Question 1 (12 Marks) **Marks**

a) Solve for x: $3^{2x} - 10(3^x) + 9 = 0.$ **3**

b) A function $f(x)$ is defined as follows.

$$f(x) = \begin{cases} 0 & \text{for} \quad x < -2 \\ -2 & \text{for} \quad -2 \le x < 0 \\ x+1 & \text{for} \quad x \ge 0 \end{cases}$$

i) Evaluate $f(-1) + f(1)$. **1**

ii) Sketch the function. **3**

c) Solve the following for x.

i) $(x-3)(5-x) < 0$ **2**

ii) $\dfrac{x}{x-1} \ge 1$ **3**

Question 2 (12 Marks) ***Start a new page*** **Marks**

a) A parabola has equation $y^2 = 8x + 2y + 7$. **6**

i) Express this equation in the form $(y - y_1)^2 = 4a(x - x_1)$.

ii) Draw a neat sketch of the parabola indicating the:

α. co-ordinates of the focus;

β. equation of the axis of symmetry;

γ. co-ordinates of all points of intersection of the parabola with the co-ordinate axes.

b) The point $P(-3, 8)$ divides the interval AB externally in the ratio $k : 1$. If A is the point $(6, -4)$ and B is the point $(0, 4)$, find the value of k. **3**

c) If $\operatorname{cosec}\theta = -\dfrac{13}{5}$ and $\cos\theta > 0$, find exact values of $\tan\theta$ and $\cos\theta$. **3**

/Q3 …Page 2

Question 3 (12 Marks) ***Start a new page***

a) Evaluate $\lim_{x \to \infty} \dfrac{3x^2 - 2x + 1}{2x^2 + x - 1}$. **2**

b) Differentiate $4x(5x^2 - 4)^3$ with respect to x leaving your answer in factorized form. **3**

c) The function $p(x)$ is defined by the rule $p(x) = (x - 1)(x^2 - 5)$.

i) Find the real roots of the equation $p(x) = 0$.

ii) Find the coordinates of the turning points of $p(x)$, and determine whether they are maxima or minima.

iii) Draw a sketch of the graph $y = p(x)$, in the domain $-3 \leq x \leq 3$. **7**

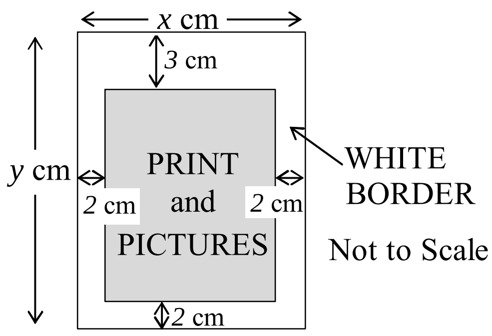
/Q4 …Page 3

Question 4 (12 Marks) ***Start a new page***

a) Sketch the function given by $f(x) = \dfrac{x}{|x|}$ **3**

b) Shade the region in the cartesian plane which satisfies $y \le \sqrt{4 - x^2}$ and $y \ge x^2 - 4$. **3**

c) A children's picture book is being designed so that each page contains *320* square centimetres of print and pictures surrounded completely by a white border as illustrated in the figure below.



Each page is to have a border of width *2* centimetres at the bottom and on each side, as well as a border of width *3* centimetres at the top. Let the width of a page be *x* centimetres and its length be *y* centimetres.

6

i) Show that the area *A* square centimetres of one such page is given by

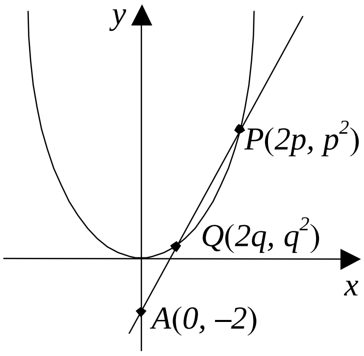
$$A = x\left(5 + \frac{320}{(x-4)}\right).$$

ii) Show that the page which fulfills all the printing requirements and which has the smallest area is *20* centimetres wide and *25* centimetres long.

Question 5 (12 Marks) ***Start a new page*** **Marks**

a) Determine the equation of the locus of the points $P(x, y)$ equidistant from the y-axis and the point $(1, 0)$. **3**

b)



i) Find the equations of the normals to the parabola $\begin{cases} x = 2t \\ y = t^2 \end{cases}$ at the points $P(2p, p^2)$ and $Q(2q, q^2)$, where $p \neq q$. Hence show that these normals intersect at the point $R(X, Y)$ where $X = -pq(p + q)$ and $Y = (p + q)^2 - pq + 2$.

ii) If the chord PQ has gradient m and passes through the point $A(0, -2)$ find, in terms of m, the equation of PQ. Hence show that p and q are the roots of the equation $t^2 - 2mt + 2 = 0$.

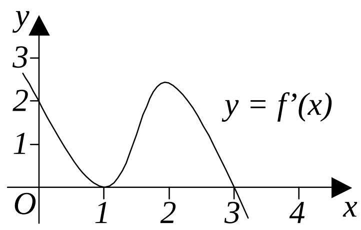
iii) By considering the sum and the product of the roots of this quadratic equation show that the point R lies on the original parabola.

iv) Find the least value of m^2 for which p and q are real. Hence find the set of possible values of the y co-ordinate of R.

9

Question 6 (12 Marks) ***Start a new page***

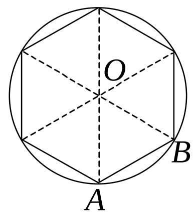
a)



3

The above diagram shows a sketch of the gradient function of the curve $y = f(x)$. In your Writing Booklet, draw a sketch of the function $y = f(x)$ given that $f(0) = 0$.

b)

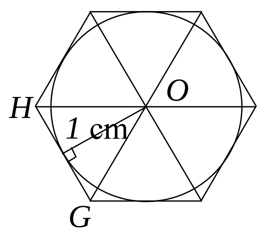


A regular hexagon is drawn inside a circle with centre O so that its vertices lie on the circumference, as shown in the diagram. The circle has radius 1 cm.

i) Prove that $\Delta\, OAB$ is equilateral. **2**

ii) Find the area of $\Delta\, OAB$ and hence find the area of this hexagon. Leave your answer in surd form. **2**

Another regular hexagon is drawn outside the circle, as shown. The altitude of $\Delta\, OGH$ is 1 cm.



iii) Find the area of $\Delta\, OGH$ and hence find the area of this outer hexagon. Leave your answer in surd form. 3

iv) By considering the results in (ii) and (iii), explain why $\frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}$. 2

END OF PAPER

YR 11 EXT ONE PRELIM 2006

(1.) a) $3^{2x} - 10(3^x) + 9 = 0$ Let $u = 3^x$

$u^2 - 10u + 9 = 0$ (1)

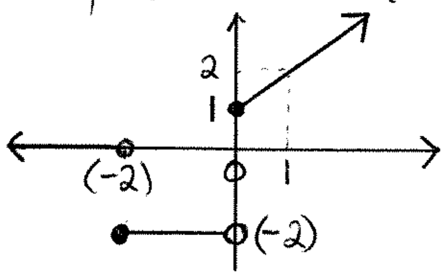
$(u-9)(u-1) = 0$

$u = 1$ or 9. (1) $3^x = 1$ or $3^x = 9$.

$x = 0$ or $x = 2$ (1)

b) (i) $f(-1) + f(1) = -2 + (1+1) = 0$ (1)

(ii)

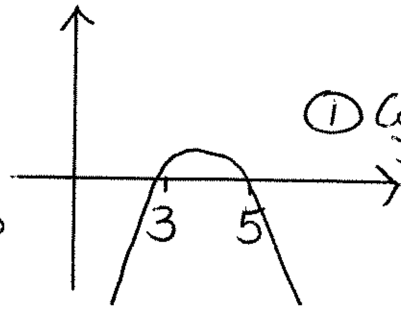


(1) mark each section including correct endpts.

c) (i) $(x-3)(5-x) < 0$

$x > 5$ or $x < 3$

(1) Correct Regions



(1) Concave Down Parabola with intercepts

(ii) $\frac{x}{x-1} \geq 1$

$\times(x-1)^2$ $\quad \times(x-1)^2$

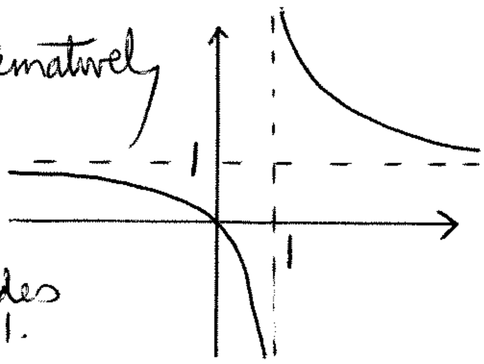
$x(x-1) \geq (x-1)^2$ (1)

$x^2 - x \geq x^2 - 2x + 1$

$x \geq 1$ (1)

BUT $x \neq 1$ $\therefore x > 1$ (1) Excludes $x = 1$.

Alternatively



MARKERS COMMENT

a) Those recognising an equation reducible to quadratic completed well. Some dropped equals zero making an expression not an equation. Some found u but failed to find x.

b) Lots failed to evaluate $f(1)$ giving answer of $x-1$!

(ii) Common mistakes on graph were failure to deal with endpoints & critical values missing from axes

c)(i) Many drew concave up parabola but then identified correctly the region below x axis. Some confuse two separate regions with writing a single inequality with finite endpoint ie $3 > x > 5$!

(ii) Most common mistake was failure to exclude $x = 1$ from solution

Q2 (a)(i) $y^2 = 8x + 2y + 7$

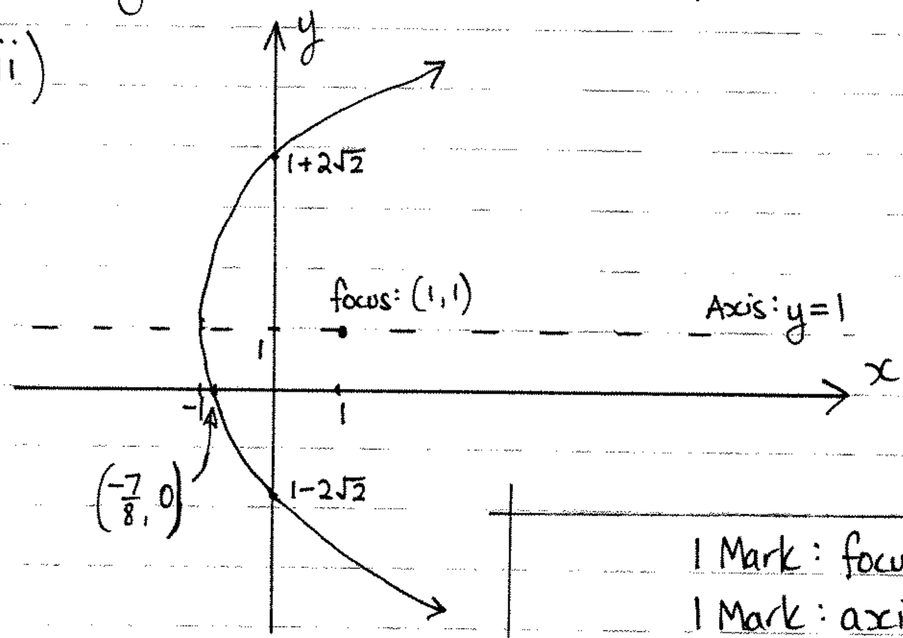
$\therefore y^2 - 2y = 8x + 7$

$\therefore (y-1)^2 = 8x + 8$

$\therefore (y-1)^2 = 8(x+1)$

1 Mark - Completing the square correctly
1 Mark - final answer

(ii)



1 Mark : focus
1 Mark : axis
1 Mark : y-intercepts
1 Mark : x-intercept
(based on answer for part (i))

(b) $x_p = \dfrac{mx_2 + nx_1}{m+n}$

$\therefore -3 = \dfrac{k.0 + 1.6}{k+1}$

$\therefore -3k - 3 = 6$

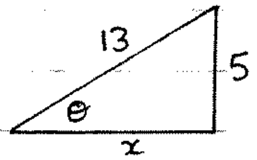
$\therefore -3k = 9$

$\therefore k = -3$

3 Marks	:	$k = \pm 3$ (either accepted)
2 Marks	:	one error (eg. swapped x_1 and x_2)
1 Mark	:	multiple errors, but tried to use correct method
0 Marks	:	no idea

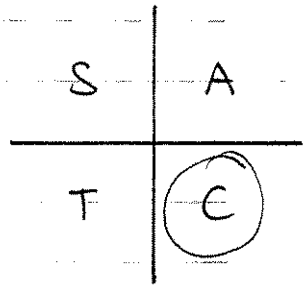
Q2. (c) $\operatorname{cosec}\theta = \frac{-13}{5}$, $\cos\theta > 0$

$\therefore \sin\theta = -\frac{5}{13}$



$x^2 = 13^2 - 5^2$

$x = 12$



$\sin\theta < 0$

$\cos\theta > 0$

$\therefore$ 4th quadrant

$\tan\theta = -\frac{5}{12}$

$\cos\theta = \frac{12}{13}$

1 Mark - Quadrant
1 Mark
1 Mark

Q3 Y11 Prelim 06 Ext 1

(a) $\lim_{x\to\infty} \dfrac{3 - \frac{2}{x} + \frac{1}{x^2}}{2 + \frac{1}{x} - \frac{1}{x^2}} = \dfrac{3}{2}$ (2)

(lim notation poor $\lim_{x\to\infty}$ = deduction of 1 mark $\frac{3}{2}$ no working 1 mark)

(b) $\frac{d}{dx}\, 4x(5x^2-4)^3 = 4x \times 3(5x^2-4)^2 \times 10x + 4(5x^2-4)^3$

$= 4(5x^2-4)^2(30x^2+5x^2-4)$

$= 4(5x^2-4)^2(35x^2-4)$ (3)

product rule (1)
1 for diff.
1 for factorization
(no product rule no marks)

(c)(i) $(x-1)(x^2-5)=0$

$x = 1, \pm\sqrt{5}$

still many $-\sqrt{5}$ excluding!

(ii) $p'(x) = (x-1)2x + (x^2-5)$

$= 2x^2 - 2x + x^2 - 5$

$= 3x^2 - 2x - 5$

$= (3x-5)(x+1)$

Stat pts when $p'(x) = 0$

ie $(3x-5)(x+1) = 0$

$x = -1, \frac{5}{3}$

$p''(x) = 6x - 2$

At $x = -1$ $p(-1) = 8$

$p''(-1) = -8 < 0$

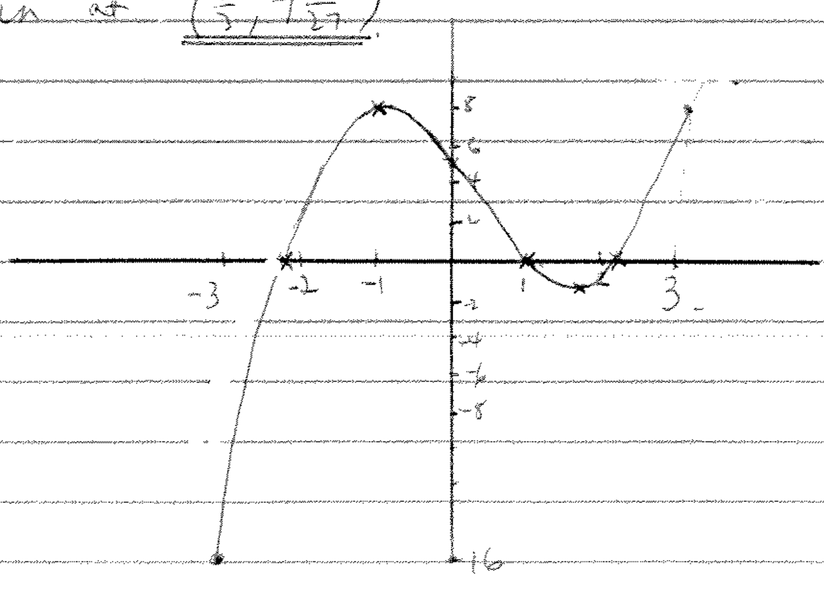
$\therefore$ max at $(-1, 8)$

(1 for nature)

At $x = \frac{5}{3}$ $p(\frac{5}{3}) = -1\frac{13}{27}$

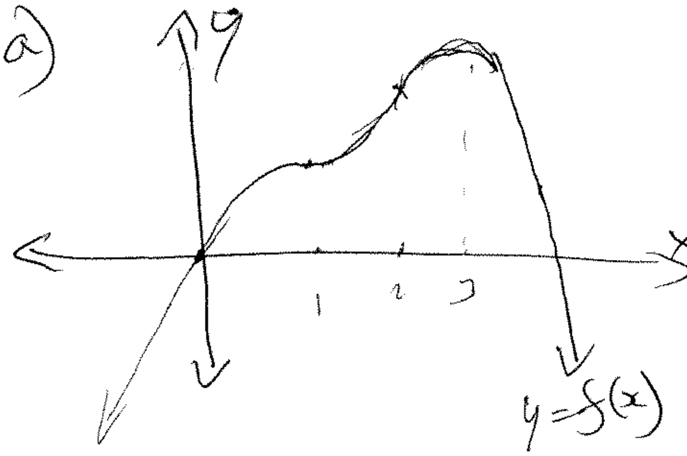
$p''(\frac{5}{3}) = 8 > 0$

$\therefore$ min at $(\frac{5}{3}, -1\frac{13}{27})$



1 turning pts
1 intercepts + end pts
(Many missed off extreme values)

Q6 a)

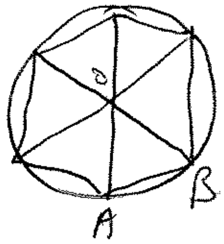


at $x=1$ St pt of inflexion

at $x=2$ Pt of inflexion

at $x=3$ MAX turning pt

(b)



i) Prove that $\triangle OAB$ is equilateral

Method 1 - Geometry

Angle at center $= \frac{360}{6}$ [regular Hexagon]

$= 60^\circ$

as $\triangle OAB$ is isosceles [$OA = OB$]

$\angle OAB = \angle OBA$ [base angles equal]

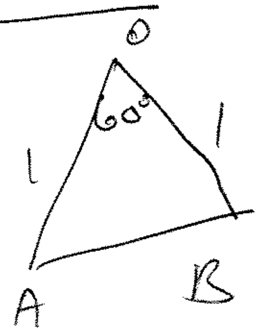
$\therefore 2\angle OBA + 60^\circ = 180$ [$\angle$ sum of $\triangle$]

$\therefore \angle OBA = 60^\circ, \angle OAB = 60^\circ$

$\therefore \triangle OAB$ is equilateral

Note: Reasons must be given.

Method 2 - Trigonometry



$AB^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \cos 60^\circ$

$= 2 - 2 \times \frac{1}{2}$

$= 1$

$\therefore \triangle OAB$ is equilateral.

(b) (ii) Area of $\triangle AOB = \frac{1}{2} ab \sin C$

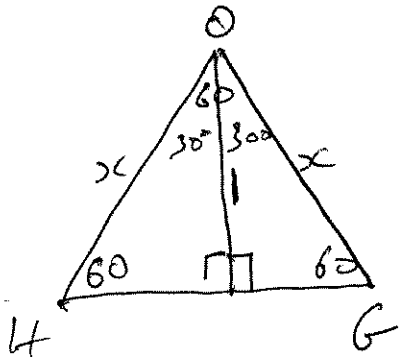
$= \frac{1}{2} \times 1 \times 1 \times \sin 60$

$= \frac{\sqrt{3}}{4} \text{ cm}^2$ ✓

$\therefore$ Area of hexagon $= 6 \times \frac{\sqrt{3}}{4}$

$= \frac{3\sqrt{3}}{2}$ # ✓

(iii)



$\frac{1}{x} = \cos 30°$

$x = \frac{1}{\cos 30°}$

$= \frac{1}{\sqrt{3}/2}$

$= \frac{2}{\sqrt{3}}$ ✓

$= \frac{2\sqrt{3}}{3}$

$\therefore$ Area of outer hexagon

$= 6 \times \frac{1}{2} \times \frac{2\sqrt{3}}{3} \times \frac{2\sqrt{3}}{3} \times \sin 60°$ ✓

$= 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3} \text{ cm}^2$ ✓

(iv) Area of Circle $= \pi r^2$

$= \pi \times 1^2$

$= \pi$ ✓

Area of Inner Hexagon < Area of Circle < Area of Outer Hexagon

$\therefore \frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}$ from (ii) & (iii)

✓