2007

Question 1.

a. Simplify

i. $\sqrt{27}+\sqrt{75}$ **1**

ii. $(3\sqrt{2}+\sqrt{5})(3\sqrt{2}-\sqrt{5})$ **1**

b. Write $0.\dot{2}\dot{9}$ as a fraction. **1**

c. Solve $2^x = 25$, writing your answer to 2 decimal places. **1**

d. Solve $-3 \le 2-3x \le 8$ **2**

e. Given $p^2 = q^2 + r^2 - 2qr\cos P$ make $\cos P$ the subject **2**

f. Simplify $(\sin x + \cos x)^2 - 2\sin x \cos x$ **2**

g. Sketch the region where $y \le 4 - x^2$ and $y \ge 2x$ **2**

Question 2

a. Express in simplest form **2**

$$(a^2+8a+15)\times\frac{a}{a+3}$$

b. Solve the simultaneous equations **2**

$$x^2+y^2=25$$

$$y=x$$

c. On separate number planes draw neat sketches of:

i. $x=y^2-1$ **1**

ii. $y=x^3$ **1**

iii. $xy=2$ **1**

d. $Q(x)$ is the function x^2-6x+8

i. Find the minimum value of $Q(x)$ **2**

ii. Solve $Q(x)>0$ **1**

e. Factorise and simplify $2^{n+1}+2^n$ **2**

Question 3

a. Solve

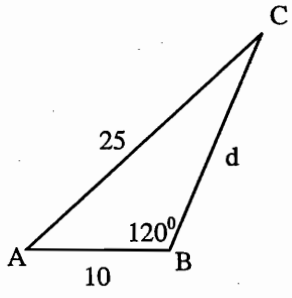
i. $\sin x = \frac{1}{2}$ for $0^0 \leq x \leq 360^0$ **2**

ii. $\tan^2 x = 1$ for all real x **2**

b. If $\sin A = \frac{5}{13}$ and A lies between 90^0 and 360^0 find the exact value(s) of $\cot A$ **2**

c. Sketch $y = 2\sin 3x$ for $0^0 \leq x \leq 360^0$ **3**

d. Find d



3

Question 4

a. For the series $0.5+0.75+1+.....$

Find the sum of the first twelve terms **2**

b. Find the limiting sum of the series

$12+4+1\frac{1}{3}+.....$ **2**

c. The first term of an arithmetic series is 31 and the sixth term is 96.

Find the twentieth term. **2**

d. If $S_n = 11n - n^2$ find T_n **2**

e. If $x+1,\ 8-x,\ x+10$ are in geometric progression

Find the progression **2**

f. Evaluate $\sum_{n=1}^{12} 5\times 2^{n-1}$ **2**

Question 5

a. Evaluate

i. $\lim_{x\to 0}\frac{x^2+x}{2x}$ **1**

ii. $\lim_{x\to 0}\frac{x+3}{x-3}$ **1**

iii. $\lim_{x\to \infty}\frac{x+1}{2x-1}$ **1**

b. What is the difference between the graphs of

$y=x+2$ and $y=\frac{(x-2)(x+2)}{x-2}$ **2**

c. Find $\frac{dy}{dx}$ given

i. $y=2x^3-7x^2+8x-1$ **1**

ii. $y=x^2(x+3)$ **2**

iii. $y=\frac{2x^2-3}{7-x}$ **2**

iv. $y=(5x^4-7)^4$ **2**

Question 6

a. For the curve

$y = 2x^3 - 3x^2 - 12x + 2$

i. Find the stationary points and determine their nature **3**

ii. Find the point(s) of inflexion **2**

iii. Sketch the curve showing all important features **2**

b. If $f(x) = 2x^3 - 9x^2 - 60x$

Solve $f'(x) = 0$ **2**

c. Find the equation of the tangent to **3**

$y = \sqrt{3x-2}$ at (2,2)

Question 7.

a. Calculate the acute angle between the lines **2**

$y = 2x - 3$ and
$3x + 5y - 1 = 0$ to the nearest minute

b. Divide A(1,7) and B(5,-2) externally in the ratio 3:2 **2**

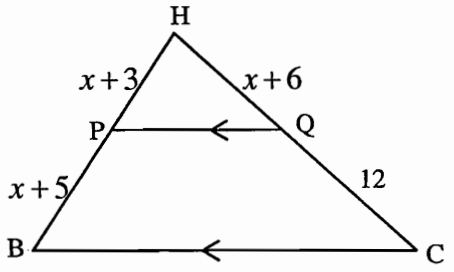
c. Solve for x: $|2x - 5| + 3x = 0$ **3**

d. Expand $\sin(x + y)$. Hence write down an expression for $\sin 2\theta$ **2**

e. Solve the inequality $\dfrac{3}{x(x-2)} < 1$ **3**

Question 8

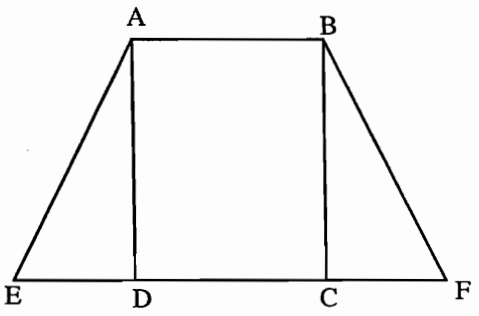
a.



3

PQ||BC. Show x has 2 values and check these.

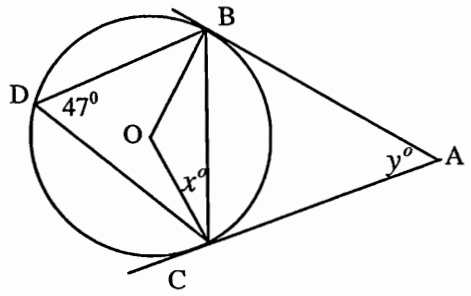
b.



3

ABCD is a rectangle. ED = CF
Prove AE = BF

c.



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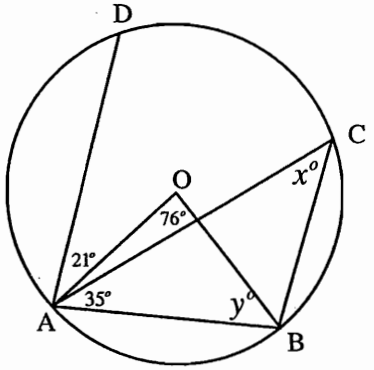
AB, AC are tangents. Find <OCB, <BAC giving reasons

d.



3

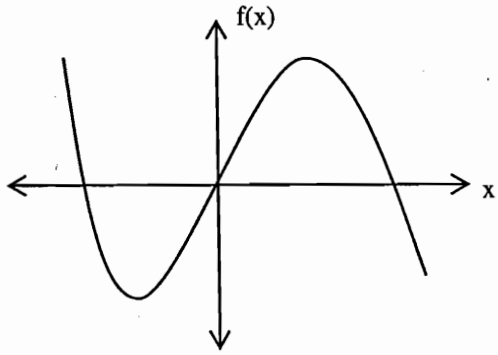
Given: $\angle DAO = 21^0$, $\angle AOB = 76^0$, $\angle ACB = x$, $\angle CAB = 35^0$ and $\angle OBA = y$

Find x, y and prove AD || BC

Question 9

a. Copy the graph of the function below onto your own paper

Directly below the graph, draw the derivative of the function, lining up important features



2

b. Find the exact value of $\cos 75^0$ **2**

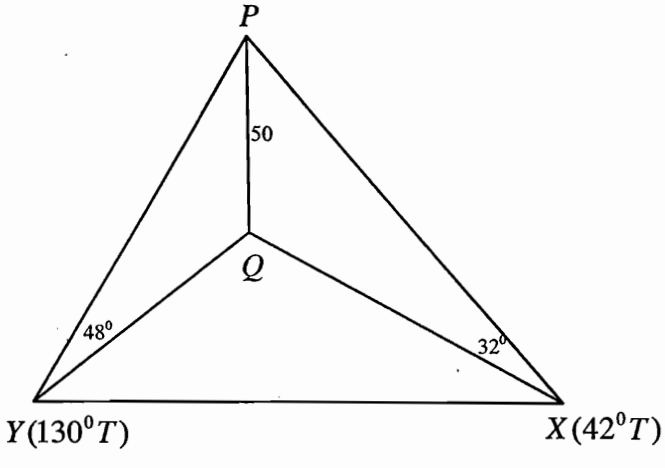
c. Find the equation of the line that passes through the point of intersection of the lines $3x+2y-6=0$ and $x-2y+5=0$, and the point (-3,-2) **3**

d. From an observation tower PQ of height 50m, two points X and Y at ground level have bearings 042^0T and 130^0T. From the top of the tower the angles of depression of X and Y are 32^0 and 48^0 respectively.

Prove that:

i. $XY^2 = 50^2(\cot^2 32^0 + \cot^2 48^0 - 2\cot 32^0 \cot 48^0 \cos 88^0)$ **3**

ii. Find XY to the nearest metre. **2**



Year 11 2007 Yearly Ext 1.

Q1) a) $2^x = 25$

$x = 4.64$ to 2D 1

b) $\frac{29}{99}$ 1

a) i/ $3\sqrt{3} + 5\sqrt{3}$

$= 8\sqrt{3}$ 1

ii/ $18 - 5$

$= 13$ 1

d) $-3 \leq 2 - 3x \leq 8$

$-5 \leq -3x \leq 6$ 1

$\frac{5}{3} \geqslant x \geqslant -2$ 1

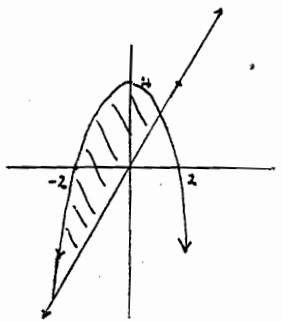
e) $\cos P = \frac{q^2 + r^2 - p^2}{2qr}$ 2

f) $= \sin^2 x + 2\sin x \cos x + \cos^2 x$ 1

$- 2\sin x \cos x$

$= 1$ 1

g)



1 graph

1 region

Q2) a) $(a+5)(a-3) \times \frac{a}{a+3}$ 1

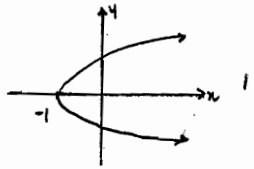
$= a(a+5)$ 1

b) $2x^2 = 25$

$x = \pm\frac{5}{\sqrt{2}}$ 1

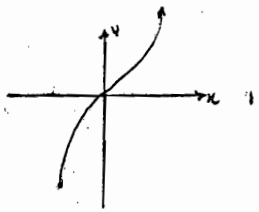
$\therefore \left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right) \left(\frac{-5}{\sqrt{2}}, -\frac{5}{\sqrt{2}}\right)$ 1

c) i/



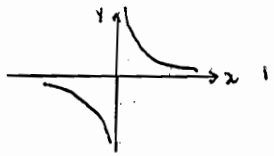
1

ii/



1

iii/



1

d) $x^2 - 6x + 8 = (x-3)^2 - 1$ 1

$= (x-4)(x-2)$

i/ min value -1 1

ii/ $Q(x) > 0$

when $x < 2$ or $x > 4$ 1

1 1

e) $2^n(2+1) = 3.2^n$

Q3) a) i/ $\sin x = \frac{1}{2}$

$\therefore x = 30°, 150°$

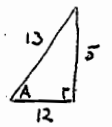
1 1

ii/ $\tan x = \pm 1$

$\therefore x = 180n \pm 45°$

1 1

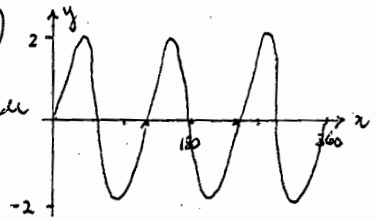
b)



1

$\therefore \cot A = \frac{12}{5}$ 1

c)



1 shape

1 amplitude

1 period

d) $\cos 120° = \frac{100 + d^2 - 625}{20d}$ 1

$10d = d^2 - 525$

Note: $d^2 - 10d - 525 = 0$

Quadratic can be done with sine rule $d = \frac{10 \pm \sqrt{100 + 2100}}{2}$

$= 5 \pm \sqrt{550}$

$= -5 \pm 5\sqrt{22}$

$d > 0$

$\therefore d = 18.45$ 1

f) $5 + 5.2 + 5.4$ 1

$S_{12} = 5(2^{12} - 1)$

$= 20475$ 1

Q4) a) $S_2 = \frac{12}{2}\left\{1 + 11 \cdot \frac{1}{4}\right\}$

$= 6 \times \frac{15}{4}$ 1

$= \frac{45}{2}$ 1

b) $S_\infty = \frac{a}{1-r}$

$= \frac{12}{1 - \frac{1}{3}}$ 1

$= 12 \times \frac{3}{2}$

$= 18$ 1

c) $a + 5d = 96$

$a = 31$

$\therefore 5d = 65$

$d = 13$

$\therefore T_{20} = 31 + 19.13$

$= 278$

a) $S_n = 11n - n^2$

$S_{n-1} = 11(n-1) - (n$

$= 11n - 11 - n^2 +$

$= 13n - n^2 -$

$\therefore T_n = 11n - n^2 - 13n +$

$= 12 - 2n$

e) $\frac{8-x}{x+1} = \frac{x+10}{8-x}$

$64 - 16x + x^2 = x^2 + 11x$

$27x = 54$

$x = 2$

$\therefore$ 3, 6, 12, 24 . . . .

Q5) a) i/ $\lim_{x \to 0} \frac{x(x+1)}{2x}$

$= \frac{1}{2}$ 1

ii/ -1 1

iii/ $\frac{1}{2}$ 1

b) straight line V's straight line with a hole at $(2,4)$ 2

c) i/ $\frac{dy}{dx} = 6x^2 - 14x + 8$ 1

ii/ $\frac{dy}{dx} = x^2 \cdot 1 + 2x(x+3)$

$= 3x^2 + 6x$ 2

iii/ $\frac{dy}{dx} = \frac{(7-x)4x + (2x^2-3)}{(7-x)^2}$

$= \frac{28x - 4x^2 + 2x^2 - 3}{(7-x)^2}$

$= \frac{28x - 2x^2 - 3}{(7-x)^2}$ 2

iv/ $\frac{dy}{dx} = 4(5x^4 - 7)^3 \cdot 20x^3$

$= 80x^3(5x^4 - 7)^3$ 2

Q6C) $y = (3x-2)^{1/2}$ 1

$\frac{dy}{dx} = \frac{1}{2}(3x-2)^{-1/2} \cdot 3$

at $x = 2$ $\frac{dy}{dx} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$ 1

$y - 2 = \frac{3}{4}(x-2)$

$4y = 3x + 2$ 1

Q6) a) $y = 2x^3 - 3x^2 - 12x + 2$

i/ $\frac{dy}{dx} = 6x^2 - 6x - 12$

Want $\frac{dy}{dx} = 0$ $6(x^2 - x - 2) = 0$

$\therefore x = -1$ or $+2$ 1

$\frac{d^2y}{dx^2} = 12x - 6$

at $x = 2$ $\frac{d^2y}{dx^2} > 0$ $\therefore$ min at $(2, -18)$

at $x = -1$ $\frac{d^2y}{dx^2} < 0$ $\therefore$ max at $(-1, 9)$

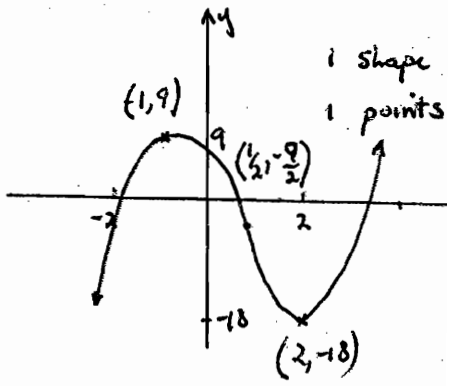
ii/ poss infl at $\frac{d^2y}{dx^2} = 0$

ie $x = \frac{1}{2}$ 1

x	0	$\frac{1}{2}$	1
$\frac{d^2y}{dx^2}$	<0	0	>1

$\therefore$ infl at $(\frac{1}{2}, -\frac{9}{2})$ 1

iii/



1 shape
1 points

b) $f'(x) = 6x^2 - 18x - 60$

$= 6(x^2 - 3x - 10)$

$= 6(x-5)(x+2)$

$f'(x) = 0$ when $x = -2$ or 5

Q7) a) $m_1 = 2 \quad m_2 = \frac{-3}{5}$

$\tan(\alpha-\beta) = \left| \frac{2+\frac{3}{5}}{1+2.\frac{-3}{5}} \right|$ 1

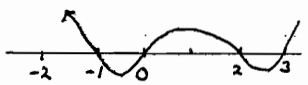
$= \left| \frac{\frac{13}{5}}{\frac{-1}{5}} \right|$

$\alpha-\beta = 85^\circ 37'$ 1

b) $3x(x-2) < x^2(x-2)^2$ 1

$0 < x(x-2)(x^2-2x-3)$

$0 < x(x-2)(x-3)(x+1)$ 1



$\therefore \quad x < -1 \text{ or } 0 < x < 2 \text{ or } x > 3$ 1

c) $|2x-5| = -3x$

$2x-5 = 3x \qquad -2x+5 = -3x$ 1

$5x = +5 \qquad x = -5$ ✓

$x = +1$ 1

Does not work

d) $\sin(x+y) = \sin x \cos y + \sin y \cos x$ 1

$\sin 2\theta = 2\sin\theta\cos\theta$ 1

b) $A(1,7) \quad B(5,-2) \quad 3:-2$

$\left(\frac{1.-2+5.3}{1}, \frac{7.-2+-2.3}{1} \right)$ 1

$(13, -20)$ 1

Q8) a) $\frac{x+3}{x+5} = \frac{x+6}{12}$ 1

$12x+36 = x^2+11x+30$

$0 = x^2-x-6$

$= (x-3)(x+2)$ 1

$\therefore \quad x = 3 \text{ or } -2$

both work 1

b) In $\triangle ADE$ & BCF

$ED = CF$ given

$AD = BC$ prop of rect.

$\hat{EDA} = \hat{FCB} = 90^\circ$ 1

as $\hat{ADE} = \hat{BCD} = 90^\circ$ prop of rect.

$\therefore \triangle ADE \equiv \triangle BCF$ 1

$\therefore AE = BF$ corresp sides of $\equiv$ $\triangle$'s equal 1

c)

$\hat{COB} = 94^\circ$ (angle at centre 2x angle at circum) 1

$\therefore \hat{OCB} = 43^\circ$ (isos $\triangle OBC$) 1

$\therefore \hat{CAB} = 86^\circ$ 1

$\hat{OBA} = 90 = \hat{OCA}$

d) $x = 38^\circ$ (angle at centre 2x angle at circumference) 1

$y = 52^\circ$ ($\triangle OAB$ isos) 1

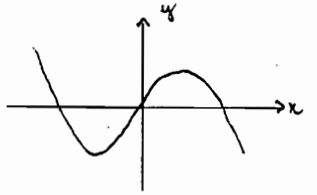
$\triangle AOC$ isos $\therefore \hat{OAC} = 14^\circ$

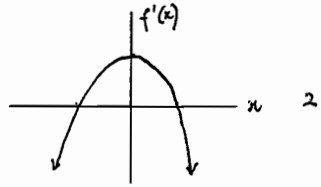
$\hat{DAB} + \hat{ABC} = 180$

$\therefore AD \parallel BC$ 1

Note: No reasons needed for x or y.

Q9) a)





2

b) $\cos(45+30) = \cos 45\cos 30 - \sin 45\sin$ 1

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$ 1

c) $(3x+2y-6) + \lambda(x-2y+5) = 0$

$-9-4-6+\lambda(-3+4+5) = 0$

$6\lambda = 19$

$\lambda = \frac{19}{6}$ 1

$6(3x+2y-6) + 19(x-2y+5) = 0$

$18x+12y-36+19x-38y+95 = 0$ 1

$37x - 26y + 59 = 0$

d) i) $\frac{50}{YQ} = \tan 48 \quad \therefore YQ = 50\cot 48^\circ$ 1 1

$\frac{50}{XQ} = \tan 32^\circ \quad \therefore XQ = 50\cot 32^\circ$ 1

In $\triangle QYX$ 1

$XY^2 = 50^2(\cot^2 48^\circ + \cot^2 32^\circ - 2\cot 32^\circ \cot 48\cos 88^\circ)$ 1

ii) $XY =$ 2