MORIAH COLLEGE

Year 11

Mathematics Extension 1

Preliminary Examination
September 2015

Time Allowed: 90 minutes

Reading Time: 5 minutes

Examiner: J.Cohen, L.Bornstein, G. Lang

General Instructions

- Write using blue or black pen.
- Board-approved calculators may be used.
- All necessary working should be shown in every question.
- Start each question on a separate sheet.

Name …………………………………………………………..

MULTIPLE CHOICE GRID

5 marks

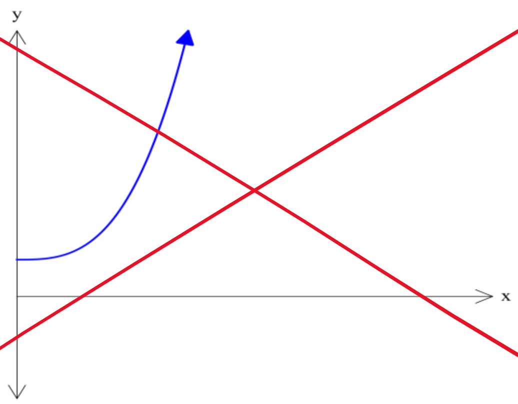
1.	**A**	**B**	**C**	**D**
2.	**A**	**B**	**C**	**D**
3.	**A**	**B**	**C**	**D**
4.	**A**	**B**	**C**	**D**
5.	**A**	**B**	**C**	**D**

Multiple choice (1 mark each)

1. Evaluate $\lim_{x \to \infty} \dfrac{3x^2 - 2x + 1}{2x^2 + x - 1}$.

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 0

2. The following graph has the following information relating to its $f'(x)$ and $f''(x)$



(A) $f'(x) < 0$, $f''(x) < 0$ (B) $f'(x) < 0$, $f''(x) > 0$ (C) $f'(x) > 0$, $f''(x) > 0$ (D) $f'(x) > 0$, $f''(x) < 0$

3. Which term best describes the roots of the equation $x^2 + 4x - 3 = 0$?

A) Real and different B) Equal
C) Definite D) Unreal

4. If $4x^2 + 3x - 5 \equiv Ax(x+1) + B(x+1) + C$, the value of B is:

(A) 3 (B) -1 (C) 2 (D) -2

5. Which of the following divides the line segment from $A(4,-3)$ to $B(-8,5)$ **externally** in the ratio $3:1$?

(A) $(-7, \frac{9}{2})$ (B) $(16,-9)$ (C) $(10,-7)$ (D) $(-14,9)$

Question 6 (start each question on a new page)   (15 marks)

a) Solve $\frac{4}{x+2} \geq \frac{1}{x}$ **3**

b) If one root of the equation $x^2 - mx + 2 = 0$ is double the other, find the value(s) of m. **2**

c) If $x = 3 - t$ and $y = 2t + 1$, find the Cartesian equation of the curve. **2**

d) If α, β are the roots of $2x^2 - 13x + 9 = 0$,

find the value of :

i) $(a+1)(b+1)$ **2**

ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ **2**

e) Solve the equation :

$\tan^2 x - 3\sec x + 3 = 0$ for $0 \leq x \leq 360^o$ **4**

Question 7 (start each question on a new page)   (15 marks)

a) For what range of values of k will the expression $kx^2 - (5k+1)x + 9k$ be positive definite. **3**

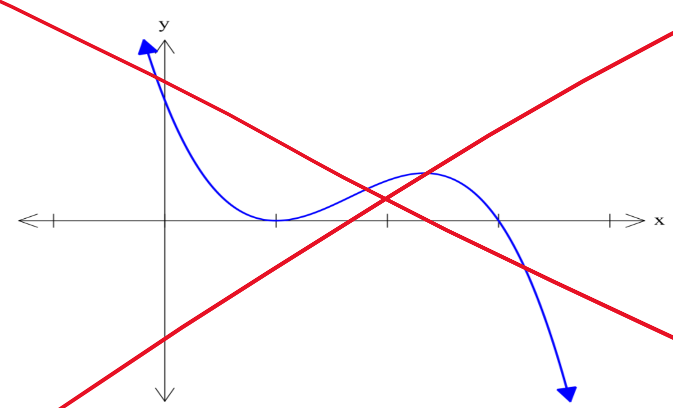
b) A parabola has equation $y^2 = 8x + 2y + 7$.

i) Express this equation in the form $(y - p)^2 = 4a(x - k)$. **1**

ii) Draw a neat sketch of the parabola, clearly showing the vertex, focus and directirix. **4**

iii) Find the endpoints of the latus rectum. **1**

c) The above diagram shows a sketch of the curve *y* = *f*(*x*).



On the attached page, draw a possible sketch of the function $y = f'(x)$ **2**

d) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

i) Derive the equations of the tangents at P and Q.

ii) Show that the point of intersection of the tangents is **4**

$$[a(p+q),\ apq]$$

Question 8 (start each question on a new page) (15 marks)

a) Consider the graph $y = \dfrac{x^2}{1 - x^2}$

i) Find the stationary point(s) and their nature **3**

ii) Find the equation of the horizontal and vertical asymptotes **2**

iii) Sketch the graph. **2**

b) Prove by mathematical induction that for all integers, $n \geq 1$ that

$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + ... + 3^n \times 2^{n+1} = \dfrac{12}{5}\left(6^n - 1\right)$ **4**

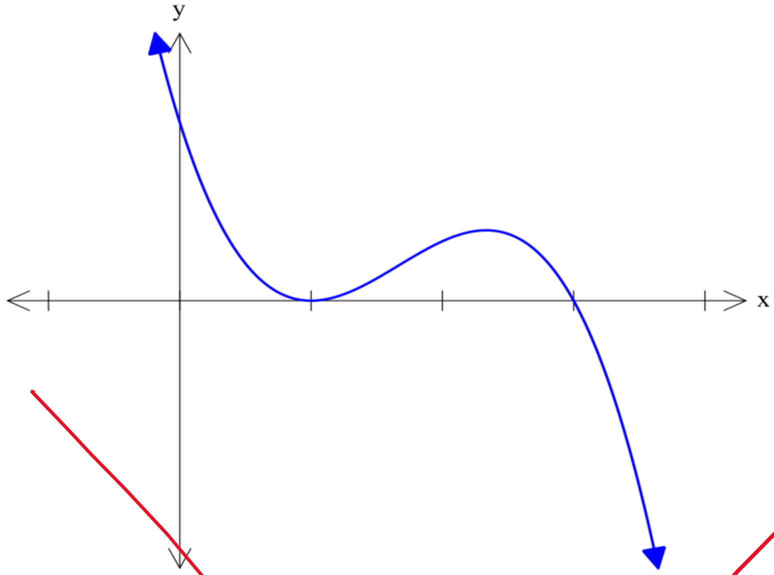
c) Solve for x:

$(x^2 + 2x)^2 = 14x^2 + 28x + 15$ **4**

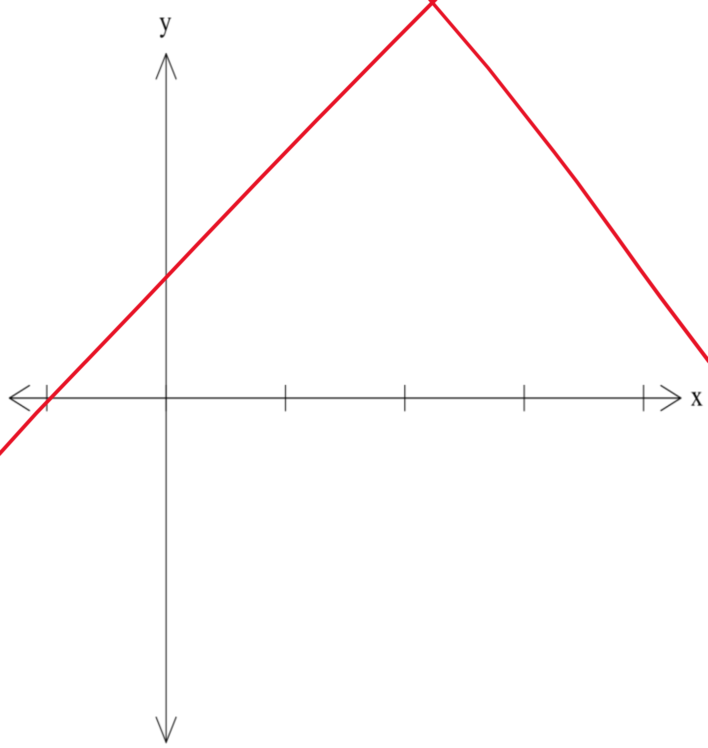
END OF TEST

Name ………………………………..

$y = f(x)$



$y = f'(x)$



30mins.

Y11 3U 2015 prelim.

1a. (A)

b. $f'(x) > 0$ increasing (C).
$f''(x) > 0$. concave up

c. $\Delta = (4)^2 - 4(1)(-3)$
$= 16 + 12$
$= 28 > 0$ (A).

d. $4x^2 + 3x - 5 \equiv Ax^2 + Ax + Bx + B + C$.
$4 = A$ | $3 = A + B$
| $3 = 4 + B$
$-1 = B$ (B)

e. $A(4, -3)$ $B(-8, 5)$
$3 : -1$

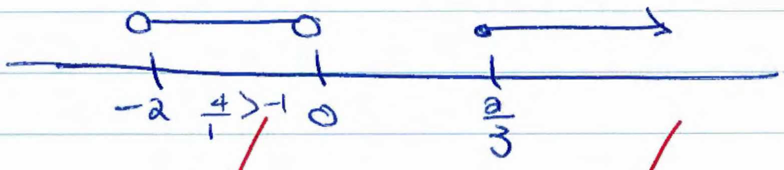
$x = \frac{-4 - 24}{2}$, $y = \frac{3 + 15}{2}$

$x = -14$, $y = 9$.
(D).

#6a. $\frac{4}{x+2} \geq \frac{1}{x}$

consider $\frac{4}{x+2} = \frac{1}{x}$ $\qquad x \neq 0$, $x \neq -2$

$$4x = x + 2$$
$$3x = 2$$
$$x = \frac{2}{3}.$$



3.

$-2 < x < 0 \qquad x \geq \frac{2}{3}.$

b. $x^2 - mx + 2 = 0.$

roots: $\alpha, 2\alpha$

sum: $\alpha + 2\alpha = \frac{-b}{a}$

$3\alpha = m.$ —— (2)

prod: $(\alpha)(2\alpha) = \frac{c}{a}.$

$2\alpha^2 = 2.$

$\alpha^2 = 1$

$\alpha = \pm 1$ —— (1)

Subst (1) in (2).

$$2(\pm 1) = m$$
$$m = \pm 3$$

2.

c. $x = 3 - t \qquad y = 2t + 1$ —— (2)

$t = 3 - x$ —— (1)

Subst (2) in (1).

2.

$$y = 2(3 - x) + 1$$
$$y = 6 - 2x + 1$$
$$y = 7 - 2x$$

d). $2x^2 - 13x + 9 = 0$

$\alpha + \beta = -\frac{b}{a} = \frac{13}{2}$ ✓

$\alpha\beta = \frac{c}{a} = \frac{9}{2}$.

i) $(\alpha+1)(\beta+1)$

$= \alpha\beta + \alpha + \beta + 1$

$= \frac{9}{2} + \frac{13}{2} + 1$

$= 12$. ✓

ii). $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$ ✓

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{13}{2}\right)^2 - 2\left(\frac{9}{2}\right)}{\frac{9}{2}}$

4.

$= \frac{133}{18}$. ✓

e) $\tan^2 x - 3\sec x + 3 = 0$. $\qquad \tan^2 x + 1 = \sec^2 x$

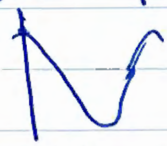
$\therefore \sec^2 x - 1 - 3\sec x + 3 = 0$.

$\sec^2 x - 3\sec x + 2 = 0$ ✓

$(\sec x - 2)(\sec x - 1) = 0$

$\sec x = 2$	$\sec x = 1$ ✓
$\frac{1}{\cos x} = 2$	$\cos x = 1$

4.



✓

$\cos x = \frac{1}{2}$

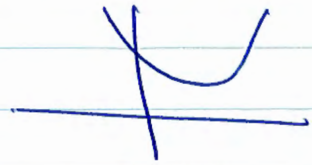
rel $\angle = 60°$ $\qquad x = 0°, 360°$

cos is + ✓

1st	4th.
$x = 60°$	$x = 300°$

#7a. $kx^2 - (5k+1)x + 9k$

for pos def



$a > 0$

$\Delta < 0.$

$k > 0.$

$\Delta = (-(5k+1))^2 - 4(k)(9k) < 0$ ✓

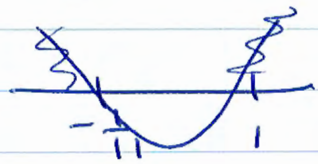
$25k^2 + 10k + 1 - 36k^2 < 0.$

$-11k^2 + 10k + 1 < 0$

$11k^2 - 10k - 1 > 0$

$(11k+1)(k-1) > 0$

$k = -\frac{1}{11} \quad , k = 1$ ✓ 3.



and $k > 0.$

$\therefore k > 1$ ✓

b. $y^2 = 8x + 2y + 7$

$y^2 - 2y + (1)^2 = 8x + 7 + (1)^2$

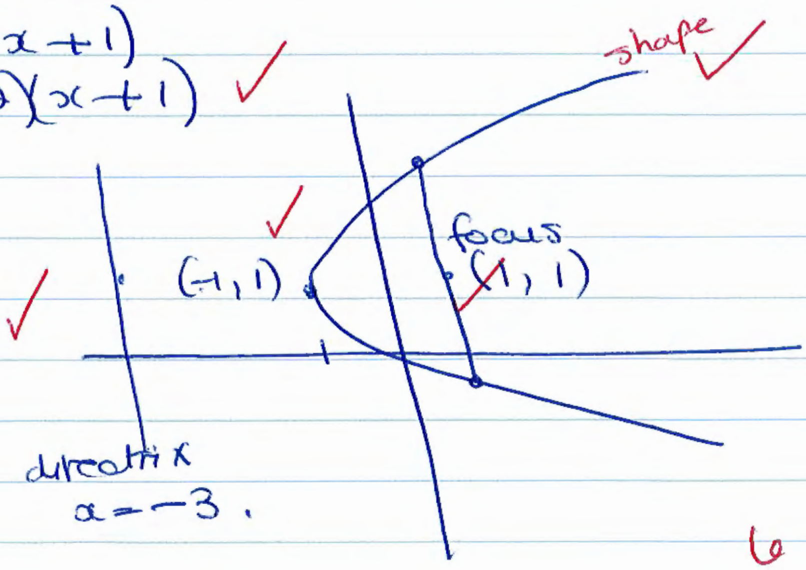
$(y-1)^2 = 8x + 8$

$(y-1)^2 = 8(x+1)$

$(y-1)^2 = 4(2)(x+1)$ ✓

V $(-1, 1)$

$a = 2.$



6

at $x = 1$

$(y-1)^2 = 16$

$y - 1 = \pm 4$

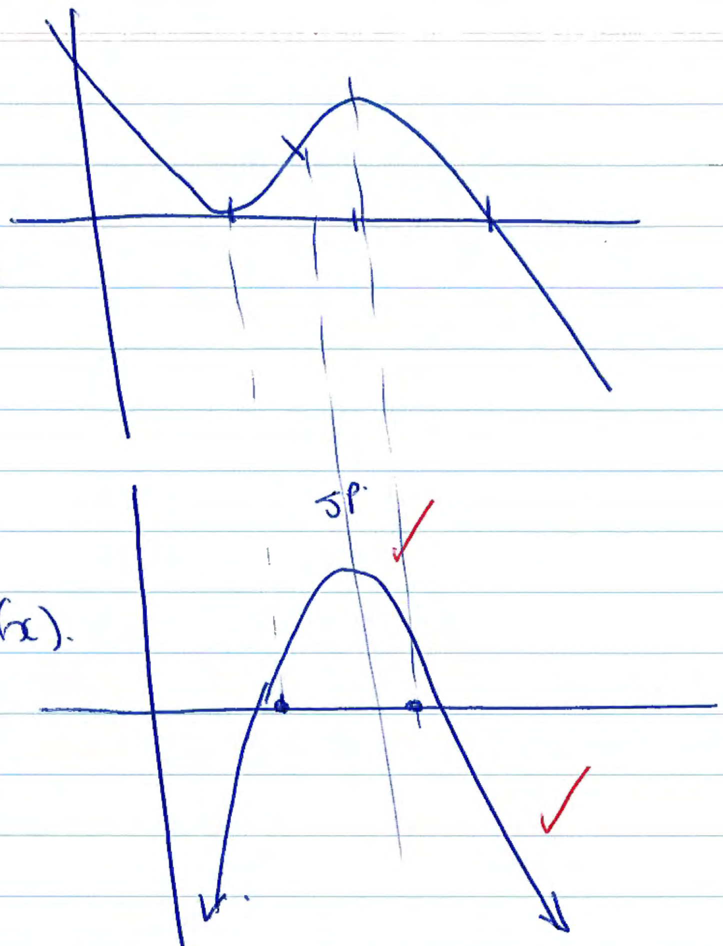
$y = 1 \pm 4$

$(1, 5)$ or $(1, -3)$. ✓

c).

$y = f(x)$

$y = f'(x)$.



p.of.
↓
SP
↓
x int.

2

d). $y = \frac{x^2}{4a}$

$m(\tan) = \frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$.

at $x = 2ap$.

$m(\tan) = \frac{2ap}{2a} = p$

eq. of tangent:

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$. ——— ①

∴ eq. at Q: $y = qx - aq^2$ ——— ②.

$px - ap^2 = qx - aq^2$

$px - qx = ap^2 - aq^2$

$x(p-q) = a(p-q)(p+q)$

$x = a(p+q)$ ——— ③.

Subst ③ in ①

$y = ap(p+q) - ap^2$

$y = ap^2 + apq - ap^2 = apq$. $(a(p+q), apq)$.

4

#8a. $y = \dfrac{x^2}{1-x^2}$

S.P: $y' = 0$.

$u = x^2 \quad | \quad v = 1-x^2$

$u' = 2x \quad | \quad v' = -2x$

$$y' = \frac{2x(1-x^2) - x^2(-2x)}{(1-x^2)^2} = 0.$$ ✓

$2x - 2x^3 + 2x^3 = 0$. 3.

$2x = 0$

$x = 0$.

$y = 0$. ✓ $(0,0)$ min S.P. ✓

i).

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y'	\	0. ◡	/

ii). $1 - x^2 \neq 0$.

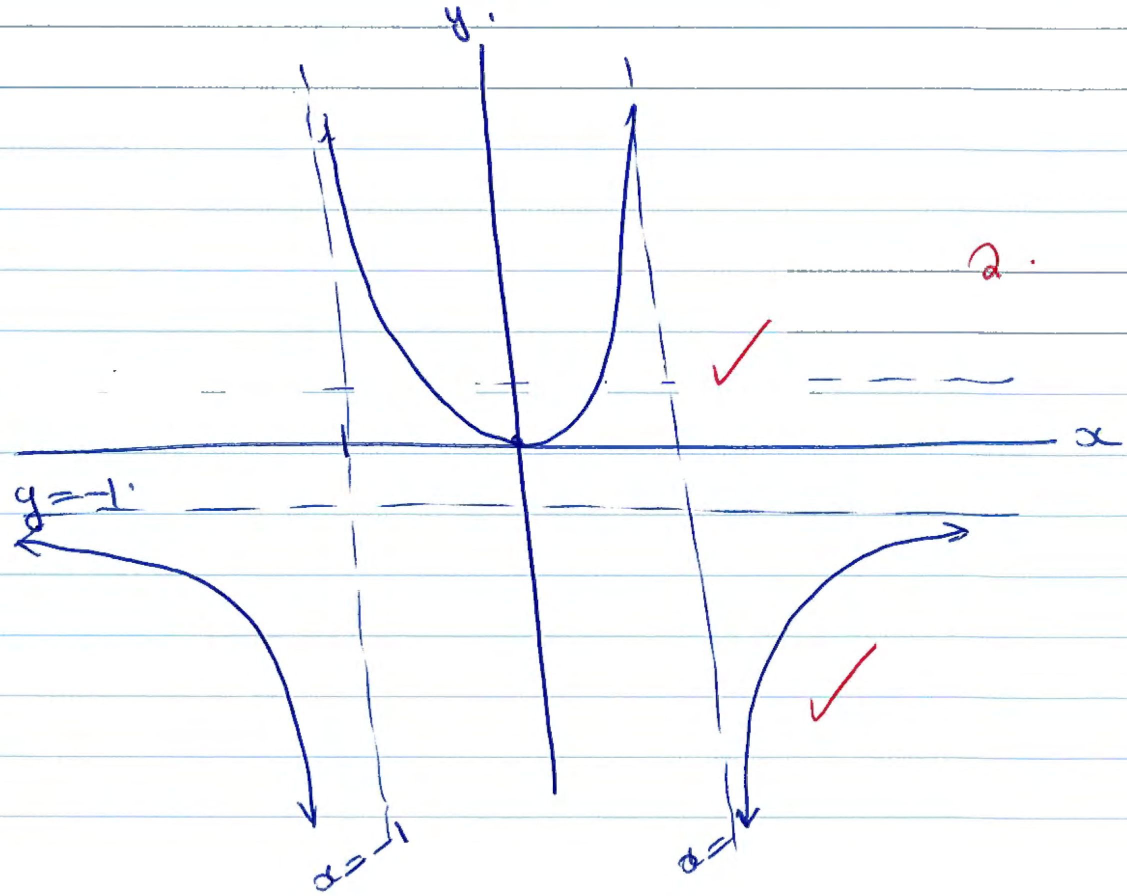
$x^2 \neq 1$

$x \neq \pm 1$. ✓

$$\lim_{x\to\infty} \frac{x^2/x^2}{\frac{1}{x^2} - x^2/x^2}$$ 2.

$= -1$.

$y = -1$. ✓



#8b. $3^1 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \dots + 3^n \times 2^{n+1}$

$= \frac{12}{5}(6^n - 1)$

prove true for $n=1$.

LHS: $3 \times 2^2 = 12$ | RHS $\frac{12}{5}(6^1 - 1) = \frac{12}{5}(5) = 12$

$\therefore$ true for $n=1$.

assume true for $n=k$. ✓ 1.

$3 \times 2^2 + 3^2 \times 2^3 + \dots + 3^k . 2^{k+1} = \frac{12}{5}(6^k - 1)$

prove true for $n = k+1$.

we want to prove:

$\underbrace{3 \times 2^2 + 3^2 \times 2^3 + \dots + 3^k . 2^{k+1}}_{\text{by assumption}} + 3^{k+1} . 2^{k+2} = \frac{12}{5}(6^{k+1} - 1)$

LHS:

$\frac{12}{5}(6^k - 1) + 3^{k+1} . 2^{k+2}$ ✓

$= \frac{12}{5}(6^k - 1) + 3^k . 3^1 . 2^k . 2^2$

$= \frac{12}{5}(6^k - 1) + 6^k . 12$.

$= \frac{12}{5} . 6^k - \frac{12}{5} + 12 . 6^k$

$= \frac{12 . 6^k - 12 + 12 . 5 . 6^k}{5}$ ✓

4

$= \frac{12}{5}(6^k - 1 + 5 . 6^k)$

$= \frac{12}{5}(6^1 . 6^k - 1)$ ✓

$= \frac{12}{5}(6^{k+1} - 1) = \underline{\text{RHS}}$.

8c). $(x^2+2x)^2 = 14x^2 + 28x + 15$

$(x^2+2x)^2 = 14(x^2+2x) + 15$

let $x^2 + 2x = k$ ✓

$k^2 = 14k + 15$

$k^2 - 14k - 15 = 0$

$(k-15)(k+1) = 0$

$k = 15$, | $k = -1$. ✓

4.

$x^2 + 2x = 15$	$x^2 + 2x = -1$
$x^2 + 2x - 15 = 0$	$x^2 + 2x + 1 = 0$
$(x+5)(x-3) = 0$	$(x+1)^2 = 0$
$x = -5, 3$ ✓	$x = -1$ ✓