Section I
Attempt Questions 1-10
All questions are equal value.
Use the multiple choice answer sheet for Questions 1-10

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1 Simplify $\dfrac{x^3-1}{x^2-1}\times\dfrac{x^2-4x-5}{4x^2+4x+4}$

(A) $\dfrac{(x-5)}{4}$

(B) $\dfrac{(x-1)}{4}$

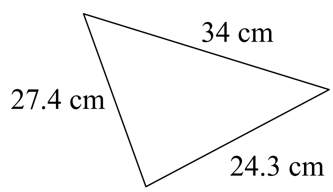
(C) $\dfrac{(x+1)}{4}$

(D) $\dfrac{(x^2+x+1)}{4}$

2 What is the solution to the equation $|x-2|=2x-1$?

(A) $x=-3$
(B) $x=-1$
(C) $x=1$
(D) $x=3$

3 The smallest angle in the triangle below is θ.

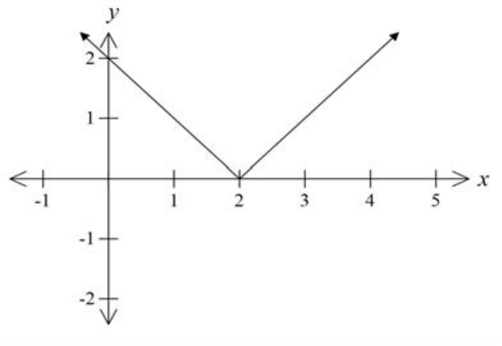


What is the value of θ to the nearest degree?
(A) 30º
(B) 45º
(C) 53º
(D) 82º

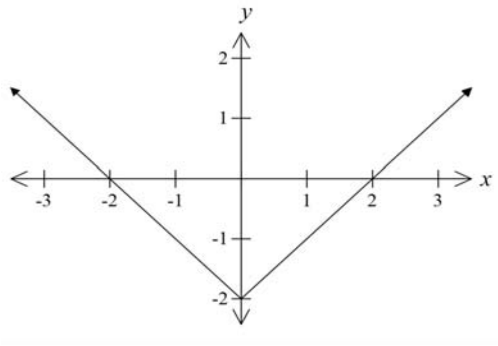
The smallest angle in the triangle below is θ.

4 Which graph best represents $y = |x| - 2$?

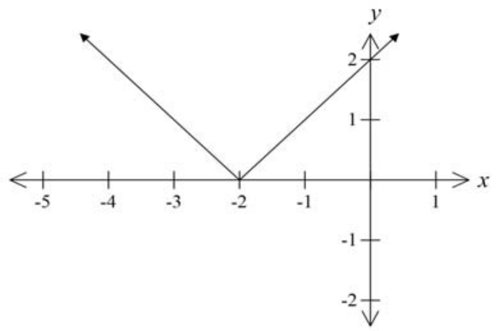
(A)



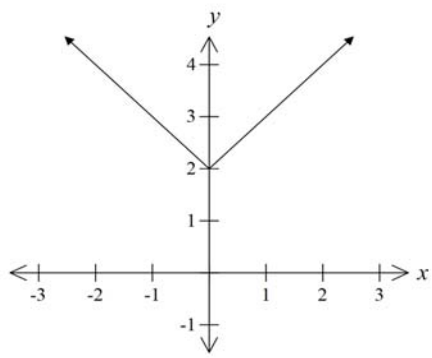
(B)



(C)



(D)



5 Which of these is the limiting sum of the geometric series $\frac{2}{5} - \frac{2}{15} + \frac{2}{45} - \frac{2}{135} + \ldots..$

(A) $\frac{3}{5}$ (B) $\frac{8}{27}$ (C) 0 (D) $\frac{3}{10}$

6 If $3\cos\theta + 2 = 0$ and $\tan\theta > 0$, what is the exact value of $\sin(\theta + 180)$?

(A) $-\frac{\sqrt{5}}{3}$

(B) $-\frac{\sqrt{5}}{2}$

(C) $\frac{\sqrt{5}}{2}$

(D) $\frac{\sqrt{5}}{3}$

7 What is the centre and radius of the circle with the equation
$x^2 + y^2 + 6x - 8y - 11 = 0$

(A) Centre $(-3,-4)$ and radius 36
(B) Centre $(-3,4)$ and radius 36
(C) Centre $(-3,-4)$ and radius 6
(D) Centre $(-3,4)$ and radius 6

8 What is the value of k if the sum of the roots of $x^2 - (k-1)x + 2k = 0$ is equal to the product of the roots?

(A) -3
(B) -2
(C) -1
(D) 1

9 Which of the following is the correct simplified expression for differentiating $f(x) = \dfrac{1}{x}$ from first principles?

(A) $f'(x) = \lim\limits_{h \to 0} \dfrac{-1}{x(x+h)}$

(B) $f'(x) = \lim\limits_{h \to 0} \dfrac{x+h-x}{h}$

(C) $f'(x) = \lim\limits_{h \to 0} \dfrac{\frac{1}{x} - \frac{1}{x+h}}{h}$

(D) $f'(x) = \lim\limits_{h \to 0} \dfrac{h}{x+h-x}$

10 What is the equation of the normal to the curve $f(x) = x^2 - 4x$ at $(1,-3)$?

(A) $x + 2y - 7 = 0$
(B) $x - 2y - 7 = 0$
(C) $2x - y - 5 = 0$
(D) $2x + y + 5 = 0$

Section II
Attempt Questions 11-14
Each question is worth 15 marks.

Answer each question in a new writing booklet. Extra booklets are available.

All necessary working should be shown in every question

Question 11 (15 Marks) Use a NEW writing booklet.

(a) Solve the following equation $x^2 + 3x = \dfrac{8}{x^2 + 3x} + 2$

by using the substitution $A = x^2 + 3x$. **3**

(b) Find A, B and C such that: **3**

$$4x^2 - x + 1 \equiv Ax(x+1) + B(x+1) + C.$$

(c) Solve **2**

$$x^6 + 26x^3 - 27 = 0$$

(d) Solve for x **3**

$$\frac{2}{x-1} \geq x.$$

(e) What are the coordinates of the point that divides the interval joining the points $A(1,1)$ and $B(5,3)$ externally in the ratio 2:3? **2**

(f) Find the equation of the straight line that passes through the point of intersection of the lines $x - 2y = 5$ and $3x - y + 1 = 0$ and the point $(2,1)$. **2**

Question 12 (15 Marks) Use a NEW writing booklet.

(a) State the domain of:

(i) $y = x + \dfrac{1}{x-2}$ **1**

(ii) $y = \sqrt{2x^2 - x - 6}$ **2**

(b) Find the horizontal asymptote of the function $y = \dfrac{2x^2 - 4x + 3}{x^2 - 5}$ **2**

(c) Find the inequalities that describe the shaded regions in the following graph. **3**

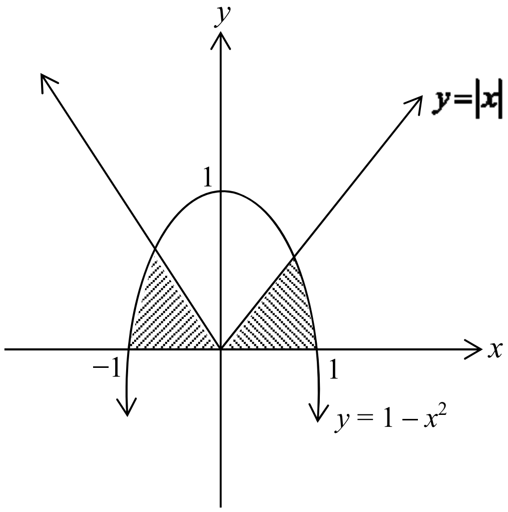


Diagram not to scale

(d) Shade the common region defined by:

$x^2 + y^2 < 25 \quad and \quad 3x - y \geq 2$ **3**

(e) What values of m is $-4x^2 + 3x + m$ a negative definite? **2**

(f) For what values of c is the line $y = x + c$ tangent to the curve $y = 2x^2 - 7x + 4$ **2**

Question 13 (15 Marks) Use a NEW writing booklet.

(a) Evaluate $\sum_{n=0}^{20}(-2)^n$ **3**

(b) Find the sum of all positive integers less than 20 000 which are divisible by 11. **3**

(c) Prove by mathematical Induction

$$\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}=\frac{n}{2n+1}$$ **4**

(d) For the parabola $x^2 = -16y + 32$.

(i) Give the vertex and focus of the parabola. **3**

(ii) Find the equation of the tangent to the parabola at the point (8, -2). **2**

Question 14 (15 Marks) Use a NEW writing booklet.

(a) Solve $2\cos 2\theta = \sqrt{3}$ for $0° \le \theta \le 360°$. **3**

(b) Prove the identity $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \sec x . \operatorname{cosec} x$ **2**

(c) (i) Show that $10\sin^2\beta + \cos\beta - 7 = (3 - 5\cos\beta)(2\cos\beta + 1)$. **2**

(ii) Hence solve $10\sin^2\beta + \cos\beta - 7 = 0$ for $0° < \beta < 360°$ to the nearest degree. **3**

(d) Differentiate $\sqrt[3]{2-3x^2}$, give your answer without negative or fractional indices. **2**

(e) Find the value of k if $f'(-3) = 1$ where **3**

$$f(x) = \frac{x^2 + k}{x^2 - k}\ .$$

END of PAPER

Yr 11 Ext 1 Prelim 2012

1	A
2	C
3	B
4	B
5	D
6	D
7	D
8	C
9	A
10	B

Ext1 Preliminary Examination 2012

Question 11

(a) $x^2+3x = \frac{8}{x^2+3x} + 2$

let $M = x^2+3x$

$M = \frac{8}{M} + 2$

$M^2 = 8 + 2M$

$M^2 - 2M - 8 = 0$

$(M-4)(M+2) = 0$

$\therefore M = 4 \text{ or } -2$ ✓

$\therefore x^2+3x = 4$ or $x^2+3x = -2$

$x^2+3x-4 = 0$ $\quad$ $x^2+3x+2=0$

$(x+4)(x-1) = 0$ $\quad$ $(x+2)(x+1) = 0$ $\quad$ 3 marks

$\therefore x = -4 \text{ or } 1$ ✓ $\quad$ $\therefore x = -2 \text{ or } -1$ ✓

(b) $4x^2 - x + 1 \equiv Ax(x+1) + B(x+1) + C$

Method 1

$\equiv Ax^2 + Ax + Bx + B + C$

$\equiv Ax^2 + (A+B)x + (B+C)$

$\therefore 4x^2 \equiv Ax^2$

$4 = A$ ✓

$-x = (A+B)x$

$-1 = (4+B)$

$\therefore -5 = B$ ✓

$1 = B+C$

$1 = -5+C$

$6 = C$ ✓

Method 2

$A = 4$ ✓

Let $x=-1$ $\quad 6 \equiv 0 + 0 + C$

$6 = C$ ✓

Let $x = 0$

$1 \equiv 0 + B + C$

$1 \equiv B + 6$

$\therefore -5 = B$ ✓

$\therefore$

$\therefore A = 4, \; B = -5, \; C = 6$ $\quad$ 3 marks

(c) $x^6 + 26x^3 - 27 = 0$

let $u = x^3$

$u^2 + 26u - 27 = 0$

$(u + 27)(u - 1) = 0$

$\therefore u = -27$ or $u = 1$ ✓

$\therefore x^3 = -27$ or $x^3 = 1$

$\therefore x = -3$ or 1 ✓ (2 marks).

(d) $\dfrac{2}{x-1} \geq x \qquad x \neq 1$

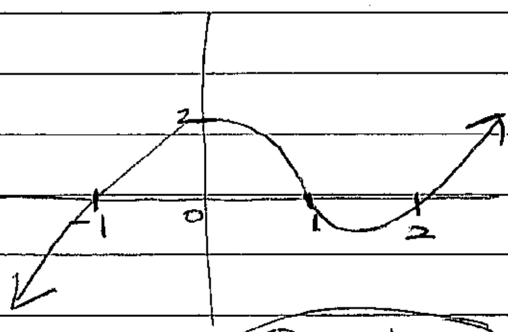
$2(x-1) \geq x(x-1)^2$ ✓

$0 \geq x(x-1)^2 - 2(x-1)$

$0 \geq (x-1)(x(x-1) - 2)$

$0 \geq (x-1)(x^2 - x - 2)$

$0 \geq (x-1)(x-2)(x+1)$ ✓



3 marks

$\therefore x \leq -1$ or $1 \leq x \leq 2 \qquad x \neq 1$

$\therefore x \leq -1$ or $1 < x \leq 2$ ✓

(e) $A(1,1) \quad B(5,3)$ $\qquad k : l$

$2 : -3$ External

$P(x,y) = \left(\dfrac{kx_2 + lx_1}{k+l}, \dfrac{ky_2 + ly_1}{k+l} \right)$

$= \left(\dfrac{2\times5 + -3\times1}{-1}, \dfrac{2\times3 + -3\times1}{-1} \right)$ ✓

$= \left(\dfrac{10-3}{-1}, \dfrac{6-3}{-1} \right)$

$= (-7, -3)$ ✓ 2 marks

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Question 11 Continued.

(f) $x - 2y = 5$

$3x - y + 1 = 0$

$(x - 2y - 5) + k(3x - y + 1) = 0$ sub in $(2,1)$

$(2 - 2 - 5) + k(6 - 1 + 1) = 0$

$-5 + k \times 6 = 0$

$6k = 5$

$k = \frac{5}{6}$ ✓

$\therefore (x - 2y - 5) + \frac{5}{6}(3x - y + 1) = 0$

$6(x - 2y - 5) + 5(3x - y + 1) = 0$

$6x - 12y - 30 + 15x - 5y + 5 = 0$ (2 marks).

$\therefore 21x - 17y - 25 = 0$ ✓

Method 2 – if the equations were solve simultaneously point of intersection $\left(\frac{-7}{5}, \frac{-16}{5}\right)$ ✓

equation of line $21x - 17y - 25 = 0$ ✓

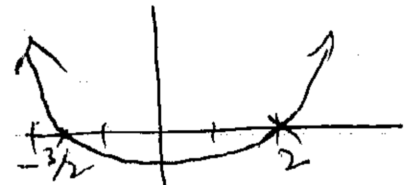
Ext 1 Y11 Prelim

Question 12 (15)

(a) (i) $x \in \mathbb{R},\ x \neq 2$ ✓

(ii) $2x^2 - x - 6 \geq 0$ ✓

$(2x+3)(x-2) \geq 0$



$x \leq -\frac{3}{2},\ x \geq 2$ ✓

(b) $y = \dfrac{2 - \frac{4}{x} + \frac{3}{x^2}}{1 - 5/x}$ ✓

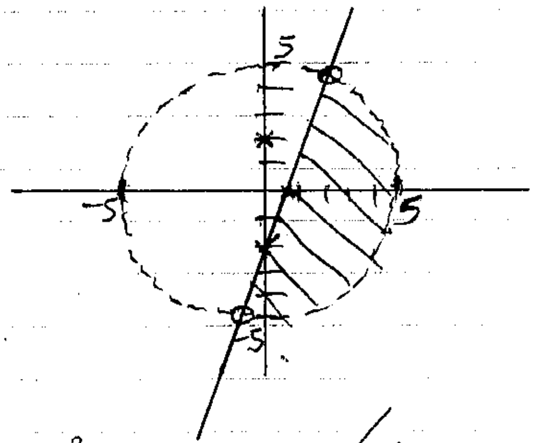
As $x \to \infty$ $y \to \frac{2}{1}$.

$\therefore$ horizontal asymptote is $y = 2$ ✓

(c). $y \leq 1 - x^2,\ y \leq |x|,\ y \geq 0$.

✓ ✓ ✓

(d).



1 broken circle

1 inside circle

1 right of correct line.

(e) $b^2 - 4ac < 0$ ✓ (and $a < 0$, which it is as $a = -4$)

$9 - 4 \times -4 \times m < 0$.

$9 + 16m < 0$.

$16m < -9$

$m < \frac{-9}{16}$ ✓

(f). solve $x + c = 2x^2 - 7x + 4$. ✓

$2x^2 - 8x + (4 - c) = 0$.

tangent when $\Delta = 0$.

$64 - 4 \times 2 \times (4 - c) = 0$.

$64 - 8(4 - c) = 0$.

$64 - 32 + 8c = 0$. ✓

$c = -4$

Yr 11 Ext 1 Prelim 2012

Section I

1. A 2. C 3. B 4. B 5. D 6. D 7. D 8. C 9. A 10. B

Section II

Question 13

a) $\sum_{n=0}^{20} (-2)^n = 1 + -2 + 4 + -8 + \ldots$

$a = 1 \quad r = -2 \quad n = 21$ ✓

$S_{21} = \dfrac{1(-2^{21} - 1)}{-2 - 1}$ ✓

$= 699051$ ✓

b) $11 + 22 + 33 + \ldots$

$T_n = a + (n-1)d$

$20000 > 11 + (n-1)11$

$20000 > 11 + 11n - 11$

$n < 1818.18$ ✓

$n = 1818$ ✓

$S_{1818} = \frac{1818}{2}(22 + 1817 \times 11)$

$= 18188181$ ✓

c) Step 1

Prove true ~~for~~ $n = 1$

$LHS = \dfrac{1}{(2(1)-1)(2(1)+1)} = \dfrac{1}{1 \times 3} = \dfrac{1}{3}$

$RHS = \dfrac{1}{2(1)+1} = \dfrac{1}{3}$

$LHS = RHS$

$\therefore$ true for $n = 1$

Step 2 Assume true for $n = k$

$\sum_{n=1}^{k} \dfrac{1}{(2n-1)(2n+1)} = \dfrac{k}{2k+1}$ ✓

Notation must be correct

Step 3 Prove true for $n=k+1$

Prove $\displaystyle\sum_{r=1}^{k+1}\frac{1}{(2r-1)(2r+1)} = \frac{k+1}{2(k+1)+1}$

$$LHS = \sum_{r=1}^{k+1}\frac{1}{(2r-1)(2r+1)}$$

$$= \sum_{r=1}^{k}\frac{1}{(2r-1)(2r+1)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2(k+1)+1)}$$ From Step 2

$$= \frac{k(2(k+1)+1)+1}{(2k+1)(2(k+1)+1}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2(k+1)+1)}$$

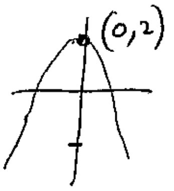
$$= \frac{(k+1)(2k+1)}{(2k+1)(2(k+1)+1)}$$

$$= \frac{k+1}{2(k+1)+1}$$

$\therefore$ true for $n=k+1$

Step 4 - - - - - - - -

d) i) $x^2=-16y+32$



$x^2=-16(y-2)$

$a=4$ vertex $(0,2)$

focus $(0,-2)$

ii) $-16y = x^2-32$

$y = \frac{-1}{16}x^2+2$

$y' = -\frac{1}{8}x$

$x=8$ $m=-1$

$y--2 = -1(x-8)$

$y+2 = -x+8$

$y = -x+6$

Q14 (a) $\cos 2\theta = \frac{\sqrt{3}}{2}$ ① 1 correct ② 2 " ③ All "

$$2\theta = 30^\circ, 330^\circ, 390^\circ, 690^\circ$$

$$\theta = 15^\circ, 165^\circ, 195^\circ, 345^\circ$$

(b) $LHS = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$ ①

$= \frac{1}{\cos x \sin x}$ ①

$= \sec x \operatorname{cosec} x$

$= RHS$

(c)(i) $10(1-\cos^2\beta) + \cos\beta - 7$ ① $1-\cos^2\beta$

$= 10(1-x^2) + x - 7$, $x = \cos\beta$

$= -(10x^2 - x - 3)$ ②

$= -(5x-3)(2x+1)$

$= (3-5x)(2x+1)$, $x = \cos\beta$

(ii) $(3-5x)(2x+1) = 0$, $x = \cos\beta$ ①

$x = \frac{3}{5}$ or $-\frac{1}{2}$ ① 2 solutions

$\cos\beta = \frac{3}{5}$ or $-\frac{1}{2}$ ① All "

$\beta = \underbrace{53^\circ, 307^\circ}$, $120^\circ, 240^\circ$

nearest degree

(d) $\frac{d}{dx}(2-3x^2)^{\frac{1}{3}} = \frac{1}{3}(2-3x^2)^{-\frac{2}{3}} \times -6x$ ①

$= \frac{-2x}{\sqrt[3]{(2-3x^2)^2}}$ ②

(e) $f(x) = \frac{x^2+k}{x^2-k}$

$f'(x) = \frac{(x^2-k).2x - (x^2+k).2x}{(x^2-k)^2}$ ①

$= \frac{-4kx}{(x^2-k)^2}$

$f(-3) = \frac{12k}{(9-k)^2} = 1$ ①

$12k = (9-k)^2 = 81 - 18k + k^2$

$k^2 - 30k + 81 = 0$

$(k-3)(k-27) = 0$, $k = 3$ or 27 ①