Total marks (84)
Attempt questions 1 – 7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Factorise $a^2 - 4b^2 + 2a + 4b$. **2**

b) Solve $x^2 > 9x$. **2**

c) Solve the following inequation

$$\frac{x+1}{x-3} \le 2 .$$ **3**

d) Using the fact that $\left(\sqrt{x} + \sqrt{y}\right)^2 = x + y + 2\sqrt{xy}$, express $11 + 4\sqrt{6}$ in the form $\left(\sqrt{x} + \sqrt{y}\right)^2$. **2**

e) Find the values of *a, b, c* given that $x^2 + x + 1 \equiv a(x-1)^2 + b(x-1) + c$. **3**

Question 2 (12 marks) Use a SEPARATE writing booklet.

a) By drawing appropriate straight lines on the same axes as the graph of $y = 2^x$, determine the number of solutions for each of the following equations: **3**

(i) $2^x = x + 2$
(ii) $2^x = x - 2$
(iii) $2^x = 2 - x$

b) (i) Sketch the graphs of $y = \left|x^2 - 2\right|$ and $y = 3$ on the same number plane. **2**

(ii) Write down the inequalities that simultaneously represent the region bounded by the two graphs. **2**

c) A parabola has the equation $x^2 - 10x + 16y + 9 = 0$.

(i) Express this equation in the form $(x-h)^2 = -4a(y-k)$. **2**

(ii) Draw a neat sketch of the parabola indicating the coordinates of the vertex and focus. **3**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) The line $3x + ky - 5 = 0$ makes an angle of 135° with the positive x axis.

(i) Show that the gradient of the line is $\frac{-3}{k}$. **1**

(ii) Find the value of k. **2**

b) Find the coordinates of the point that divides the join of (-3, 2) and (5, 7) **externally** in the ratio 1 : 3. **3**

c) Find the perpendicular distance between the pair of parallel lines $3x - 4y + 12 = 0$ and $3x - 4y - 6 = 0$. **3**
[Hint: Find the coordinates of a point on one of the lines.]

d) Prove that $x^2 + (k-3)x - k = 0$ has real roots for all values of k. **3**

Question 4 (12 marks) Use a SEPARATE writing booklet.

a) A point $P(x,y)$ moves in such a way that its distance from $A(-2,1)$ is twice its distance from the point $B(2,1)$. Determine and describe the locus of the point $P(x,y)$. **4**

b) Consider the point $P\left(2ap, ap^2\right)$ on the parabola $x^2 = 4ay$.

(i) Show that the equation of the normal to the parabola $x^2 = 4ay$ at the point P is given by $x + py = 2ap + ap^3$. **2**

(ii) Find the equation of the line which passes through the focus $S(0, a)$ and is perpendicular to the normal. **2**

(iii) If the line found in part (ii) meets the normal at N, find the coordinates of N. **2**

(iv) Show that the locus of N is a parabola and find its vertex. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Solve $\sin 2\theta = \frac{\sqrt{3}}{2}$ for $0° \le \theta \le 360°$. **3**

b) If $\sin\theta = p$, where θ is acute, find expressions for $\cos\theta$ and $\tan\theta$ in terms of p. **3**

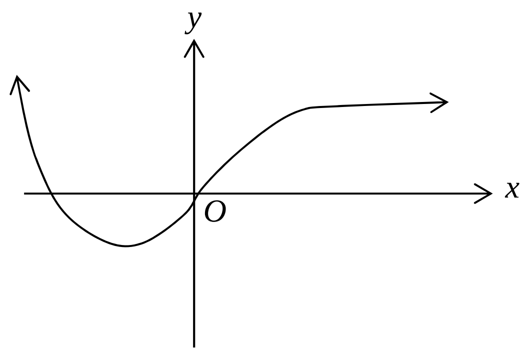
c) Solve for $0° \le \theta \le 360°$, to the nearest degree: $4\sec^2\theta - \tan\theta = 7$. **3**

d) Solve for *x:*

$$x^2 - 5x + 2 + \frac{4}{5x - x^2 - 2} = 0.$$ **3**

Question 6 (12 marks) Use a SEPARATE writing booklet.

a) **2**



The diagram shows the graph of $y = f(x)$.
Copy or trace this graph into your Writing Booklet.
On the same set of axes draw the graph of $y = f'(x)$.

b) Let $y = x\sqrt{x}$.

(i) Find $\frac{dy}{dx}$, leaving your answer in surd form. **2**

(ii) Find the coordinates of the point on the graph of $y = x\sqrt{x}$ at which the normal has a slope of $-\frac{1}{4}$. **3**

Question 6 continued. **Marks**

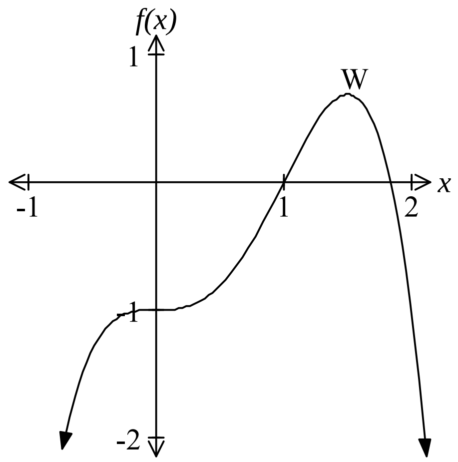
c) If $f(x) = \dfrac{x-1}{\sqrt{2x+1}}$,

(i) show that $f'(x) = \dfrac{x+2}{\sqrt{(2x+1)^3}}$. **2**

(ii) Show that *f(x)* is an increasing function in the domain of x. **3**

Question 7 (12 marks) Use a SEPARATE writing booklet.

a) Study the graph of $y = ax^4 + bx^3 + c$ shown below.



(i) Given that (1, 0) is a point of inflection, evaluate the constants *a, b* and *c*. **3**

(ii) Find the co-ordinates of W (the absolute maximum of the function). **2**

b) Given that $y = \dfrac{2}{(x-1)(x-3)}$,

(i) find the equations of all (horizontal and vertical) asymptotes **2**

(ii) find the y-intercept **1**

(iii) find the co-ordinates of any stationary point(s) and the nature of the stationary point(s). (You need not calculate the second derivative.) **2**

(iv) sketch the function showing all significant information. **2**

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \,, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

Yr11 Ext1 2007

Q1

a) $(a-2b)(a+2b) + 2(a+2b)$

$= (a+2b)(a-2b+2)$

b) $x^2 > 9x$ $\qquad x<0,\ x>9$

$x^2 - 9x > 0$

$x(x-9) > 0$

c) $\dfrac{x+1}{x-3} \le 2$ $\qquad x<3$ or $x \geq 7$

$\times (x-3)^2 \qquad \times (x-3)^2$

$(x+1)(x-3) \le 2(x-3)^2$

$0 \le 2(x-3)^2 - (x+1)(x-3)$

$0 \le (x-3)(2x-6-x-1)$

$0 \le (x-3)(x-7)$

d) $11 + 4\sqrt{6} = 11 + 2\sqrt{24}$ $\qquad x = 8 \quad y = 3$

$x + y = 11 \qquad xy = 24$

e) $x^2 + x + 1 \equiv a(x-1)^2 + b(x-1) + c$

Let $x=0$ $\quad 1 = a - b + c$

Let $x=2$ $\quad 7 = a + b + c$

Let $x=1$ $\quad 3 = c$

$a - b = -2$ ①

$a + b = 4$ ②

$2a = 2$

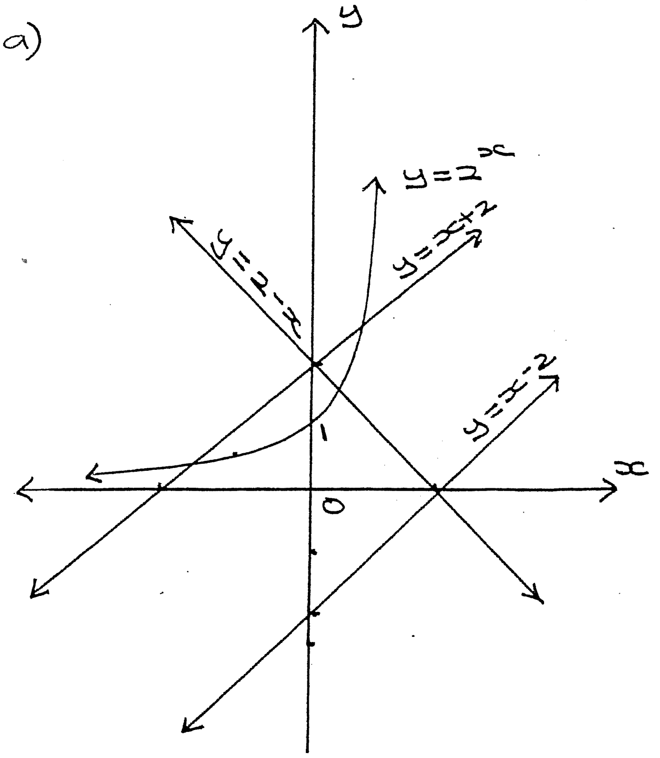
$a = 1$

$b = 3$

* one mark is allocated to method

Question 2

a)



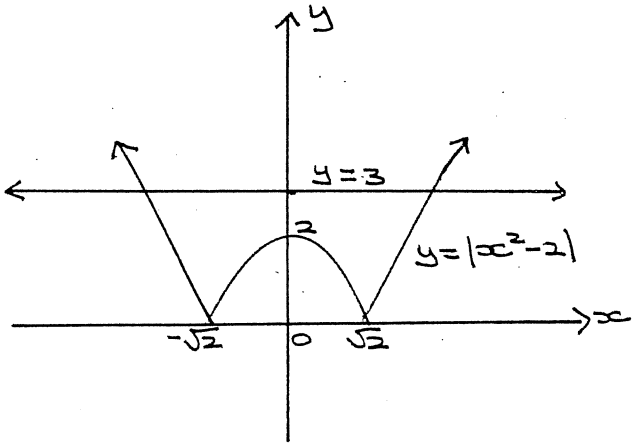
(0) Marks if answers were given without diagrams

(1) Mark if correct diagram was given without answers

(3) Marks for correct answers + diagram

(i) 2 (ii) 0 (iii) 1

b) i)



(1) Mark for each graph

For the graph $y=|x^2-2|$ $-\sqrt{2}$ and $\sqrt{2}$ had to be shown and the y-intercept

$$y=|x^2-2|=3$$
$$x^2-2=3$$
$$x^2=5$$
$$x=\pm\sqrt{5}$$

$0\leq y\leq 3$ $\quad -\sqrt{5}\leq x\leq\sqrt{5}$

(1) Mark for each inequality

c) $x^2 - 10x + 16y + 9 = 0$

$x^2 - 10x + 25 = -16y - 9 + 25$ — ① Mark

$(x-5)^2 = -16y + 16$

$(x-5)^2 = -16(y-1)$ — ① Mark

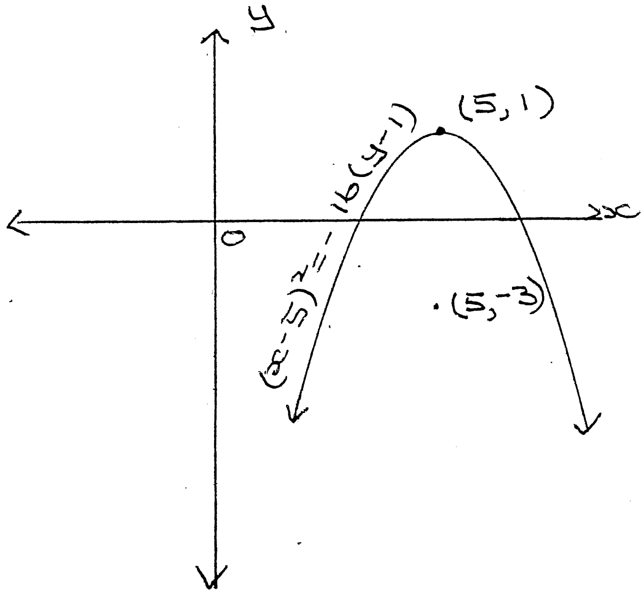
vertex $= (5,1)$

focal length $= 4$

① Mark for vertex

① Mark for and shape

① Mark for vertex



2007 Year 11 Preliminary

Question 3

(a) $3x + ky - 5 = 0$

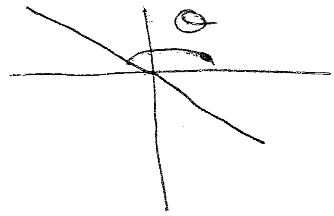
(i) $3x + ky - 5 = 0$

$$ky = -3x + 5$$

$$y = \frac{-3}{k}x + \frac{5}{k}$$ ✓ (1 mark) answer in gradient-intercept form.

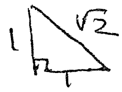
$\therefore$ The gradient is $-\frac{3}{k}$

(ii)



$\theta = 135^\circ$

$\tan\theta = m$



$\tan 135^\circ = -\tan 45^\circ$

$= -1$ ✓ 1 mark for $\tan 135^\circ = -1$

$\therefore \frac{-3}{k} = -1$

$\therefore \underline{k = 3}$ ✓ (2 marks) for k.

(b) $(-3, 2)$ $(5, 7)$ externally $1 : -3$ ✓ 1 mark for neg value.

$$P(x,y) = \left(\frac{kx_2 + lx_1}{k+l}, \frac{ky_2 + ly_1}{k+l}\right)$$

$$= \left(\frac{1\times5 + -3\times-3}{1+-3}, \frac{1\times7 + -3\times2}{1+-3}\right)$$ ✓ 1 mark correct sub.

$$= \left(\frac{5+9}{-2}, \frac{7-6}{-2}\right)$$

$$= \left(-7, -\frac{1}{2}\right)$$ ✓ 1 mark correct evaluation of your values sub. (3 marks)

(c) $3x - 4y + 12 = 0$ $\quad 3x - 4y - 6 = 0$

when $y = 0$ $x = -4$

$\therefore$ passes through $(-4, 0)$ ✓ (1 mark Finding a point on a line)

Perpendicular distance from $(-4, 0)$ to $3x - 4y - 6 = 0$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|3\times-4 + -4\times0 - 6|}{\sqrt{3^2 + (-4)^2}}$$ ✓ 1 mark for correct sub of your values.

1 mark correct evaluation of your values

3 (3 marks)

Question 3

(c) $d = \frac{|-12-6|}{5}$

$d = \frac{18}{5}$ ✓

(d) $x^2+(k-3)x-k=0$

(3 marks)

Real roots $\Delta \geq 0$

$\therefore b^2-4ac \geq 0$

1 mark correct sub into Δ

$(k-3)^2-(4\times1\times-k) \geq 0$ ✓

$k^2-6k+9+4k \geq 0$

$k^2-2k+9 \geq 0$

$k^2-\cancel{2k}+1+8 \geq 0$

$(k-1)^2+8 \geq 0$ ✓ 1 mark correct formation of perfect square.

$(k-1)^2 \geq 0$ For all values of k

So $(k-1)+8 \geq 0$ For all values of k

$\therefore x^2+(k-3)x-k=0$ has real roots for all values of k.

✓ 1 mark correct conclusion

Question 4

(a) $A(-2,1)$ $B(2,1)$

Let $P(x,y)$ be any point of the locus.

Condition of locus

$AP = 2BP$ — 1 mark correct condition of locus.

$$\sqrt{(x+2)^2+(y-1)^2} = 2\sqrt{(x-2)^2+(y-1)^2}$$

$$(x+2)^2+(y-1)^2 = 4\left((x-2)^2+(y-1)^2\right)$$

$$x^2+4x+4+y^2-2y+1 = 4(x^2-4x+4+y^2-2y+1)$$

$$x^2+4x+y^2-2y+5 = 4x^2-16x+16+4y^2-8y+4$$

$$3x^2-20x+3y^2-6y+15=0$$

1 mark for correct expansion + putting on LHS equal to 0

$$x^2-\frac{20}{3}x+y^2-2y+5=0$$

$$x^2-\frac{20}{3}x+\frac{100}{9}+y^2-2y+1 = -5+\frac{100}{9}+1$$

$$\left(x-\frac{10}{3}\right)^2+(y-1)^2 = -4+11\tfrac{1}{9}$$

$$\left(x-\frac{10}{3}\right)^2+(y-1)^2 = 7\tfrac{1}{9} = \frac{64}{9}$$

1 mark complete both squares correctly.

$\therefore$ circle centre $\left(\frac{10}{3}, 1\right)$ radius $=\frac{8}{3}$

1 mark for description and details.

(4)

(b)(i) $x^2=4ay$

$$y=\frac{x^2}{4a}$$

$$\frac{dy}{dx}=\frac{x}{2a}$$

Gradient of tangent at $P(2ap, ap^2)$

$\therefore m=p$

Gradient of normal at P

$\therefore m=\frac{-1}{p}$

1 mark for finding correctly the gradient.

Equation of a normal

$$y-ap^2=\frac{-1}{p}(x-2ap)$$

$$py-ap^3=-x+2ap$$

1 mark for correct derivation of the normal.

$\therefore x+py = 2ap+ap^2$ is the required equation.

(2)

Question 4

(b) (ii) equation of line through $S(0,a)$ $m = p$ ✓ 1 mark for gradient

$y - a = p(x - 0)$

$\therefore y = px + a$ ✓ 1 mark for correct equation.

2

(iii) Solve simultaneously

$y = px + a$ — ①

$x + py = 2ap + ap^3$ — ②

sub ① into ②

1 mark for unsimplified x value.

$x + p(px + a) = 2ap + ap^3$

$x + p^2x + pa = 2ap + ap^3$

$x(1 + p^2) = ap^3 + ap$

$x = \dfrac{ap(p^2+1)}{(1+p^2)}$

2

$\therefore x = ap$ — ③ ✓

check in ②

$ap + p(ap^2 + a)$

$= ap + ap^3 + ap$ ✓

Sub ③ in ①

$y = p(ap) + a$

$y = ap^2 + a$ ✓

1 mark for y value but ~~x~~ y had to be simplified.

$\therefore$ N has the coordinates $(ap, ap^2 + a)$

IV Locus of N $(ap, ap^2 + a)$

1 mark for the equation of the locus.

$\therefore x = ap$ $y = ap^2 + a$

$p = \dfrac{x}{a}$ $y = a\left(\dfrac{x}{a}\right)^2 + a$

$a = \dfrac{x}{p}$ $y = \dfrac{x^2}{a} + a$

✓

$\therefore$ The locus is a parabola.

The vertex is $(0, a)$

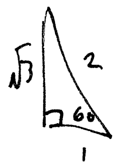
1 mark for the vertex. ✓

2

Yr 11 Ext 1 Final 2007

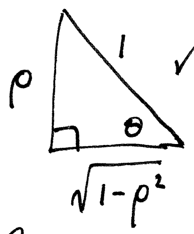
Q5

a) $2\theta = 60, 120, 420, 480$



$\therefore \theta = 30, 60, 210, 240$

b) $\sin\theta = p$



$\cos\theta = \sqrt{1-p^2}$

$\tan\theta = \dfrac{p}{\sqrt{1-p^2}}$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

c) $4\sec^2\theta - \tan\theta = 7$

$4(\tan^2\theta + 1) - \tan\theta - 7 = 0$

$4\tan^2\theta - \tan\theta - 3 = 0$

$(4\tan\theta + 3)(\tan\theta - 1) = 0$

$\tan\theta = \frac{-3}{4}$

$\theta = 143^\circ$ or 323°

$\tan\theta = 1$

$\theta = 45^\circ$ or 225°

d) Let $u = x^2 - 5x + 2$

$u + \dfrac{4}{-u} = 0$

$-u^2 + 4 = 0$

$u^2 - 4 = 0$

$u = \pm 2$

$x^2 - 5x + 2 = 2$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0$ or 5

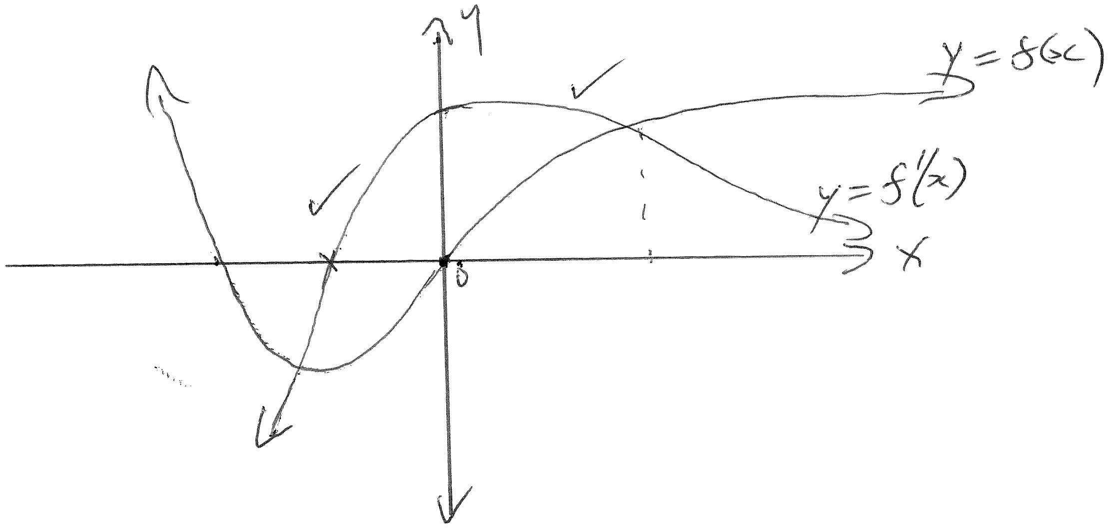
$x^2 - 5x + 2 = -2$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4$ or 1

Q6 (a)



(b) (i) $y = x\sqrt{x}$

$= x^{3/2}$ ✓

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2}$ ✓

(ii) If Normal has $m = -\frac{1}{4}$ ✓

Tangent has $m = 4$

$\therefore$ let $\frac{3\sqrt{x}}{2} = 4$

$\sqrt{x} = \frac{8}{3}$

$x = \frac{64}{9}$ ✓

$\therefore$ Coordinates are $\left(\frac{64}{9}, \frac{512}{27}\right)$ ✓

(18.96)

(c) (i) $f(x) = \frac{x-1}{\sqrt{2x+1}}$ $\begin{matrix} u \\ v \end{matrix}$

$f'(x) = \frac{\sqrt{2x+1} \cdot 1 - (x-1) \cdot \frac{1}{\sqrt{2x+1}}}{(\sqrt{2x+1})^2}$ ✓

$= \frac{(2x+1) - (x-1)}{(2x+1)\sqrt{2x+1}}$

$= \frac{x+2}{\sqrt{(2x+1)^3}}$ ✓

(ii) Show that $f'(x) > 0$ in the domain of x

Domain $2x+1 > 0$

$x > -\frac{1}{2}$ ✓

Now as $x + 2 > 0$ if $x > -\frac{1}{2}$ ✓

$2x + 1 > 0$ if $x > -\frac{1}{2}$ ✓

$\therefore f'(x) = \frac{x+2}{\sqrt{(2x+1)^3}} > 0$

Note: Marks were awarded if students correctly attempted to find Stat pts & nature.

7(a) $y = ax^4 + bx^3 + c$

(i) $\frac{dy}{dx} = 4ax^3 + 3bx^2$

$\frac{d^2y}{dx^2} = 12ax^2 + 6bx$

let $\frac{d^2y}{dx^2} = 0$ for Pt of inflection

$12ax^2 + 6bx = 0$

$6x(2ax + \cancel{6}b) = 0$

$x = -\frac{b}{2a}$

let $x = 1$ $\therefore 2a = -b$ or $b = -2a$

as $c = -1$ (from graph)

$\therefore P(1,0)$ satisfies $y = ax^4 + bx^3 + c$

$\therefore 0 = a + b - 1$

$\therefore a + b = 1$

$\therefore a - 2a = 1$

$a = -1$ #

$b = 2$ #

$c = -1$ #

[If $(2,0)$ is used then $c = -1, a = -\frac{7}{8}, b = \frac{15}{8}$

(ii) let $\frac{dy}{dx} = 0$

$4ax^3 + 3bx^2 = 0$

$\therefore -4x^3 + 6x^2 = 0$

$-2x^2(2x - 3) = 0$

$\therefore x = 0, \frac{3}{2}$

Take $x = \frac{3}{2}$

$\therefore y = -\cancel{1}\left(\frac{3}{2}\right)^4 + 2\left(\frac{3}{2}\right)^3 - 1$

$= -\frac{81}{16} + \frac{27}{4} - 1$

$\therefore W\left(\frac{3}{2}, \frac{11}{16}\right)$

7b) $y = \dfrac{2}{(x-1)(x-3)}$

(i) Vertical asymptotes: $x = 1$, $x = 3$ ✓ (Both)

Horizontal asymptote: $y = 0$ ✓ $\boxed{2}$

(ii) y-int occurs when $x = 0$

$$y = \frac{2}{-1 \times -3}$$
$$= \frac{2}{3}$$ ✓ $\boxed{1}$

(iii) $$\frac{dy}{dx} = \frac{(x-1)(x-3)\cdot 0 - 2[(x-1)\cdot 1 + (x-3)\cdot 1]}{(x-1)^2(x-3)^2}$$

$$= \frac{-2[2x-4]}{(x-1)^2(x-3)^2}$$

$$= \frac{-4[x-2]}{(x-1)^2(x-3)^2}$$ ✓ $\boxed{2}$

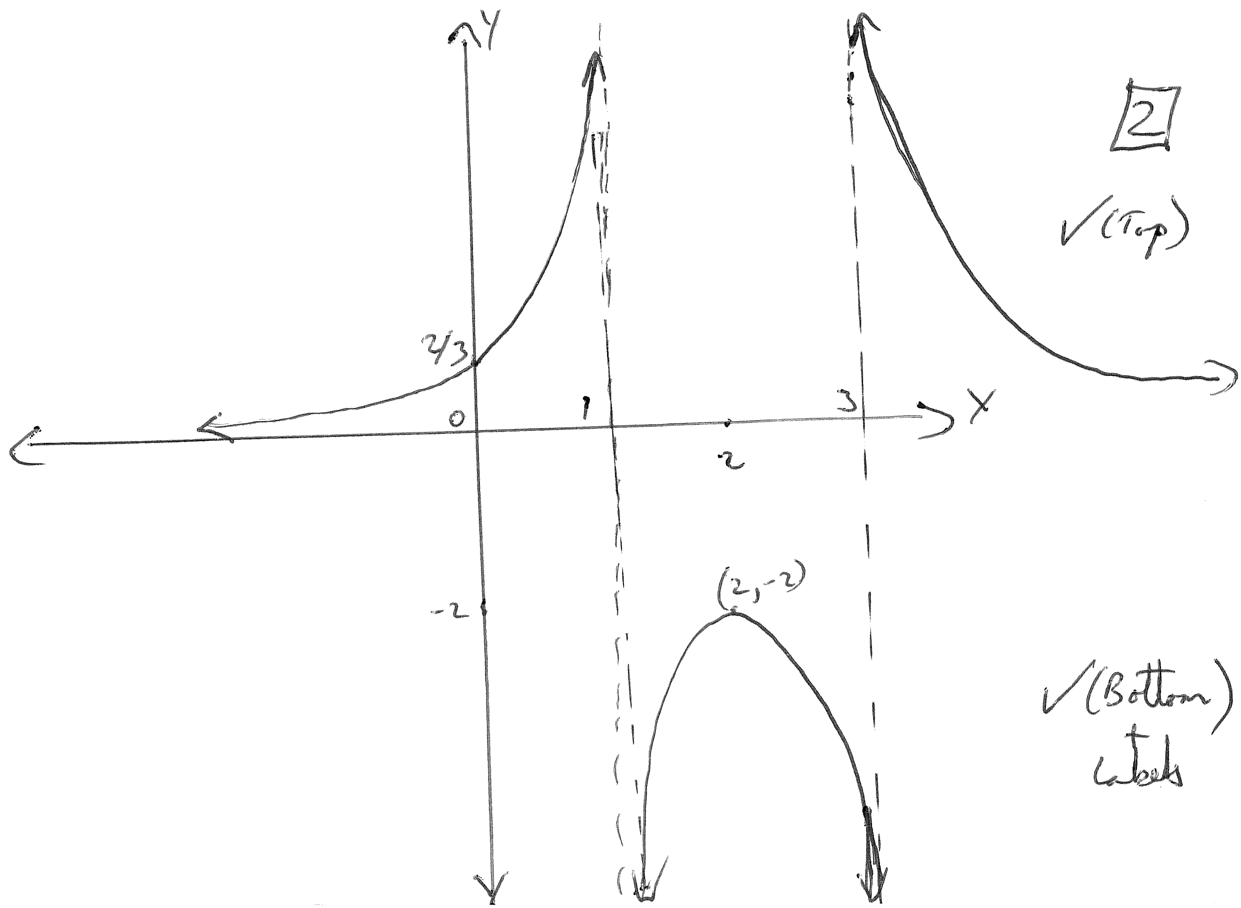
St pts occur when $\frac{dy}{dx} = 0$ $\therefore x = 2$

$\therefore$ Coordinates $(2, -2)$

$\therefore$ Max T.P.

Test: ✓

x	1.9	2	2.1
$\frac{dy}{dx}$	+	0	−
	/	—	\



$\boxed{2}$

✓ (Top)

✓ (Bottom) + labels