 1 = 0$ has a multiple root at $x = 1$. **3**

(c) The parametric equations of a curve are $x = \ln(1+t^2)$ and $y + 1 = \ln(1+2t^2)$. **4**

Find the cartesian equation of the curve, and hence show that the x −intercept of the curve is $\ln\left(\frac{1+e}{2}\right)$.

(d) The graph of $y = f(x)$ is shown.



Not to scale

On separate systems of axes, draw the graph of

(i) $y = f(|x|)$ **2**

(ii) $y = \sqrt{(f(x))^2}$ **2**

(e) From seven girls and five boys, a committee of seven is to be chosen. **2**
What is the probability of choosing a committee containing at least four girls?

(f) How many people would have to be in a school before it contained at least two people with the same first and last initials. **3**

Question 8 (20 Marks) **START A NEW PAGE** **Marks**

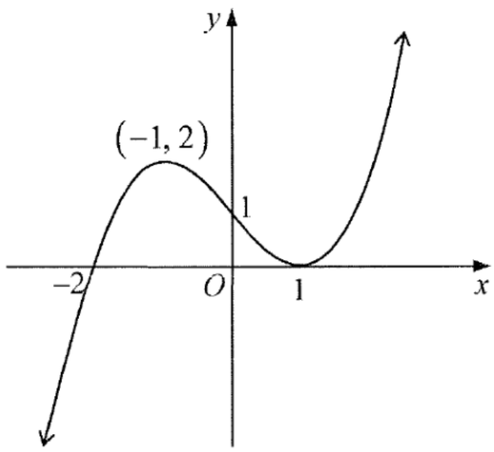
(a) Solve for x and y: **3**

$$2\tan^{-1}x - \cos^{-1}y = \frac{\pi}{2}$$

$$3\cos^{-1}y + \tan^{-1}x = \frac{5\pi}{6}$$

(b) Solve $\cos x - \cos 3x = 0$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. **3**

(c) The graph of $y = f(x)$ is drawn below.



Draw a separate half-page graph for each of the following functions, showing all important features, asymptotes and intercepts.

(i) $y^2 = f(x)$ **3**

(ii) $y = \frac{1}{f(x)}$ **3**

(d) At a particular dinner, each rectangular table has nine seats, five facing the stage and four with their backs to the stage.

(i) In how many ways can 9 people be seated at the table if John and Mary sit on the same side? **2**

(ii) What is the probability of John and Mary sitting on opposite sides of the table? **3**

(e) 10 points are placed randomly in a 1 by 1 square. Show that there must be some pair of points that are within $\frac{\sqrt{2}}{3}$ of each other. **3**

Question 9 (20 Marks) **START A NEW PAGE** **Marks**

(a) (i) If $t = \tan\theta$, show that $\tan 4\theta = \frac{4t(1-t^2)}{1-6t^2+t^4}$ **3**

(ii) Given the roots of $\tan 4\theta = \cot\theta$ are $\theta = \frac{\pi}{10}$ and $\theta = \frac{3\pi}{10}$. **3**

Find the exact value of $\tan\frac{\pi}{10}$.

(b) If $\alpha, \beta\ and\ \gamma$ are the roots of $3x^3 + 8x^2 - 1 = 0$, find the value of **3**

$\left(\beta + \frac{1}{\gamma}\right)\left(\gamma + \frac{1}{\alpha}\right)\left(\alpha + \frac{1}{\beta}\right)$.

(c) A metal rod is taken from a freezer at -8^0C into a room where the air temperature is 22^0C. The rate at which the rod warms follows Newton's law, that is $\frac{dT}{dt} = -k(T-22)$ where k is a positive integer, time t is measured in minutes and temperature T in ^0C.

(i) Show that the function $T = 22 - Ae^{-kt}$, where A is a constant, provides this rate of change. **1**

(ii) Hence find the value of A **2**

(iii) The temperature of the rod reaches 4^0C in 90 minutes. Find the exact value of k. **2**

(iv) Find the temperature of the rod after another 90 minutes. **1**

(d) Suppose a particular population of bacteria obeys the growth formula $P(t) = \frac{6000}{3+\ 7\,e^{-0.2\,t}}$ where P is measured in milligrams and time t, in hours.

(i) Predict what the population will be as t gets very large. **1**

(ii) If the population grows the fastest when $P(t) = 1000$, find when this occurs, to 4 significant figures. **2**

(c) Sketch the graph of $P(t)$, showing all important features. **2**

END OF EXAMINATION

Question	6		7		8		9		Total
Functions		/4		/18		/6		/3	/31
Combinatorics		/2		/2		/8		-	/12
Trigonometric Functions		/10		-		/6		/6	/22
Calculus		/4		-		-		/11	/15
MCQ									/5
Total	/5	/20		/20		/20		/20	/85

Q1. B **Q2.** D **Q3.** A **Q4**. D **Q5**. C

Justification for Q4: Students: X X X X X X X X A B
If neither A or B: 28 possibilities (8C6 from non A and B)
If A or B: 112 possibilities (8C5 x 2)

Question 6

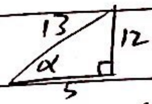
a) $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{5}{13}\right)$

$= \sin\left(\frac{\pi}{6} + \cos^{-1}5/13\right)$

$= \sin\frac{\pi}{6} \cdot \cos\left(\cos^{-1}5/13\right) + \sin\left(\cos^{-1}5/13\right) \cdot \cos\pi/6$

1 m $= \frac{1}{2} \cdot \frac{5}{13} + \frac{12}{13} \cdot \frac{\sqrt{3}}{2}$

1 m $= \frac{5+12\sqrt{3}}{26}$.



$\alpha = \cos^{-1}\frac{5}{13}$

b) $\frac{\sin\theta}{1+\cos\theta} + \frac{\sin\theta}{1-\cos\theta} = 4$

$\frac{\sin\theta - \sin\theta\cos\theta + \sin\theta + \sin\theta\cos\theta}{1-\cos^2\theta} = 4$

1 m domain restriction $\cos\theta \neq 1 \therefore \theta \neq 0$ (or similar)

$\frac{2\sin\theta}{\sin^2\theta} = 4$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

1m $\sin\theta = 0$ or $\sin\theta = \frac{1}{2}$ (for domain, $\theta = 0°, \frac{\pi}{6}$)

$\therefore \cot 2\theta = \frac{1}{\tan 2\theta} = \frac{1-\tan^2\theta}{2\tan\theta}$

1 m $= \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{2/\sqrt{3}}$

1 m $= \sqrt{3}/3$ only as $\cot 2\theta$ undefn for $\theta = 0$

Student Number -Page 5-

(c) $\cos 37\frac{1}{2}^\circ \cdot \sin 7\frac{1}{2}^\circ$

1m $= \frac{1}{2}[\sin 45 - \sin 30]$

$= \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{\sqrt{2}-1}{2}\right)$

1 m $= \frac{1}{4}(\sqrt{2}-1)$

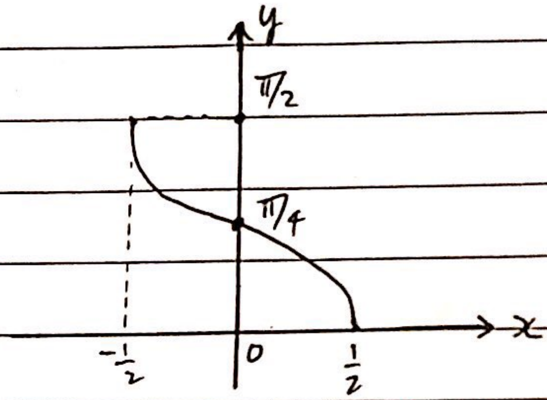
d) $2y = \cos^{-1} 2x$

1 m
Range: $0 \le 2y \le \pi$

$0 \le y \le \frac{\pi}{2}$

Domain: $-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$



1 m for shape

Note vertical tangents at $x = \pm\frac{1}{2}$

e) (i) $x = \frac{y-1}{y-2}$

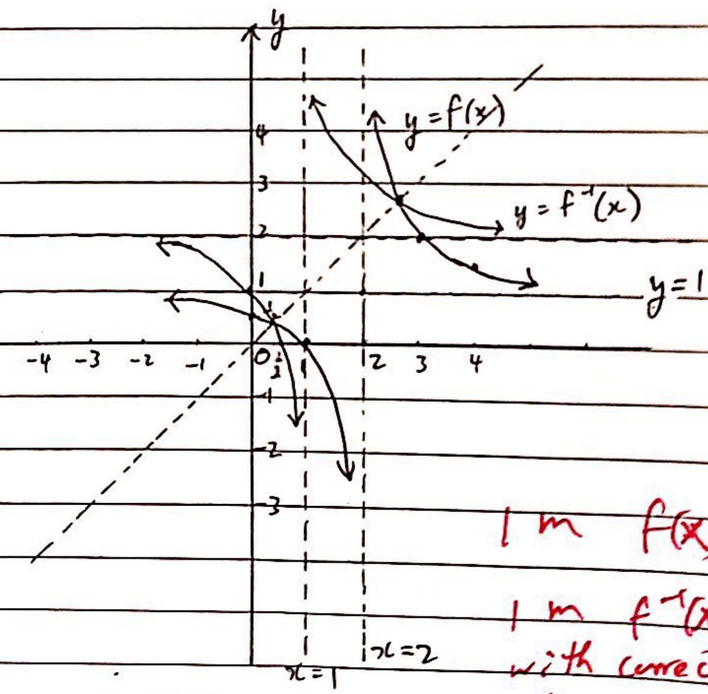
$xy - 2x - y + 1 = 0$

$y(x-1) = 2x-1$

1 m $\therefore y = \frac{2x-1}{x-1}$

$\therefore f^{-1}(x) = \frac{2x-1}{x-1}$

(ii)



1 m $f(x)$

1 m $f^{-1}(x)$ with correct intercepts & asymptotes

1 m symmetrical about $Y = x$

f) $V = 42\left(1 - e^{-t/\pi}\right)$

1 m (i) $a = \frac{dV}{dt} = \frac{42}{\pi} e^{-t/\pi}$ v. poorly done

$\therefore a(\pi) = \frac{42}{\pi} e^{-1}$

$= \frac{42}{\pi e}$

$= 4.9181\ldots$ Need to be in

1 m $= 4.92 \text{ m/s}^2$ 2 dp.

1 m (ii) $21 = 42\left(1 - e^{-t/\pi}\right)$

$-e^{-t/\pi} = -\frac{1}{2}$

$\therefore t/\pi = \ln 2$ Some students

1 m $\therefore t = \pi \ln 2$ sec. continue to give

decimal places

won't get the second

mark

1. (g) Student Number

If there are 366 students and 366 days, then no student may share a birthday

However with 1 more, at least 2 students may.

Hence if $2 \times 366 = 732$ students at least 2 may.

1m However with 1 more than 732, at least 3 may

So with 900 students at least 3 may share a birthday, by the PHP.

[Similar argument if 365 days in a year are considered.

OR From $n = dq + r$,

$$900 = 2 \times 366 + 168$$

By PHP, at least $(2+1)$ students may share a birthday

MATHEMATICS Extension 1 : Question 7

Suggested Solutions	Marks	Marker's Comments

a) $\dfrac{x}{x+1} \geq \dfrac{2}{x+3}$

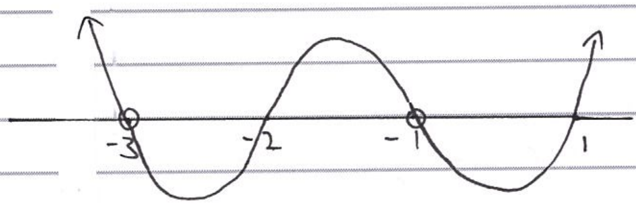
$x(x+1)(x+3)^2 \geq 2(x+3)(x+1)^2$ — 1

$x(x+1)(x+3)^2 - 2(x+3)(x+1)^2 \geq 0$

$(x+1)(x+3)(x^2+3x-2x-2) \geq 0$

$(x+1)(x+3)(x^2+x-2) \geq 0$

$(x+1)(x+3)(x+2)(x-1) \geq 0$ — 1



Since $x \neq -3$ or -1

$x<-3$ or $-2 \leq x < -1$ or $x \geq 1$ — 2

1 Inequality without the restrictions

2 correct answer with restrictions

or

$\dfrac{x}{x+1} \geq \dfrac{2}{x+3}$

$\dfrac{x}{x+1} - \dfrac{2}{x+3} \geq 0$

$\dfrac{x^2+3x-2x-2}{(x+1)(x+3)} \geq 0$

$\dfrac{x^2+x-2}{(x+1)(x+3)} \geq 0$ — 1

$\dfrac{(x+2)(x-1)}{(x+1)(x+3)} \geq 0$ — 1

Since LHS has the same sign as $(x+3)(x+2)(x+1)(x-1)$ then draw polynomials like before

Anyone who multiplied both sides by terms that are not necessarily positive to begin with, will receive 2 maximum provided they did everything else correctly.

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments
a) Continued.		
Don't square both sides !!!		
When solving $a^2 \geq b^2$ you are solving for		
$a > b$ (when a is positive) AND		
$a < b$ (when a is negative)		
b) let $f(x) = 2x^3 - (2a+1)x^2 + (2+b)x - 1$		
$f'(x) = 6x^2 - (4a+2)x + (2+b)$		
$f'(1) = 0 \Rightarrow 6(1)^2 - (4a+2)(1) + (2+b) = 0$		
$6 - 4a - 2 + 2 + b = 0$		
$4a - b = 6$ – Eqn 1		1
$f(1) = 0 \Rightarrow 2(1)^3 - (2a+1)(1)^2 + (2+b)(1) - 1 = 0$		
$2 - 2a - 1 + 2 + b - 1 = 0$		
$2a - b = 2$ – Eqn 2		1
Eqn 1 – Eqn 2 $\rightarrow$ $2a = 4$		If someone did the exact same steps but made an algebraic error in each of the 3 steps, they still received 1/3.
$a = 2$		
Sub $a = 2$ into Eqn 1.		
$2(2) - b = 2$		
$4 - b = 2$		
$b = 2$		
$\therefore a = 2$		1
$b = 2$		

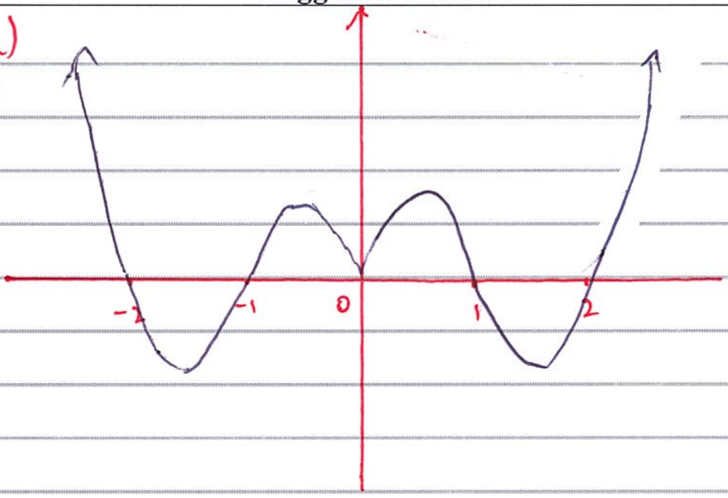
MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments
c) $x = \ln(1+t^2)$ $\quad y+1 = \ln(1+2t^2)$		
$1+t^2 = e^x$ $\quad 1+2t^2 = e^{y+1}$		① Raising e to the power of both sides
$t^2 = e^x - 1$ $\quad 2t^2 = e^{y+1} - 1$		
$t^2 = \dfrac{e^{y+1}-1}{2}$		① Make t^2 the subject for both equations
$\therefore e^x - 1 = \dfrac{e^{y+1}-1}{2}$		
$2e^x - 2 = e^{y+1} - 1$		
$2e^x - e^{y+1} - 1 = 0$		① Simplifying the constants from both sides
When $y=0$		
$2e^x - e - 1 = 0$		
$2e^x = e+1$		
$e^x = \dfrac{e+1}{2}$		
$x = \ln\left(\dfrac{e+1}{2}\right)$		① for getting to the result without skipping steps
Note: Cartesian equation can be simplified to $y = \ln(2e^x - 1) - 1$ but not necessary.		

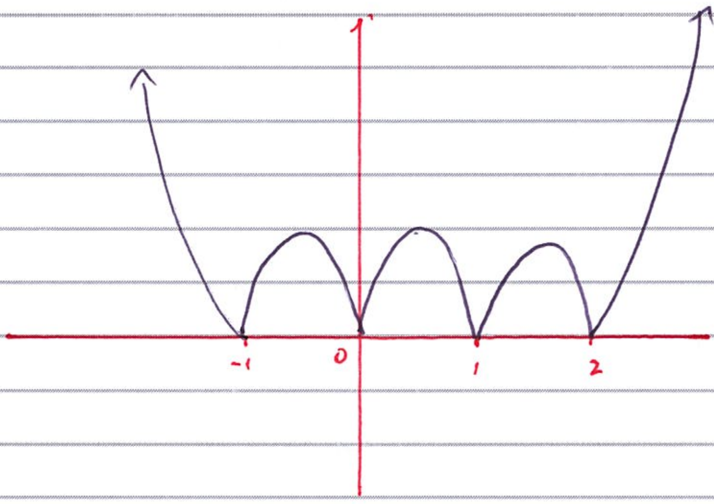
MATHEMATICS Extension 1 : Question……..		
Suggested Solutions	**Marks**	**Marker's Comments**

d)

i)

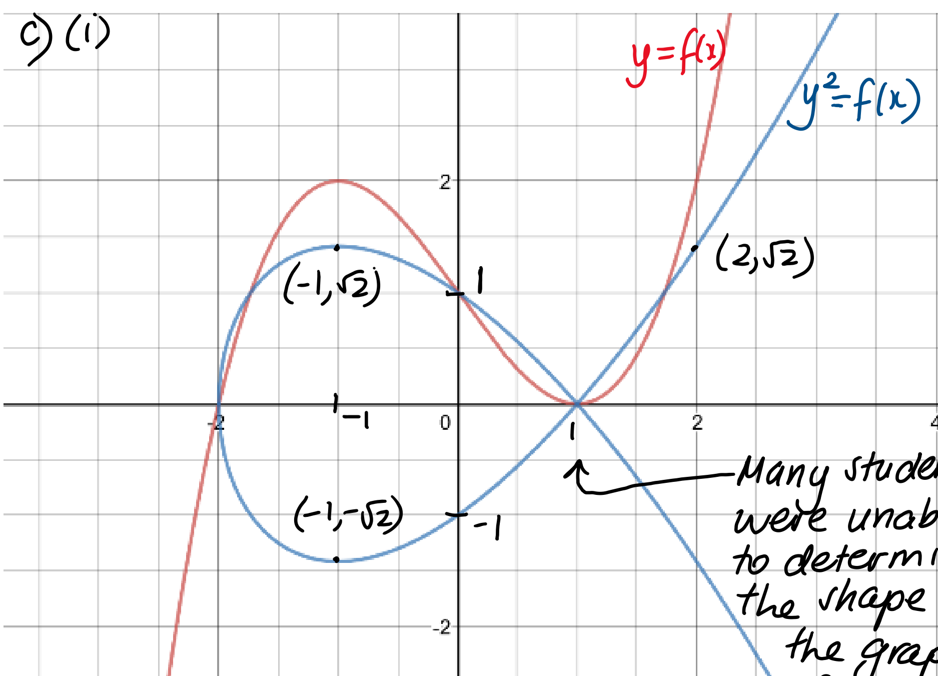


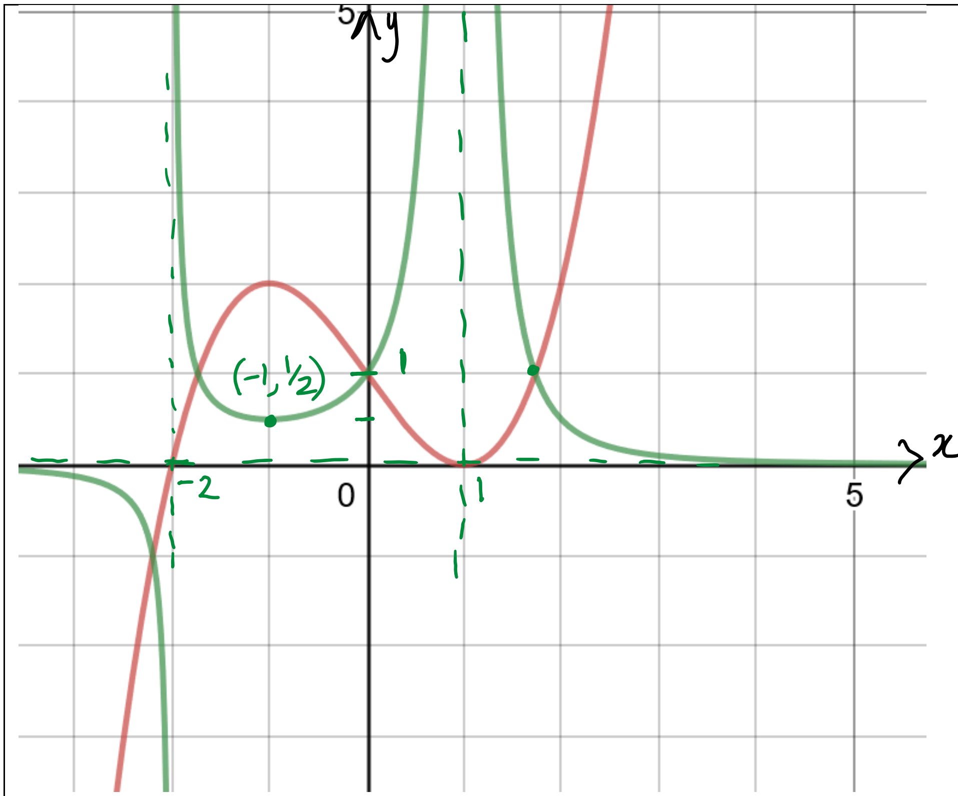
ii)



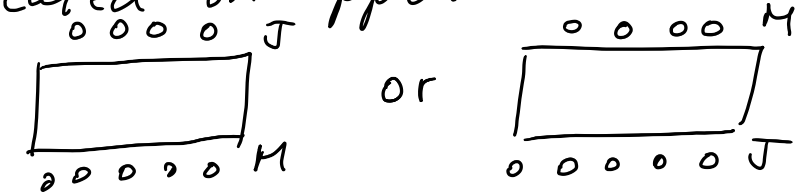
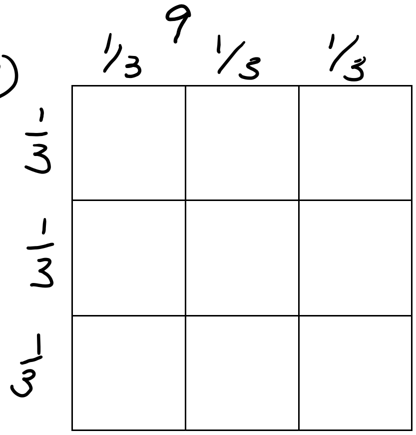
① Mark subtracted per mistake

MATHEMATICS Extension 1 : Question……..		
Suggested Solutions	**Marks**	**Marker's Comments**
e) Total = N(4 girls) + N(5 girls) + N(6 girls) + N(7 girls)		*Students also received 1 mark for
$= ({}^7C_4 \times {}^5C_3) + ({}^7C_5 \times {}^5C_2) + ({}^7C_6 \times {}^5C_1) + ({}^7C_7 \times {}^5C_0)$		$P = \dfrac{(4\text{ girls}) + (5\text{ girls}) + (6\text{girls}) + (7\text{girls})}{{}^{12}C_7}$
$= 350 + 210 + 35 + 1$		
$= 596$		1
N(no restrictions) $= {}^{12}C_7 = 792$		
$\therefore$ Probability (≥ 4 girls) $= \dfrac{596}{792} = \dfrac{149}{198}$		1
f) There are 26 letters in the alphabet		
$\therefore$ There are 26^2 different initials combinations		1
$\therefore$ Number of students required		
$= 26^2 + 1$		
$= 676 + 1$		1
$= 677$	1	

MATHEMATICS Extension 1: Question 8___		
Suggested Solutions	**Marks**	**Marker's Comments**
a) $2\tan^{-1}x - \cos^{-1}y = \pi/2$ (1)		
$2\tan^{-1}x + 6\cos^{-1}y = 5\pi/3$ (2)		
(2) − (1) $7\cos^{-1}y = 7\pi/6$	1	eliminating x
$\cos^{-1}y = \pi/6$		
$\therefore y = \sqrt{3}/2$	1	exact value of y
Sub into (1):		
$2\tan^{-1}x = \pi/2 + \pi/6$		
$\tan^{-1}x = \pi/3$		
$\therefore x = \sqrt{3}$	1	exact value of x
b) $\cos x - \cos 3x = 0$		
$\cos(2x - x) - \cos(2x + x) = 0$		
$2\sin 2x \cdot \sin x = 0$	1	identity
$\therefore \sin 2x = 0$ or $\sin x = 0$		
$2x = 0, \pi, -\pi$ or $x = 0$	1	$x = 0$
$\therefore x = 0$ or $\pm\pi/2$	1	$x = \pm\pi/2$
c) (i)  Many students were unable to determine the shape of the graph for $x \geq 1$	1 1 1	symmetry about x-axis $(-1, \pm\sqrt{2})$ intercepts at $x = 1, -2$ $y = 1, -1$ shape particularly at $x = 1$ and as $x \to \infty$

	1 1 1	asymptotes and y-intercept of 1 (−1, ½) as a minimum shape and axes labelled Mostly well done.
d) facing stage: J has 5 to choose from M has 4 to choose from backs to stage: J has 4 to choose from M has 3 to choose from other 7 arranged in 7! ways ∴ J&M can be seated in $4 \times 3 \times 7! + 5 \times 4 \times 7!$ $= 7!(12+20)$ $= 161280$ ways	1 1	no. of ways facing stage no. of ways back to stage
(ii) Method 1: Total no. of arrangements $= 9!$ P(J&M on opposite sides) $= 1 - P(\text{J\&M on same side})$ $= 1 - \frac{161280}{9!}$ $= 5/9$	1 1 1	

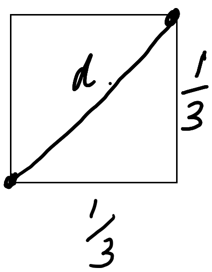
MATHEMATICS Extension 1: Question __8_

Suggested Solutions	Marks	Marker's Comments
Method 2: No. of ways J & M can be seated on opposite sides:  No. of ways $= 4\times5\times7! + 4\times5\times7!$ $= 40\times7!$ $= 201600$	1	
P(J & M on opposite sides) $= \frac{201600}{9!}$	1	total arrangements
$= \frac{5}{9}$	1	
e)  Divide the 1x1 square into 9 smaller squares of size $\frac{1}{3}\times\frac{1}{3}$. Let the 10 points be the pigeons	1	explanation of dividing 1x1 square into smaller squares of size $\frac{1}{3}\times\frac{1}{3}$ units
and the 9 smaller squares be the pigeonholes. Worst case scenario, 9 points are placed randomly inside the 1x1 square so that there is one point in each smaller square. When a 10th point is placed inside the square there must now be	1	Use of Pigeonhole principle: 10 pigeons into 9 pigeonholes

MATHEMATICS Extension 1: Question 8_

Suggested Solutions	Marks	Marker's Comments

one smaller square which contains 2 points. The furthest these 2 pts can be apart is when they are placed at either end of the diagonal in a $\frac{1}{3} \times \frac{1}{3}$ square



$d^2 = \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2$ by Pythagoras' Theorem

$= \frac{2}{9}$

$d = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$

$\therefore$ There will be 2 points within $\frac{\sqrt{2}}{3}$ of each other

Marks: 1

Marker's Comments: Pythagoras' Theorem with explanation and conclusion

Notes: Many students were not consistent in the language used, interchanging between squares/spaces/boxes/minisquare. Consistent language is necessary. Many students did not account for the points being placed randomly inside the 1x1 square and arranged the points around the square and then many fudged their answer. Many explanations need to be worded better.

MATHEMATICS Extension 1 : Question....9...

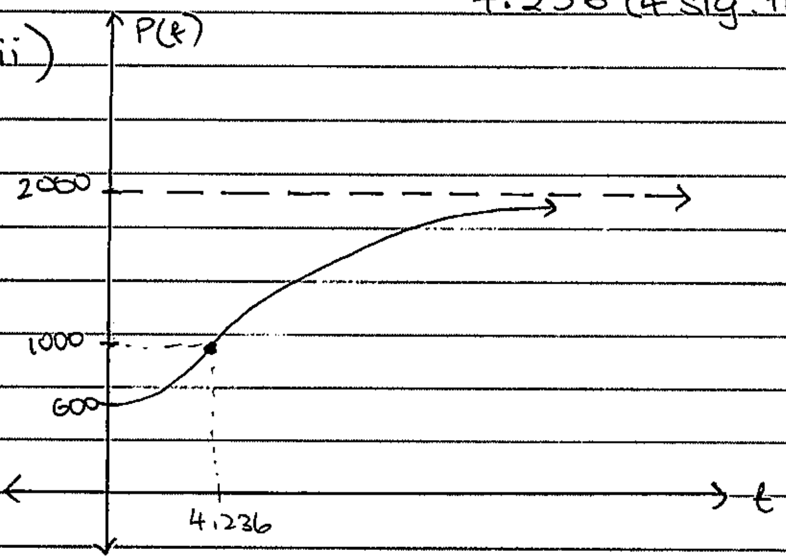
Suggested Solutions	Marks	Marker's Comments
a) i) RTP: $\tan 4\theta = \dfrac{4t(1-t^2)}{1-6t^2+t^4}$		
LHS $= \tan 4\theta$		
$= \dfrac{2\tan 2\theta}{1-\tan^2 2\theta}$	1	
$= \dfrac{2 \times \frac{2t}{1-t^2}}{1-\left(\frac{2t}{1-t^2}\right)^2}$		
$= \dfrac{\frac{4t}{1-t^2}}{1-\frac{4t^2}{(1-t^2)^2}}$	1	⇒ shown some working towards final step.
$= \dfrac{\frac{4t}{1-t^2}}{\frac{(1-t^2)^2-4t^2}{(1-t^2)^2}}$		
$= \dfrac{4t(1-t^2)^2}{(1-t^2)\left((1-t^2)^2-4t^2\right)}$		
$= \dfrac{4t(1-t^2)}{1-2t^2+t^4-4t^2}$	1	
$= \dfrac{4t(1-t^2)}{t^4-6t^2+1}$		
ii) $\tan 4\theta = \cot\theta$		
$\dfrac{4t(1-t^2)}{t^4-6t^2+1} = \dfrac{1}{t}$		
$4t^2(1-t^2) = t^4-6t^2+1$		
$4t^2-4t^4 = t^4-6t^2+1$		
$5t^4-10t^2+1=0$		
$t^2 = \dfrac{10 \pm \sqrt{100-4(5)}}{10}$		
$= \dfrac{10 \pm \sqrt{80}}{10}$		

MATHEMATICS Extension 1 : Question 9

Suggested Solutions	Marks	Marker's Comments
$\therefore \tan^2\theta = \dfrac{5 \pm 2\sqrt{5}}{5}$	1	
$\tan\theta = \pm\sqrt{\dfrac{5 \pm 2\sqrt{5}}{5}}$		
since $\theta = \frac{\pi}{10}$ is a solution, $\tan\frac{\pi}{10}$ is in the 1st quadrant	1	$\Rightarrow$ reasoning needed
$\therefore \tan\frac{\pi}{10} > 0$		
$\therefore \tan\frac{\pi}{10} = \sqrt{\dfrac{5 \pm 2\sqrt{5}}{5}}$		
Now, $\tan\theta$ is an increasing function for $0 \leq \theta < \frac{\pi}{2}$,		
$\therefore \tan\frac{3\pi}{10} > \tan\frac{\pi}{10}$		
$\therefore \tan\frac{\pi}{10} = \sqrt{\dfrac{5 - 2\sqrt{5}}{5}}$	1	$\Rightarrow$ correct solution!
b) $3x^3 + 8x^2 - 1 = 0$		
$\alpha + \beta + \gamma = -\frac{8}{3}$ $\alpha\beta + \alpha\gamma + \beta\gamma = 0$ $\alpha\beta\gamma = \frac{1}{3}$	1	
$\left(\beta + \frac{1}{\gamma}\right)\left(\gamma + \frac{1}{\alpha}\right)\left(\alpha + \frac{1}{\beta}\right)$		
$= \left(\dfrac{\gamma\beta + 1}{\gamma}\right)\left(\dfrac{\alpha\gamma + 1}{\alpha}\right)\left(\dfrac{\alpha\beta + 1}{\beta}\right)$		
$= \dfrac{1}{\alpha\beta\gamma}\left(\alpha\beta\gamma^2 + \beta\gamma + \alpha\gamma + 1\right)\left(\alpha\beta + 1\right)$		
$= \dfrac{1}{\alpha\beta\gamma}\left(\alpha^2\beta^2\gamma^2 + \alpha\beta\gamma^2 + \alpha\beta^2\gamma + \beta\gamma + \alpha^2\beta\gamma + \alpha\gamma + \alpha\beta + 1\right)$		
$= \dfrac{\alpha\beta\gamma + \alpha\beta\gamma(\alpha + \beta + \gamma) + \alpha\beta + \alpha\gamma + \beta\gamma + 1}{\alpha\beta\gamma}$	1	
$= \dfrac{\frac{1}{3} + \frac{1}{3}\left(-\frac{8}{3}\right) + 0 + 1}{\frac{1}{3}}$ $= \frac{2}{3}$	1	

MATHEMATICS Extension 1 : Question 9		
Suggested Solutions	Marks	Marker's Comments
c) i) $T = 22 - Ae^{-kt}$		
$\frac{dT}{dt} = (-k)(-Ae^{-kt})$		
$= (-k)(22 - Ae^{-kt} - 22)$	1	
$= (-k)(T - 22)$		
$\therefore T = 22 - Ae^{-kt}$ is a solution to $\frac{dT}{dt} = (-k)(T-22)$		
ii) when $t=0$, $T=-8$		
$-8 = 22 - Ae^{-k(0)}$	1	$\Rightarrow$ show substitution
$A = 22 + 8$		
$A = 30$	1	
iii) when $t = 90$, $T = 4$		
$\therefore 4 = 22 - 30e^{-k(90)}$	1	
$30e^{-90k} = 18$		
$e^{-90k} = \frac{18}{30}$		
$e^{-90k} = \frac{3}{5}$		
$-90k = \ln\frac{3}{5}$		
$k = \frac{-\ln\frac{3}{5}}{90}$		
or $k = \frac{\ln\frac{5}{3}}{90}$	1	
iv) when $t = 180$,		
$T = 22 - 30e^{-\frac{\ln\frac{5}{3}}{90}(180)}$		
$= 11.2$	1	
d) i) $P(t) = \frac{6000}{3+7e^{-0.2t}}$		
as $t \to \infty$, $e^{-0.2t} \to 0$		
$\therefore P(t) \to \frac{6000}{3}$		
$= 2000$		
$\therefore$ as $t \to \infty$, $P(t)$ approaches 2000	1	

MATHEMATICS Extension 1 : Question 9

Suggested Solutions	Marks	Marker's Comments
ii) when $P(t) = 1000$,		
$1000 = \dfrac{6000}{3+7e^{-0.2t}}$		
$3+7e^{-0.2t} = 6$		a lot of students wrote that
$7e^{-0.2t} = 3$	1	$6-3=2$ ✗
$e^{-0.2t} = \dfrac{3}{7}$		
$-0.2t = \ln\frac{3}{7}$		
$t = \dfrac{\ln\frac{3}{7}}{-0.2}$		
$= 4.236489\ldots$		
$= 4.236$ (4 sig. fig)	1	
iii) 		1 ⇒ asymptote and starting point 1. ⇒ having (4.236, 1000) as steepest point ie. POI, change in concavity must be shown