Question 1 (12 marks) **Marks**

a) Solve: $2\sin x = \sqrt{3}$ $\quad 0° \leq x \leq 360°$ **2**

b) Express $0{\cdot}424242\ldots\ldots$ as a fraction (showing working) in the form $\dfrac{p}{q}$, where p and q are integers. **2**

c) Solve for x: i) $|x + 7| = 3x - 2$ **3**

ii) $2^{5x-2} = 32$ **2**

d) If $(5 - 3\sqrt{2})(4 + \sqrt{2}) = a + b\sqrt{c}$, find the values of a, b and c. **3**

Question 2 (12 marks)

a) Evaluate i) $\displaystyle\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$ **1**

ii) $\displaystyle\lim_{x \to \infty} \frac{3x^2 - 2x + 1}{5 - 4x - 2x^2}$ **1**

b) Find $\dfrac{dy}{dx}$, given that **7**

i) $y = \sqrt{x^3}$

ii) $y = 3x^3 - 7x + 6$

iii) $y = 7x(4x + 8)^6$

iv) $y = \dfrac{x - x^2}{5x + 1}$

c) Differentiate from First Principles $y = x^2 - 3x$ **3**

Question 3 (12 marks)

a) Sketch the following functions on separate number planes. Identify any intercepts and asymptotes where appropriate. **6**

i) $y = \dfrac{-1}{x+3}$

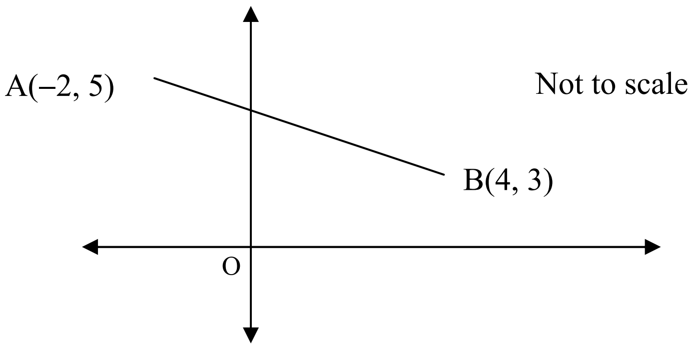
ii) $y = \sqrt{5 - x}$

b) Solve: $2\log_a(x-4) - \log_a(x-5) = \log_a(x-2)$ **3**

c) Solve: $\dfrac{5x}{2x-1} \geq 3$ **3**

Question 4 (12 marks)

a)



The diagram shows the points A(−2, 5), B(4, 3) and O(0, 0).
The point C is the fourth vertex of the parallelogram OABC.

i) Show by derivation that the equation of AB is $x + 3y - 13 = 0$ **1**

ii) Find in exact form the length of AB. **1**

iii) Calculate the perpendicular distance from O to AB. **2**

iv) Calculate the area of parallelogram OABC. **1**

v) Find the coordinates of C. **2**

b) Sketch the region represented by $x^2 + (y-1)^2 \leq 9$ **2**

c) Find the equation of the line through the point of intersection of the lines $2x - 3y + 6 = 0$ and $5x + y - 4 = 0$ and the point (1, 4). **3**

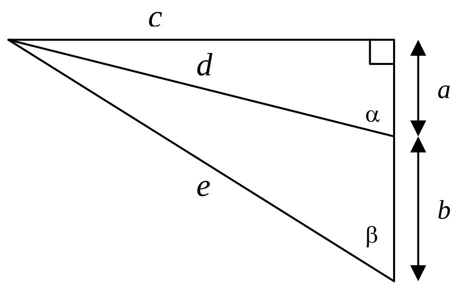
Question 5 (12 marks)

a) Sketch the curve $y = 3\cos 2x$ $\quad 0° \leq x \leq 360°$ **3**

b) A, B and C are three towns. B is 20km from A in the direction 330°T. **3**
C is 30km from A in the direction 205°T. Find the distance from B to C.
(Hint: draw a diagram)

c) Prove the identity: $\dfrac{1}{\sec\theta - \tan\theta} - \dfrac{1}{\sec\theta + \tan\theta} = 2\tan\theta$ **3**

d)



Show that $a = \dfrac{b\cos\alpha\sin\beta}{\sin(\alpha - \beta)}$ **3**

Question 6 (12 marks)

a) Find the sum of the first 20 multiples of 7. i.e. $7 + 14 + 21 + \ldots\ldots$ **2**

b) A gardener weeded his lawn over summer. The first year he dug out 2 wheelbarrows full. Each successive year he dug out $\frac{3}{4}$ of the previous years total.

i) How many barrows full will he have dug out in the first 10 years? **2**

ii) Over his lifetime, what is the limiting number of barrow loads he will end up removing? **2**

c) If $2p + 1$, $5p$, $12p - 4$ are the first 3 terms of a geometric sequence, find the value of p and hence find T_n. **3**

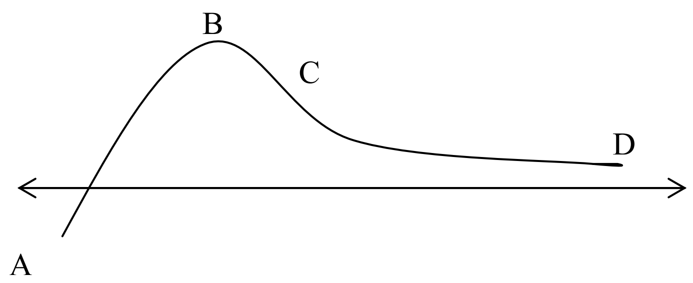
d) Consider the series $\log_a 36 + \log_a 18 + \log_a 9 + \ldots\ldots\ldots + \log_a \frac{9}{8}$ **3**

i) Show that it is an arithmetic series.

ii) Find the sum of the series.

Question 7 (12 marks)

a) Sketch the derivative of the curve below and clearly label it **2**



b Find the equation of the normal to $y = 9 - x^2$ at P(1, 8). **3**

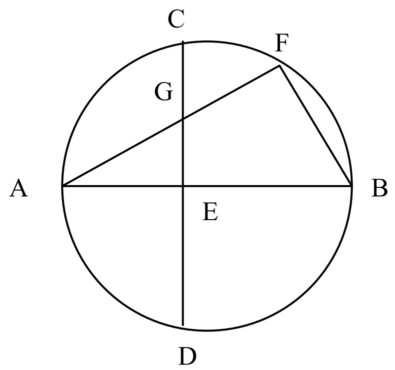
c) Given the curve $y = x^4 - 16x^3 + 72x^2 + 10$, find **7**

i) All stationary points and determine their nature

ii) Any points of inflexion, justifying your answers
.
iii) Sketch the curve showing this information.

Question 8 (12 marks)

a)

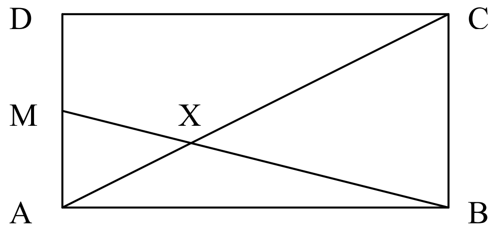


AB is a diameter and CD⊥AB.

Prove that E, G, F and B are concyclic

3

b)

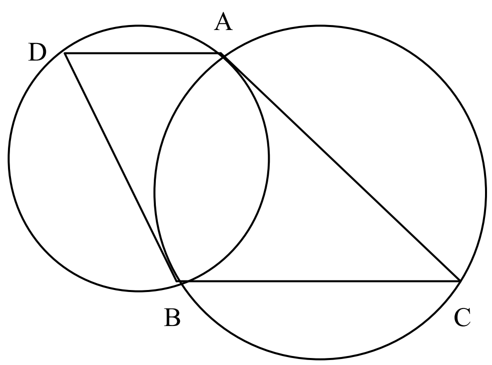


ABCD is a rectangle. AB = 2AD. M is the midpoint of AD. The line BM meets AC at X.

i) Show that ΔAXM and ΔBXC are similar. **3**

ii) Show that 3CX = 2AC **2**

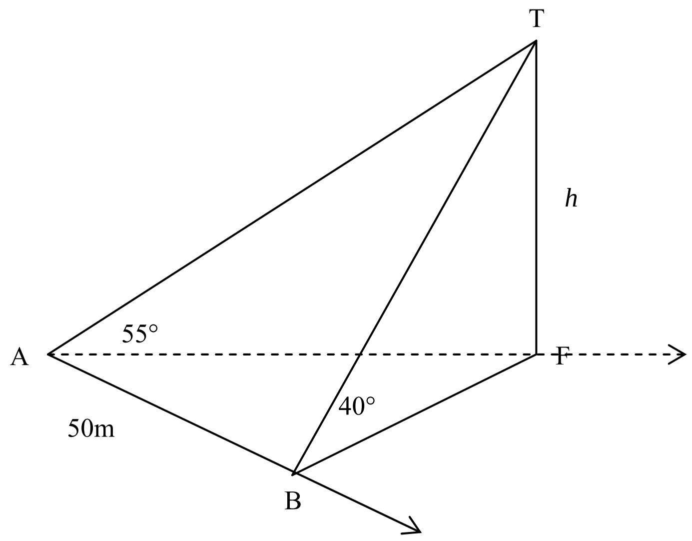
c) Two circles meet each other at A and B. AC and DB are tangents.
Prove that AD || BC **4**



Question 9 (12 marks)

a)

The diagram shows a tower TF of height h metres standing due north of A on level ground. The angles of elevation of the top T of the tower from two points A and B (due east of A), on the ground nearby are 55° and 40° respectively. The distance AB is 50m.



i) What is the measure of $\angle$FAB? **1**

ii) Find AF and BF in terms of h. **2**

iii) Hence find the height of the tower to the nearest metre. **3**

b) A cylinder, open at one end has a volume of 1000cm^3.

i) Show that the surface area S is given by $S = \pi r^2 + \dfrac{2000}{r}$ **2**

ii) Find the value of r, to 4 significant figures that minimises the surface area. **4**

Question 10 (12 marks)

a) A ball is thrown vertically up in the air with its height x metres above the ground at any time t seconds given by $x = 4t(5 - t)$.

i) When does it reach maximum height? **2**

ii) What is the maximum height reached? **1**

iii) What is its acceleration then? **1**

iv) What is the speed of the ball when it returns to the ground? **2**

v) Find the distance travelled during the 3^{rd} second. **2**

b) Sketch a continuous curve $y = f(x)$ having the following properties.

$f(-3) = 12$,
$f(0) = 6$,
$f(3) = f'(3) = f'(-3) = 0$
$f'(x) < 0$ for $-3 < x < 3$ and $f'(x) > 0$ for $x < -3$ or $x > 3$

c) Give an example by sketching, of a function which has a minimum at $x = 0$, but which is not differentiable at $x = 0$.

END OF EXAMINATION

2008 YR11 ~~YRLY~~ SUGGESTED SOLUTIONS

Q1

a) $2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$x = 60^\circ$ or 120° 1, 1

b) $x = 0.4242\ldots$

$100x = 42.42\ldots$ 1

$99x = 42$

$x = \frac{42}{99}$

$= \frac{14}{33}$ 1

c) i) $|x+7| = 3x-2$

(1) $x+7 = 3x-2$

$9 = 2x$ 1

$x = 4\frac{1}{2}$

ii) $2^{5x-2} = 32 \Rightarrow 5x-2 = 5$, $x = \frac{7}{5}$ 2.

(2) $-x-7 = 3x-2$

$4x = -5$ 1

$x = -\frac{5}{4}$

Test Solns

$4\frac{1}{2}$ ✓ $-\frac{5}{4}$ ✗

So only one 1

soln $x = 4\frac{1}{2}$

$(\ldots\sqrt{2})(4+\cancel{2}\sqrt{2}) = 20+5\sqrt{2}-12\sqrt{2}-6$

$\cancel{12\sqrt{2}} - 12$ 1 $= 14-7\sqrt{2}$

1 $a = 14$

$b = -7$

1 $c = 2$

Q2

a) i) $\lim_{x \to 4} \frac{(x-4)(x+4)}{(x-4)} = 8$ 1

ii) $\frac{3}{-2}$ 1

b) i) $y = x^{3/2}$ Accept $\frac{3x^2}{2\sqrt{x^3}}$.

$y' = \frac{3}{2}x^{1/2}$ 1

ii) $y' = 9x^2 - 7$ 2

transcript error $6x-7$ (Award 2)

iii) $y = 7x(4x+8)^6$ product rule otherwise 0.

$y' = 7x \times 6 \times 4(4x+8)^5 + (4x+8)^6 \times 7$ 1

$= 168x(4x+8)^5 + 7(4x+8)^6$ 1

$= 7(4x+8)^5 [\ldots] (28x+8$ mistake after not counted

iv) $y = \frac{x-x^2}{5x+1}$ wrong rule = 0. !!

$y' = \frac{(5x+1)\times(1-2x) - (x-x^2)\times 5}{(5x+1)^2}$ ✓ 1, 1

$= \frac{-5x^2-2x+1}{(5x+1)^2}$ ✓ ← has to be right

e) $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2-3x)}{h}$

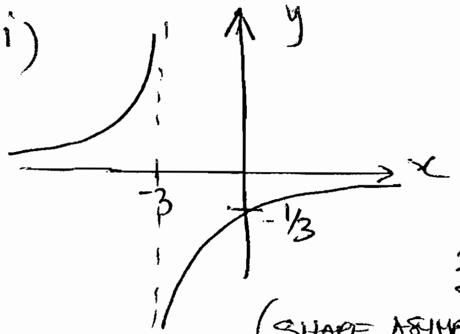
$= \lim_{h \to 0} \frac{2xh - 3h + h^2}{h}$ 1

$= 2x-3$ 1

lose a mark for no $\lim_{h \to 0}$

Q3

a) i)



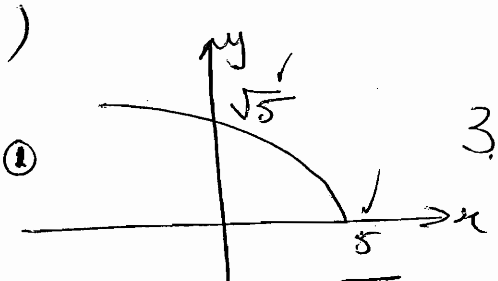
3

(SHAPE, ASYMPTOTE INTERCEPT)

ii)

what if

①



3

$y = \sqrt{x-5}$

$-x+5 \geq 0$

b) $2\log(x-4) - \log(x-1) = \log(x\ \ )$

$\frac{(x-4)^2}{x-1} = x+2$ 1

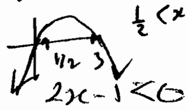
$(x-4)^2 = (x-1)(x-5)$

$x^2 - 8x + 16 = x^2 - 7x + 10$ 1

$x = 6$

Ignoring denominator $= 0$ 1

c) $\frac{5x}{2x-1} \geq 3$. $(2x-1)(-x+3) \geq 0$. $\frac{1}{2} < x \leq 3$.



also $-2x^2+7x-3 \geq 0$.
or $2x^2-7x+3 \leq 0$.

Case 1 $2x-1 > 0$ | case 2 $2x-1 < 0$

$x > \frac{1}{2}$ | $x < \frac{1}{2}$

$5x \geq 6x-3$ | $5x \leq 6x-3$.

$-x \geq -3$ | $-x \leq -3$

$x \leq 3$ | $x \geq 1$ No soln

1 | 1

$\frac{1}{2} < x \leq 3$

Only soln

lose mark for $\frac{1}{2} \leq x \leq 3$ $\frac{1}{2} < x \leq 3$ 1

Q4

a) i) $m_{AB} = \frac{5-3}{-2-4} = -\frac{1}{3}$.

$y - 5 = -\frac{1}{3}(x+2)$

$3y - 15 = -x - 2$

$x + 3y - 13 = 0$ 1

ii) $AB = \sqrt{(-2-4)^2 + (5-3)^2}$

$= \sqrt{40}$ 1

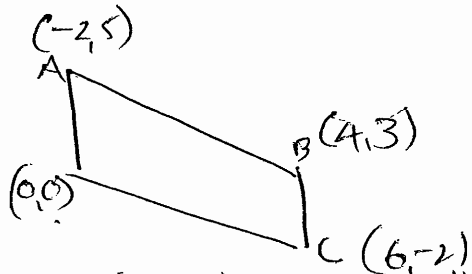
$= 2\sqrt{10}$

iii) $d = \left| \frac{0 + 3\times 0 - 13}{\sqrt{1+9}} \right|$ 1

$= \left|\frac{-13}{\sqrt{10}}\right| = \frac{13}{\sqrt{10}}$ 1

iv) $A = bh = 2\sqrt{10} \times \frac{13}{\sqrt{10}} = 26u^2$ 1

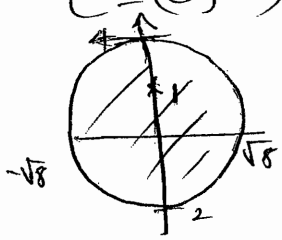
v)



$C = (6,-2)$

what if additional pt $(-6,2)$ 1 mark ½ take mark off

b)



1 circle

1 region

Note centre necessary

maybe 4 or 2

√8 Not necessary

c) $(2x-3y+6) + K(5x+y-4) = 0$ 1

Subst $(2-12+6) + K(5+4-4) = 0$

$5K = 4$ 1

$K = 4/5$

$\Rightarrow 5(2x-3y+6) + 4(5x+y-4) = 0$ 1

Q4-

c) $2x - 3y + 6 = 0$
$5x + y - 4 = 0. \quad \times 3$

$2x - 3y = -6$
$15x + 3y = 12$

$17x = 6.$
$x = 6/17.$

$2\left(\frac{6}{17}\right) - 3y + 6 = 0.$

$12 - 51y + 102 = 0.$
$y = \frac{114}{51} = 2\frac{4}{17}.$

$\left(\frac{6}{17}, 2\frac{4}{17}\right)$ ✓
$(1, 4)$

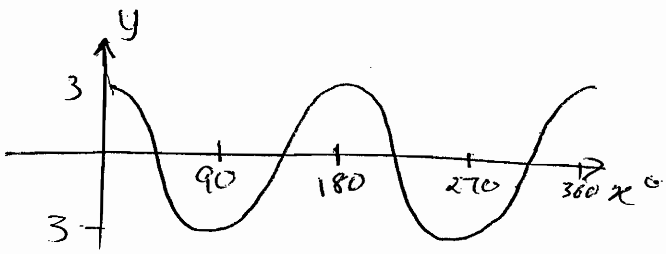
$m = \frac{4 - 2^4/17}{1 - 6/17} = 2\frac{8}{11} = \frac{30}{11}.$ ✓

$y - 4 = \frac{30}{11}(x - 1)$
$11y - 44 = 30x - 30.$
$0 = 30x - 11y + 14$

$\rightarrow y = \frac{30x}{11} + \frac{14}{11}$ ✓

Q5

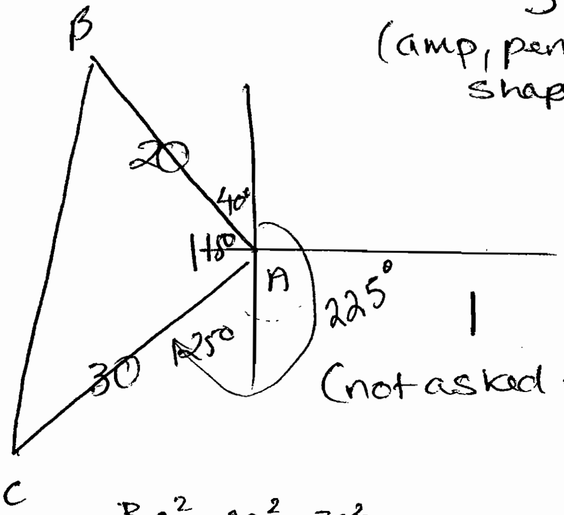
a)



3

(amp, period shape)

b)



1

(not asked for)

1

$BC^2 = 20^2 + 30^2 - 2 \times 20 \times 30 \times \cos 125^\circ$

$= 1867.142$

$BC \doteqdot$ ~~42.5 km~~ 44.6 | km.

(Note: If no diagram 2 marks at point 2)

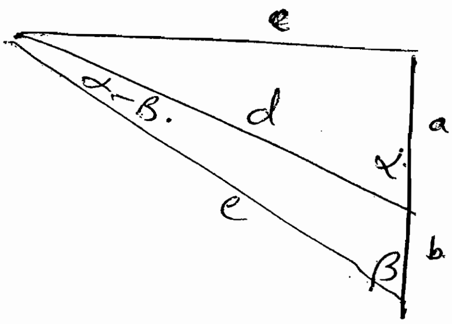
c) $LHS = \dfrac{1}{\sec\theta - \tan\theta} - \dfrac{1}{\sec\theta + \tan\theta}$

$= \dfrac{\sec\theta + \tan\theta - \sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$ | 1

$= \dfrac{2\tan\theta}{1}$ 1

$= RHS.$

d)



$\cos\alpha = \dfrac{a}{d}$

$d = \dfrac{a}{\cos\alpha}.$ 1

$\dfrac{b}{\sin(\alpha-\beta)} = \dfrac{d}{\sin\beta}$

$\dfrac{b}{\sin(\alpha-\beta)} = \dfrac{a}{\cos\alpha \sin\beta}$ 1

$a = \dfrac{b\cos\alpha\sin\beta}{\sin(\alpha-\beta)}$ 1

$\sin\alpha\cos\beta - \cos\alpha.\sin\beta.$

Q6

a) $a=7$

$l=140$ 1

$n=20$

$S=\frac{20}{2}(7+140)$

$=1470$ 1

b) i) $r=\frac{3}{4}$ $a=2$ $n=10$

$S_{10}=\frac{2\left(1-\left(\frac{3}{4}\right)^{10}\right)}{1-3/4}$ 1

$\doteqdot 7.5$ loads 1

7 loads (2)

ii) $S=\frac{a}{1-r}=\frac{2}{1-3/4}=8$ loads

1 1

c) $\frac{5p}{2p+1}=\frac{12p-4}{5p}$

$25p^2=24p^2+4p-4$ 1

$(p-2)^2=0$

$p=2$ 1

$T_2=10$ $T_3=20$)

asked for

$T_n=5\times 2^{n-1}$ 1

d)

i) $T_3-T_2=\log 9-\log 18$

$=\log\frac{1}{2}=-\log 2$

$T_2-T_1=\log 18-\log 36$

$=\log\frac{1}{2}=-\log 2$

$=-0.301$

$\therefore$ arithmetic. 1

+ log 2 no marks.

ii) $n=6$

$a=\log 36$

$l=\log\frac{9}{8}$

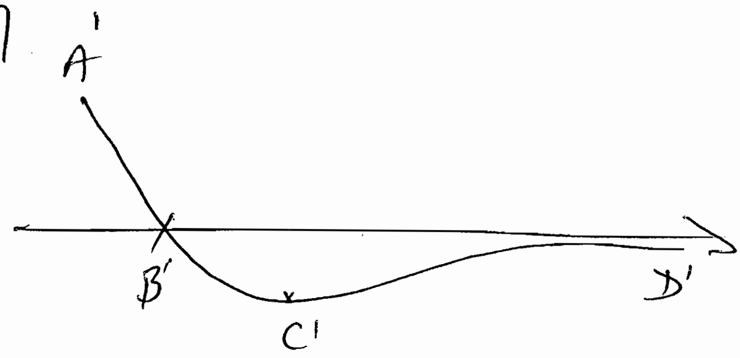
$S=\frac{6}{2}\left[\log 36+\log\frac{9}{8}\right]$ 1

$=3\log\left(\frac{81}{2}\right)$ 1

$\cancel{36}^{9}\times\frac{9}{\cancel{8}_{2}}$

Q7

a)



2

b) $y = 9 - x^2$

$y' = -2x$ 1

at $x = 1$, $m = -2$ $\therefore \perp m = \frac{1}{2}$

for tangent: 2 marks if correct

$y - 8 = \frac{1}{2}(x - 1)$ 1

$2y - 16 = x - 1$

$x - 2y + 15 = 0$ 1

---

c) $y = x^4 - 16x^3 + 72x^2 + 10$

$y' = 4x^3 - 48x^2 + 144x$

$= 4x(x^2 - 12x + 36)$ 1

$= 4x(x - 6)^2$

$y' = 0 \Rightarrow x = 6$ or 0

$y = 432$ $\quad 10$ 1

$(6, 432)$ $\quad (0, 10)$

$y'' = 12x^2 - 96x + 144$

$= 12(x - 2)(x - 6)$ 1

$y'' \Rightarrow 0 \Rightarrow x = 2$ or 6 1

$(2, 176)$ $(6, 432)$

186 $\quad$ 442 $\quad$ 7354

Test

x	-1	0	1	6	8
y'	<0	0	>0	0	>0

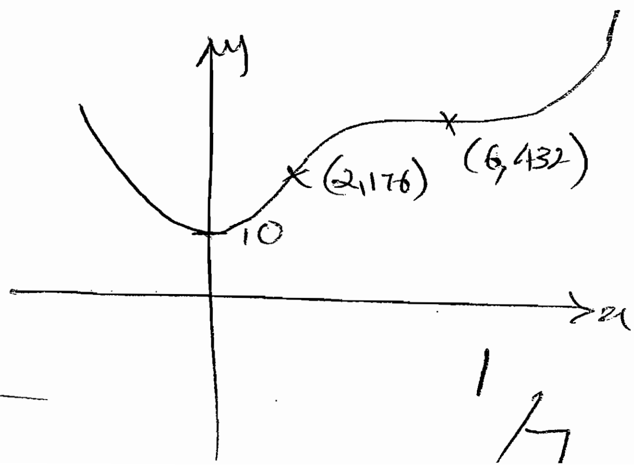
1

$\therefore$ Min at $x = 0$

horiz inflex at $x = 6$

Test x	0	2	4	6	8
y''	>0	0	<0	0	>0

1



1

/7

Q8

a) $\angle AFB = 90°$ (diameter / circ. thm) 1

$\angle CEB = 90°$ (given) 1

$\therefore$ EGFB is a cyclic Quad. (opp $\angle$s Σ to $180°$) 1

b) i) $\angle MXA = \angle CXB$ (Vert opp) 1

$\angle DAX = \angle ACB$ (alts =, DC$\parallel$AB) 1

$\therefore \triangle MXA \parallel\!\!| \triangle CXB$ (AA) 1

ii) $2MA = CB$ (given)

$2AX = XC$ (Corrs sides Sim $\triangle$s)

$AX + CX = AC$
$3AX = AC$.

$\therefore \dfrac{CX}{AX} = 2$ 1

how?

$CX = 2AX$
$3CX = 6AX$

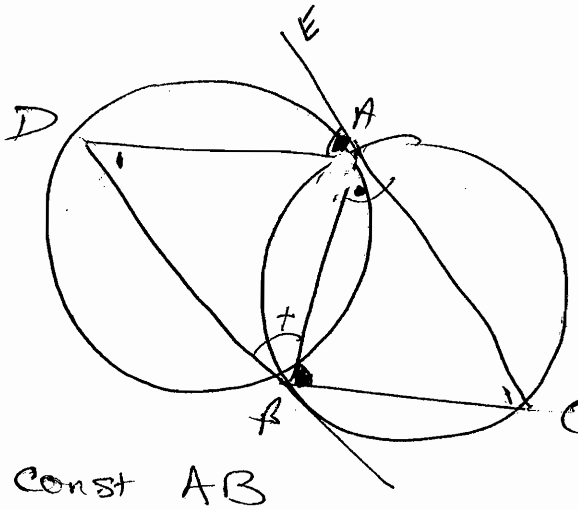
$AC = 3AX$.
$2AC = 6AX$.

$6AX = 3CX = 2AC$.

$CX = \dfrac{2}{3}AC$

$3CX = 2AC$ 1

c)



$\angle DAC + \angle ACB = 180°$.
$\therefore DA \parallel BC$ (co-interior) gets 0 marks!

Const AB

let $\angle BCA$ α.

$\therefore \angle ABD = \alpha$ (tangent alt seg thm) 1

Let $\angle BDA = \beta$

$\therefore \angle CAB = \beta$ (" " ") 1

$\therefore \angle ABC = 180 - (\alpha + \beta)$ ($\angle$ sum $\triangle$) 1

$\angle DAB = 180 - (\alpha + \beta)$ (") 1

$\angle ABC = \angle DAB$ 1

Q9

a) i) $\angle FAB = 90°$ 1

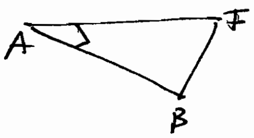
ii) $\frac{AF}{h} = \cot 55°$ $BF = h\cot 40°$ 1, 1

h in terms of m

$AF = h\cot 55$

iii) $AF^2 + AB^2 = BF^2$

$(h\cot 55)^2 + 50^2 = (h\cot 40)^2$ 1



$h^2\cot^2 55 + 2500 = h^2\cot^2 40$

$h^2 = \frac{2500}{\cot^2 40 - \cot^2 55} = \frac{250}{\tan^2 50 - \tan^2 35}$ 1

$\doteqdot 2688.2$ 1

$h \doteqdot 51.8\text{ m} \doteqdot 52\text{ m}$

b) i) $V = \pi r^2 h \Rightarrow 1000 = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$ 1

$SA = \pi r^2 + 2\pi r h$

$= \pi r^2 + \frac{2000}{r}$ 1

ii)

$\frac{d(SA)}{dr} = 2\pi r - \frac{2000}{r^2}$ 1

$\frac{dSA}{dr} = 0 \Rightarrow 2\pi r = \frac{2000}{r^2}$ 1

$r^3 = \frac{1000}{\pi}$

$r = \frac{10}{\sqrt[3]{\pi}}$

Test

r	$\frac{10}{\sqrt[3]{\pi}}^-$	$\frac{10}{\sqrt[3]{\pi}}$	$\frac{10}{\sqrt[3]{\pi}}^+$
SA'	$-$	0	$+$

1

$\therefore$ Min $r \doteqdot 6.8278 = \sqrt[3]{\frac{1000}{\pi}}$

4 sig figs 6.828. 1

Q 10

a) i) $x = 20t - 4t^2$

$\dot{x} = 20 - 8t$ (1) or $t = \frac{-b}{2a} = \frac{-20}{-8} = 2\frac{1}{2}$

$\dot{x} = 0 \Rightarrow t = 2\frac{1}{2}$ (1)

ii) $x(2\frac{1}{2}) = 20 \times 2\frac{1}{2} - 4 \times (2\frac{1}{2})^2 = \cancel{39\frac{3}{4}}$ $25m.$ 1

$4(2.5)(5-2.5)$

iii) $\ddot{x} = -8 \ m/s^2$ 1

iv) at $t = 5$ 1

$\dot{x}(5) = 20 - 8 \times 5 = -20 \ m/s$ 1

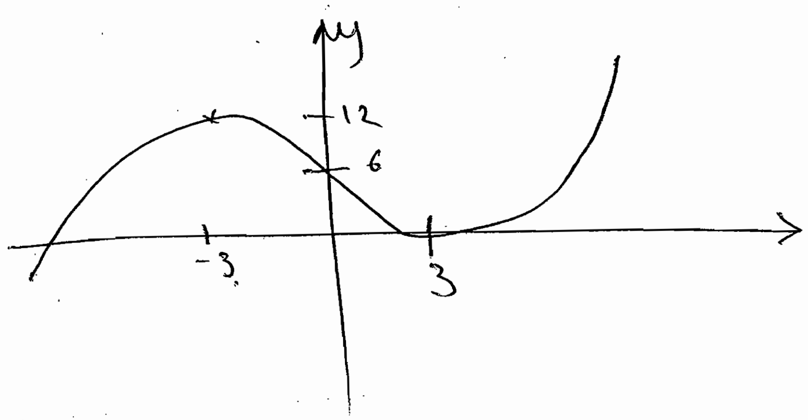
speed = 20 m/s.

v) $x(2) = 24$ $x(2\frac{1}{2}) = \cancel{39\frac{3}{4}}$ 25 $x(3) = 24$

~~Limit of 5.~~ $\therefore$ distance = $\cancel{31\frac{1}{2}}$ 2 m. 2

3 → 4 :

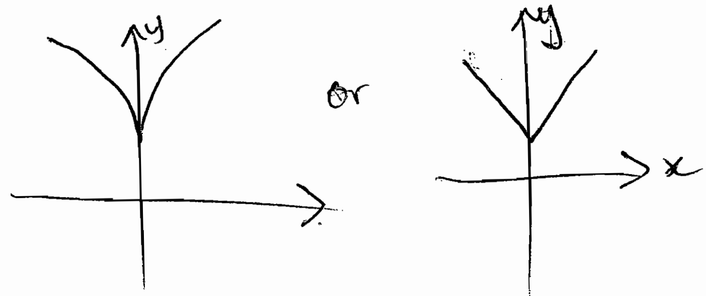
b)



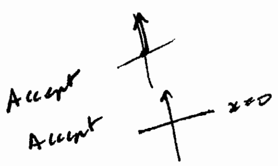
3
(Max, Min, Inflex)

c)

eg



1



or anything else that meets the criteria