Set By: IS

**KNOX GRAMMAR SCHOOL**
MATHEMATICS DEPARTMENT

**2002**
**YEARLY EXAMINATION**

# Mathematics Extension 1
# Year 11

Total marks (**84**)

## General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- All necessary working should be shown in every question

- Attempt Questions 1–7
- All questions are of equal value
- Use a **SEPARATE** Writing Booklet for each question

If no attempt is made on a question, write the Question Number and your Student Number in the appropriate place on the cover, and write NIL ATTEMPT in the Mark Column.
HAND IN THIS BOOKLET.

NAME:______________________ TEACHER:______________________

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**Total marks (84)**
**Attempt questions 1 – 7**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

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**Question 1 (12 marks)** Use a SEPARATE writing booklet. **Marks**

(a) Solve $\dfrac{3}{x-2} > 4.$ **3**

(b) Factorise $a^2 - 4b^2 + 2a + 4b.$ **2**

(c) Solve $\sin 2\theta = \dfrac{\sqrt{3}}{2}$ for $0° \le \theta \le 360°.$ **2**

(d) If $x = 4 + 2\sqrt{3}$, simplify $x - \dfrac{1}{x}.$ **3**

(e) Using the fact that $\left(\sqrt{x}+\sqrt{y}\right)^2 = x + y + 2\sqrt{xy}$, express $11 + 4\sqrt{6}$ in the form $\left(\sqrt{x}+\sqrt{y}\right)^2.$ **2**

**Question 2 (12 marks)** Use a SEPARATE writing booklet.

(a) $f(x) = x^2 + 3x - 2.$ Simplify $\dfrac{f(h) - f(-h)}{2h}.$ **3**

(b) Find the inverse function of the function $y = \dfrac{2x-3}{x-1}$ in the form $y = f(x).$ **3**

(c) Solve $x^2 > 9x.$ **3**

(d) Solve $|x-2| = 2x + 4.$ **3**

**Question 3 (12 marks)** Use a SEPARATE writing booklet. **Marks**

(a) By drawing appropriate straight lines on the same axes as the graph of $y = 2^x$, determine the number of solutions for each of the following equations: **3**

(i) $2^x = x + 2$

(ii) $2^x = x - 2$

(iii) $2^x = 2 - x$

(b) Find $\lim_{x \to 2} \frac{x^2 - 4}{2x - 4}$ **2**

(c) If $\sin\theta = p$, where $\theta$ is acute, find expressions for $\cos\theta$ and $\tan\theta$ in terms of $p$. **2**

(d) Solve for $0° \le \theta \le 360°$, to the nearest degree: $4\sec^2\theta - \tan\theta = 7$. **5**

**Question 4 (12 marks)** Use a SEPARATE writing booklet.

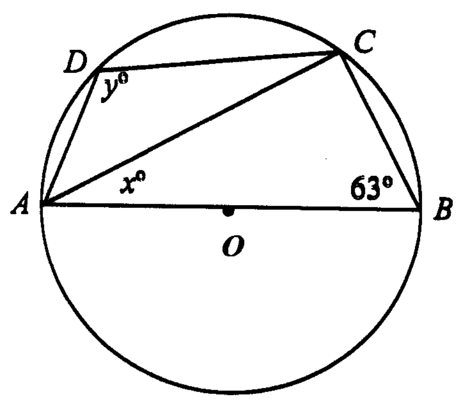
(a) Find the coordinates of the point that divides the join of (-3, 2) and (5, 7) **externally** in the ratio 1 : 3. **3**

(b) Find the perpendicular distance between the pair of parallel lines $3x - 4y + 12 = 0$ and $3x - 4y - 6 = 0$. **3**

[Hint: Find the coordinates of a point on one of the lines.]

(c) If $f(x) = \frac{x - 1}{\sqrt{2x + 1}}$, show that $f'(x) = \frac{x + 2}{\sqrt{(2x + 1)^3}}$. **4**

(d) **2**



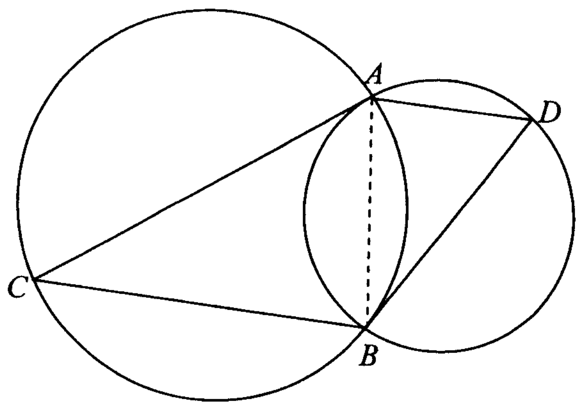

$ABCD$ is a cyclic quadrilateral, and $AB$ is the diameter of the circle, with centre $O$.
$\angle ABC = 63°, \angle CAB = x°$, and $\angle ADC = y°$.
Find the values of $x$ and $y$, giving reasons.

**Question 5 (12 marks)** Use a SEPARATE writing booklet. **Marks**

(a) Find the values of $a$, $b$, $c$ given that $x^2 + x + 1 \equiv a(x-1)^2 + b(x-1) + c$. **3**

(b) Find the maximum value of the function $f(x) = 8 + 4x - x^2$. **2**

(c) **3**




Two circles intersect at $A$ and $B$. $AC$ is a tangent to one circle at $A$ and $BD$ is a tangent to the other circle at $B$.
**Copy or trace** the diagram into your writing booklet.
Prove that $AD$ is parallel to $CB$.

(d) (i) Sketch the graphs of $y = |x^2 - 2|$ and $y = 3$ on the same number plane. **2**

(ii) Write down the inequalities that simultaneously represent the region bounded by the two graphs. **2**

**Question 6 (12 marks)** Use a SEPARATE writing booklet.

(a) Find the value of $m$ if the quadratic equation $x^2 + mx + (m+2) = 0$:

(i) has one root equal to 3. **2**

(ii) has its roots reciprocals of each other. **2**

(b) Prove that $x^2 + (k-3)x - k = 0$ has real roots for all real values of $k$. **3**

(c) (i) Find the equation of the tangent to the curve $y = x^2$ at the point $(a, a^2)$ on the curve. **2**

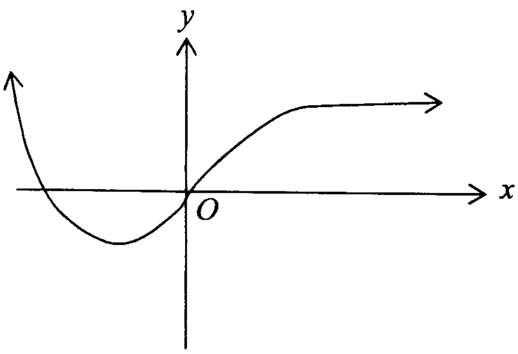
(ii) Find the values of $a$ if this tangent passes through the point (2, -5). **1**

(iii) Hence find the equations of the two tangents to the curve $y = x^2$ that pass through the point (2, -5). **2**

**Question 7 (12 marks)** Use a SEPARATE writing booklet. **Marks**

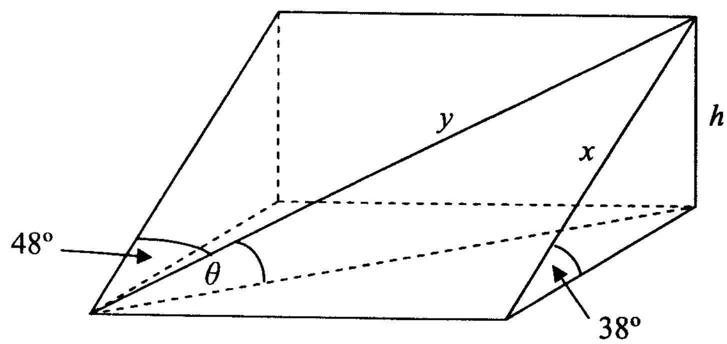
(a) Find the coordinates of the points on the curve $y = x(x-2)^3$ where the tangent to the curve is horizontal (i.e. has a gradient of zero). **4**

(b) **2**




The diagram shows the graph of $y = f(x)$.
**Copy or trace** this graph into your Writing Booklet.
On the same set of axes draw the graph of $y = f'(x)$.

(c)




A hill is inclined at an angle of 38° to the horizontal. A road on the hill makes an angle of 48° with a line of greatest slope.

(i) Show that the angle of inclination of the road to the horizontal ($\theta$) is given by: $\sin\theta = \sin 38° \cos 48°$. **2**

(ii) Hence find the value of $\theta$. **1**

(d) Sketch the graph of $|xy| \geq 4$ on a number plane. **3**

**End of Examination**

# YEAR 11 EXTENSION I MATHEMATICS YEARLY EXAM - 2002.

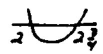
**1.(a)** $\frac{3}{x-2} > 4$

$3(x-2) > 4(x-2)^2$

$3x - 6 > 4x^2 - 16x + 16$

$4x^2 - 19x + 22 < 0$

$(4x-11)(x-2) < 0$




$2 < x < 2\frac{3}{4}$ ③

**(b)** $a^2 - 4b^2 + 2a + 4b$

$= (a-2b)(a+2b) + 2(a+2b)$

$= (a+2b)(a-2b+2)$ ②

**(c)** $\sin 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = 60°, 120°, 420°, 480°$

$\theta = 30°, 60°, 210°, 240°$ ②

**d)** $x = 4 + 2\sqrt{3}$

$\frac{1}{x} = \frac{1}{4+2\sqrt{3}} \times \frac{4-2\sqrt{3}}{4-2\sqrt{3}}$

$= \frac{4-2\sqrt{3}}{4}$

$x - \frac{1}{x} = 4 + 2\sqrt{3} - \frac{4-2\sqrt{3}}{4}$

$= \frac{16 + 8\sqrt{3} - (4 - 2\sqrt{3})}{4}$

$= \frac{12 + 10\sqrt{3}}{4}$

OR $\frac{6+5\sqrt{3}}{2}$ ③

**:)** $11 + 4\sqrt{6} = 11 + 2\sqrt{24}$

$x + y = 11, \quad xy = 24$

$\therefore x = 8, y = 3$

$\therefore 11 + 4\sqrt{6} = (\sqrt{8} + \sqrt{3})^2$ ②

**2 (a)** $f(x) = x^2 + 3x - 2$

$\frac{f(h) - f(-h)}{2h} = \frac{(h^2+3h-2) - (h^2 - 3h - 2)}{2h}$

$= \frac{6h}{2h}$

$= 3$ ③

**(b)** $y = \frac{2x-3}{x-1}$

Inverse is $x = \frac{2y-3}{y-1}$

$xy - x = 2y - 3$

$xy - 2y = x - 3$

$y(x-2) = x - 3$

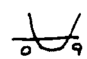
$y = \frac{x-3}{x-2}$ ③

**(c)** $x^2 > 9x$

$x^2 - 9x > 0$

$x(x-9) > 0$

$x < 0, \ x > 9$




③

**(d)** $|x-2| = 2x + 4$

Note: $2x + 4 \geq 0$, i.e. $x \geq -2$.

$x - 2 = 2x + 4$ or $x - 2 = -(2x+4)$

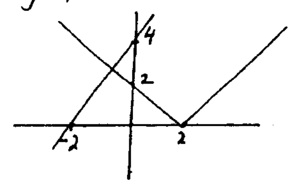
$-6 = x$ or $3x = -2$

$x = -6$ or $x = -\frac{2}{3}$

But $x \geq -2$ $\therefore$ $x = -\frac{2}{3}$ is only solution

OR Test $x = -6$, $x = -\frac{2}{3}$ into equation

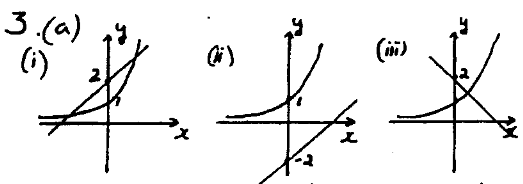
OR Draw graphs and solve:




③

# Task 5.

**3.(a)**




(i) 2 solutions (ii) 0 solutions (iii) 1 solution ③

**(b)** $\lim_{x\to 2} \frac{x^2-4}{2x-4} = \lim_{x\to 2} \frac{(x-2)(x+2)}{2(x-2)}$

$= \lim_{x\to 2} \frac{x+2}{2}$

$= \frac{4}{2}$

$= 2$ ②

**(c)** $\sin\theta = p$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1-p^2}$ ✓

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{p}{\sqrt{1-p^2}}$ ✓ ②

**(d)** $4\sec^2\theta - \tan\theta = 7$

$4(\tan^2\theta + 1) - \tan\theta - 7 = 0$

$4\tan^2\theta - \tan\theta - 3 = 0$

$(4\tan\theta + 3)(\tan\theta - 1) = 0$

$\tan\theta = -\frac{3}{4}$ or $\tan\theta = 1$

$\theta = 180° - 37°, 360° - 37°, 45°, 180° + 45°$

$= 143°, 323°, 45°, 225°$ ⑤

**4 (a)** $(-3, 2) \ (5, 7)$ — $x_1, y_1$; $x_2, y_2$; $m_1 : m_2 = -1 : 3$

$\left(\frac{(-1)\times 5 + 3\times(-3)}{-1+3}, \frac{(-1)\times 7 + 3\times 2}{-1+3}\right)$

$\left(-7, -\frac{1}{2}\right)$. ③

**(b)** $3x - 4y + 12 = 0$, $3x - 4y - 6 = 0$

$(2, 0)$ lies on $3x - 4y - 6 = 0$

Perp. dist. $= \left|\frac{3\times 2 - 4\times 0 + 12}{\sqrt{3^2+4^2}}\right|$

$= \frac{18}{5}$ units ③

**(c)** $f(x) = \frac{x-1}{\sqrt{2x+1}}$

$f'(x) = \frac{\sqrt{2x+1}\times 1 - (x-1)\times\frac{1}{2}(2x+1)^{-\frac{1}{2}}\times 2}{2x+1}$

$= \frac{\sqrt{2x+1} - \frac{x-1}{\sqrt{2x+1}}}{2x+1} \times \frac{\sqrt{2x+1}}{\sqrt{2x+1}}$

$= \frac{2x+1-(x-1)}{(2x+1)^{3/2}}$

$= \frac{x+2}{\sqrt{(2x+1)^3}}$ ④

**(d)** $\angle ACB = 90°$ (angle in semi-circle)

$\therefore x = 180 - (90 + 63)$ (angle sum of triangle)

$= 27$

$y = 180 - 63$ (opposite angles of cyclic quad.)

$= 117$ ②

5(a) $x^2+x+1 \equiv a(x-1)^2+b(x-1)+c$

$= ax^2+(-2a+b)x+a-b+c$

$\therefore a = 1$

$-2a+b=1 \quad \therefore b=3$

$a-b+c=1 \quad \therefore c=3$ (3)

(b) $f(x) = 8+4x-x^2$

EITHER: Axis of symmetry: $x = \frac{-4}{2\times -1} = 2$

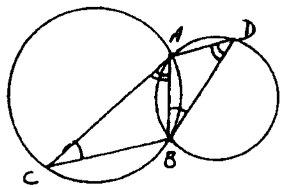
$f(2) = 8+8-4 = 12$

OR: $f(x) = -(x^2-4x+4)+8+4$

$= 12-(x-2)^2$

Maximum value is 12 (2)

(c)




$\angle ABD = \angle ACB$ (alt. seg. thm)

$\angle CAB = \angle ADB$ (alt. seg. thm)

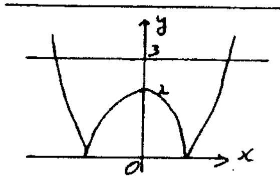
Since two pairs of angles of a triangle are equal, the third pair are equal

$\therefore \angle ABC = \angle BAD$

$\therefore AD \parallel CB$ (corresponding angles are equal.) (3)

(d)

(i)




(2)

(ii) $y \le 3, \ y \ge |x^2-2|$ (2)

6(a) $x^2+mx+(m+2)=0$

(i) one root 3: $9+3m+(m+2)=0$

$4m=-11$

$m = -2\frac{3}{4}$ (2)

(ii) reciprocals: Product = 1

$m+2=1$

$m=-1$ (2)

(b) $x^2+(k-3)x-k=0$

$\Delta = (k-3)^2-4\times1\times(-k)$

$= k^2-6k+9+4k$

$= k^2-2k+9$

$= (k-1)^2+8$

$\Delta > 0$ for all $k$

$\therefore$ roots are real for all real $k$. (3)

(c)(i) $y = x^2$

$\frac{dy}{dx} = 2x$.

At $(a, a^2)$ $\frac{dy}{dx} = 2a$

Tangent: $y-a^2 = 2a(x-a)$

$y-a^2 = 2ax-2a^2$

$y = 2ax-a^2$ (2)

(ii) Subst. $x=2, y=-5$:

$-5 = 2\times a\times 2 - a^2$

$a^2-4a-5=0$

$(a-5)(a+1)=0$

$a=5$ or $a=-1$ (1)

(iii) $a=5$: $y = 10x-25$

$a=-1$: $y=-2x-1$ (2)

7. (a) $y = x(x-2)^3$

$\frac{dy}{dx} = (x-2)^3\times 1 + x\times 3(x-2)^2$

$= (x-2)^3+3x(x-2)^2$

$= (x-2)^2[(x-3)+3x]$

$= (x-2)^2(4x-3)$

Curve is horizontal when $\frac{dy}{dx}=0$,

i.e. $x=2$, $x=\frac{3}{4}$.

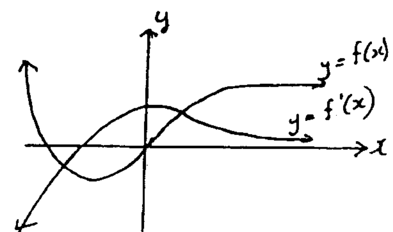
When $x=2$, $y=0$.

When $x=\frac{3}{4}$, $y=\frac{3}{4}\times\left(-\frac{5}{4}\right)^3$

$= \frac{375}{256}$

Points are $(2,0)$, $\left(\frac{3}{4}, \frac{375}{256}\right)$ (4)

(b).




(2)

(c) (i) $\sin 38° \cos 48° = \frac{h}{x} \times \frac{x}{y}$

$= \frac{h}{y}$

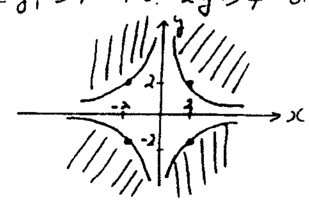
$= \sin\theta$ (2)

(ii) $\sin\theta = \sin 38° \cos 48°$

$\doteqdot 0.4119\ldots$

$\theta = 24°20'$ (1)

(d) $|xy| \ge 4$ i.e. $xy \ge 4$ or $xy \le -4$

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