NORTH SYDNEY BOYS HIGH SCHOOL

2009
PRELIMINARY EXAMINATION

Mathematics
Extension 1

Examiner: R. Lowe

General Instructions

- Working time – 2 hours and a half
- Attempt all questions
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new **Question** is to be started on a **new page.**

Class Teacher:
(Please tick or highlight)
- O Mr Barrett
- O Mr Ireland
- O Mr Lam
- O Mr Lowe
- O Mr Rezcallah
- O Mr Trenwith
- O Mr Weiss

Student Name:

(To be used by the exam markers only.)

Question No	1	2	3	4	5	6	7	8	9	10	11	Total	Total
Mark	—/10	—/11	—/12	—/12	—/5	—/7	—/13	—/11	—/12	—/12	—/13	—/118	—/100

Question 1

a) Evaluate $\dfrac{3.24^2 - 2.1^2}{\sqrt{36+2.1}}$ correct to 3 significant figures. 1

b) Simplify $\dfrac{2x^2-1}{1-2x^2}$ 1

c) Simplify $\dfrac{3^{n+2}+3^n}{3^n}$ 2

d) Given that tan $\alpha = 2$ where α is an acute angle, 2

find the value of tan 2α in exact form.

e) Solve $\dfrac{4}{x-3} \le 2$ 3

f) Find correct to 2 decimal places $log_3 12$ 1

Question 2 Start a new page

a) Find the acute angle between the lines $4x - y = 0$ and $y = x$ to the nearest degree. 2

b) Solve $|15+4x| \le 3$ 2

c) Given that R(x, y) divides the interval joining P(– 4, 5) and 2

Q (– 1, 9) externally in the ratio 3: 5, find the coordinates of R.

d) Find the period and amplitude for the graph of $y = 3\sin(2\pi x)$. 2

e) i) Without calculus sketch the graph $y = x^3(2-x)$ 2

ii) Hence, or otherwise, solve the inequality $x^3(2-x) < 0$. 1

Question 3 Start a new page

a) Write $0.1\dot{2}$ as a fraction in simplest form **2**

b) Simplify fully $\frac{5x+1}{(x+1)(x-1)} + \frac{2}{1-x} - \frac{3}{1+x}$ **2**

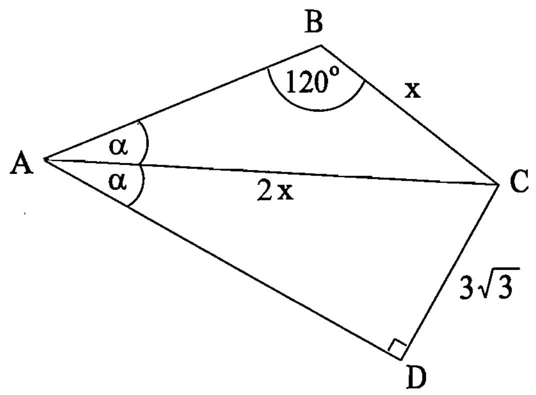
c) i) Prove that $(\operatorname{cosec}^2 x - 1)\sin^2 x = \cos^2 x$ **2**

ii) Hence or otherwise solve: **3**

$(\operatorname{cosec}^2 x - 1)\sin^2 x = \frac{3}{4}$ *for* $0 \le x \le 360^0$

d)

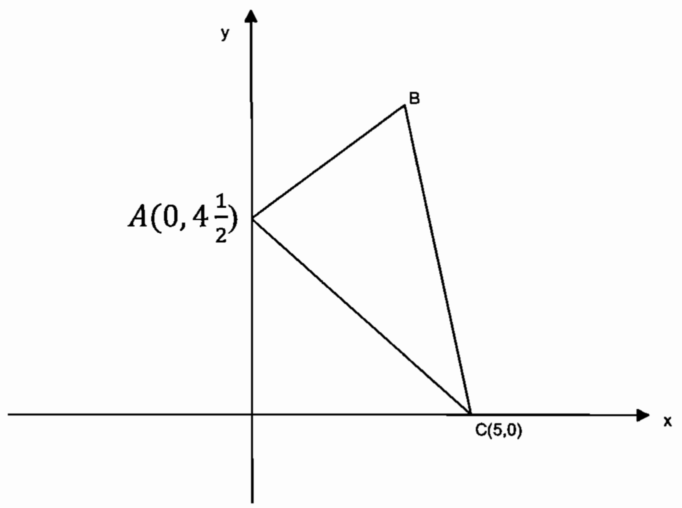
In the quadrilateral ABCD shown in the diagram, ∠BAC = ∠CAD = α.
∠ABC = 120°, ∠ADC = 90°, BC = x, AC = 2x, and CD = $3\sqrt{3}$ cm



i) Use the triangle ABC to find the value of sin α. **2**

ii) Hence, find the value of x. **1**

Question 4 Start a new page



The lines AB and CB have equations $x-2y+9=0$ and $4x-y-20=0$ respectively.

i) Find the coordinates of the point B. 3

ii) Show that the equation of the line AC is $9x+10y-45=0$. 2

iii) Calculate the distance AC in exact form. 2

iv) Find the equation of the line perpendicular to BC which passes passes through A. 2

v) Calculate the shortest distance between the point B and the line AC. 3

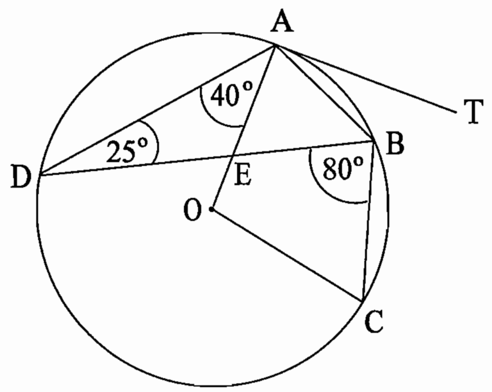
Hence find the area of the triangle ABC.

Question 5 Start a new page

A, B, C, and D are four points on a circle with centre O, such that

$\angle ADB = 25°$, $\angle DBC = 80°$ and $\angle OAD = 40°$.

AT is a tangent at A to the circle.



i) Show that the triangle AEB is isosceles. 3

ii) Show that quadrilateral OEBC is cyclic. 2

Question 6 Start a new page

a) The first and last terms of an arithmetic series are 5 and 165 respectively, and the sum of these terms is 1785. Find the number of terms in the series. 2

b) i) Write down a single expression for the sum of the first n terms of the geometric series $1 + x^2 + x^4 + x^6 + \cdots$ 2

ii) When does the sum to infinity exist (in terms of x^2) ? 1

iii) What is the sum to infinity when $x = \frac{1}{3}$? 2

Question 7 Start a new page

a) Differentiate with respect to x

i) $y = 4x^8 - \frac{1}{\sqrt{x}}$ 2

ii) $y = (3x^2 + 7)^8$ 2

iii) $y = \frac{x^3}{x^2+1}$ 2

b) A particle moves in a straight line so that its position, x m, from the origin at time, t seconds, is given by $x = t^3 + 3t^2 - 9t$

Find i) its initial displacement, velocity and acceleration. 3

ii) when the particle is stationary and its position at that time. 2

iii) the distance travelled in the first 2 seconds. 2

Question 8 Start a new page

a) For the curve $y = x^3 + 3x^2 - 24x + 2$

i) Find the stationary points and determine their nature 3

ii) Find any points of inflexion 2

iii) Sketch the curve 2

b) A farmer has 30m of wire fencing. He wishes to make a rectangular enclosure using the side of the barn as one wall. What is the largest area that the fenced enclosure can have ? Justify your answer by using calculus. 4

Question 9 Start a new page

a) If $sin\emptyset = \frac{4}{5}$ $and\ sin \propto = \frac{5}{13}, find$ $\cos(\emptyset - \propto)$ if α and ∅ are acute 3

b) Find the exact value of $\sin\ 105^0$. Express your answer with a rational denominator. 3

c) Solve $\cos\left(2x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ $where\ 0 \leq x \leq 2\pi$ 3

d) A ship is sailing on a bearing of 205^0 from port A to port B. If port B is 70 nautical miles west of port A, find how far the ship has sailed. 3

Question 10 Start a new page

a) Consider the function *y=f(x)*. If the point (4,8) is a maximum turning point on the function what is the value of *f '(4)* ? 1

b) On the same curve *y=f(x)* a tangent is drawn from the point (3,6) and this tangent has a *y* intercept of 10. What is the value of *f '(3)* ? 2

c) On separate axes sketch the following graphs, showing all important features, and state their domain and range

i) $y = 4sin2x$ $0^0 \leq x \leq 360^0$ 3

ii) $y = \sqrt{4 - x^2}$ 3

iii) $y = 4^{-x}$ 3

Question 11 Start a new page

a) Points A, B and C lie on horizontal ground so that $\angle ACB$ is 90^0 and A and B are 200m apart. Point D is h metres vertically above C. The angles of elevation of D from A and B are 20^0 *and* 23^0 respectively. Draw a half page diagram to show this information and find the value of h correct to 2 significant figures. 4

b) Solve $\log_2 x - \log_2(x-2) = \frac{2}{3}\log_2 27$ 3

c) The radius of curvature R of a function $y = f(x)$ is a measure of how sharply a curve is bending.

It is defined as $R = \dfrac{\left(1+(\frac{dy}{dx})^2\right)^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$.

Use this formula to find the exact radius of curvature of the curve $y = \sqrt{x}$ at $x = 1$. 3

d) Find the domain and range of the function $f(x) = \sqrt{3 - \sqrt{x}}$ 3

Suggested Solutions

Question 1 (Weiss)

(a) (1 mark)

$$\frac{3.24^2 - 2.1^2}{\sqrt{36 + 2.1}} = 0.986$$

(b) (1 mark)

$$\frac{2x^2 - 1}{1 - 2x^2} = -1$$

(c) (2 marks)

$$\frac{3^{n+2} + 3^n}{3^n} = \frac{\cancel{3^n}(3^2 + 1)}{\cancel{3^n}} = 9 + 1 = 10$$

(d) (2 marks)

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Substitute $\tan\alpha = 2$ results in

$$\tan 2\alpha = \frac{2 \times 2}{1 - 2^2} = -\frac{4}{3}$$

(e) (3 marks)

$$\underset{\times(x-3)^2}{\frac{4}{x-3}} \le \underset{\times(x-3)^2}{2}$$

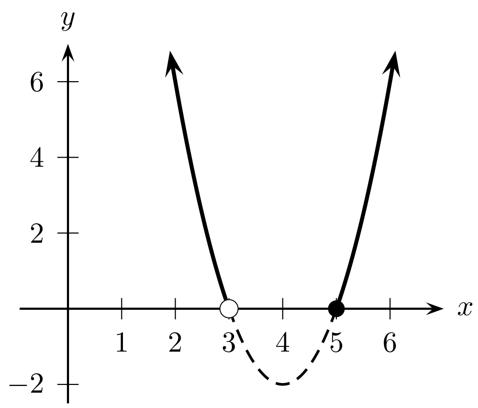
$$4(x-3) \le 2(x-3)^2$$

$$2(x-3)^2 - 4(x-3) \ge 0$$

$$2(x-3)\,(x-3-2) \ge 0$$

$$2(x-3)(x-5) \ge 0$$

$$x < 3 \text{ or } x \ge 5$$



(f) (1 mark)

$$\log_3 12 = \frac{\log_{10} 12}{\log_{10} 3} = 2.26$$

Question 2 (Lowe)

(a) (2 marks)

$$\begin{aligned} 4x - y = 0 &\quad \Rightarrow m_1 = 4 \\ y = x &\quad \Rightarrow m_2 = 1 \\ \tan\theta &= \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \\ &= \left|\frac{4-1}{1+4}\right| = \frac{3}{5} \\ \therefore \theta &= \tan^{-1}\frac{3}{5} = 30^\circ 58' \end{aligned}$$

(b) (2 marks)

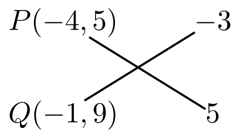
$$|15 + 4x| \le 3$$

$$\underset{-15}{-3} \le \underset{-15}{15} + 4x \le \underset{-15}{3}$$

$$\underset{\div 4}{-18} \le \underset{\div 4}{4x} \le \underset{\div 4}{-12}$$

$$-\frac{9}{2} \le x \le -3$$

(c) (2 marks)



As R divides PQ externally,

$$R(x,y) = \left(\frac{5(-4) + -3(-1)}{-3+5}, \frac{5(5) + -3(9)}{-3+5}\right) = \left(-\frac{17}{2}, -\frac{2}{2}\right) = \left(-8\frac{1}{2}, -1\right)$$

(d) (2 marks)

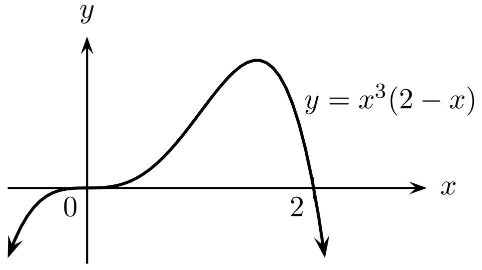
$$y = 3\sin(2\pi x)$$

- amplitude $a = 3$.
- period:

$$f = \frac{2\pi}{T}$$

$$f = 2\pi \quad \Rightarrow \quad T = \frac{2\pi}{2\pi} = 1$$

(e) i. (2 marks)



ii. (1 mark)
$x^3(2 - x)$ is below the x axis when $x < 0$ or $x > 2$.

Question 3 (Rezcallah)

(a) (2 marks)

$$\begin{aligned} x &= 0.122\cdots \\ 10x &= 1.222\cdots \\ 100x &= 12.222\cdots \\ \hline 90x &= 11 \\ \therefore x &= \frac{11}{90} \end{aligned}$$

(b) (2 marks)

$$\begin{aligned} &\frac{5x+1}{(x+1)(x-1)} + \frac{2}{1-x} - \frac{3}{1+x} \\ &= \frac{5x+1}{(x+1)(x-1)} - \frac{2}{x-1} - \frac{3}{x+1} \\ &= \frac{(5x+1) - 2(x+1) - 3(x-1)}{(x+1)(x-1)} \\ &= \frac{5x+1-2x-2-3x+3}{(x+1)(x-1)} \\ &= \frac{2}{(x+1)(x-1)} \text{ or } -\frac{2}{(1-x)(x+1)} \end{aligned}$$

(c) i. (2 marks)

$$\begin{aligned} &\left(\operatorname{cosec}^2 x - 1\right)\sin^2 x \\ &= \left(\frac{1}{\sin^2 x} - 1\right)\sin^2 x \\ &= 1 - \sin^2 x \\ &= \cos^2 x \end{aligned}$$

ii. (3 marks)

$$\begin{aligned} \left(\operatorname{cosec}^2 x - 1\right)\sin^2 x &= \frac{3}{4} \\ \cos^2 x &= \frac{3}{4} \\ \cos x &= \pm\frac{\sqrt{3}}{2} \\ x &= 30^\circ, 150^\circ, 210^\circ, 330^\circ \end{aligned}$$

(d) i. (2 marks)
Applying the sine rule in $\triangle ABC$,

$$\begin{aligned} \frac{\sin\alpha}{x} &= \frac{\sin 120^\circ}{2x} \\ \therefore \sin\alpha &= \frac{\sin 120^\circ}{2} = \frac{\sqrt{3}}{4} \end{aligned}$$

ii. (1 mark)

$$\text{In } \triangle ACD,\ \sin\alpha = \frac{3\sqrt{3}}{2x}$$

But $\sin\alpha = \frac{\sqrt{3}}{4}$,

$$\begin{aligned} \therefore \frac{\sqrt{3}}{4} &= \frac{3\sqrt{3}}{2x} \\ 2x &= 12 \\ \therefore x &= 6 \end{aligned}$$

Question 4 (Lam)

(a) (3 marks)

$$\begin{cases} x - 2y + 9 = 0 & (1) \\ 4x - y - 20 = 0 & (2) \\ 8x - 2y - 40 = 0 & (2') \end{cases}$$

Subtract (2') from (1),

$$\begin{aligned} -7x + 49 &= 0 \\ 7x &= 49 \\ x &= 7 \to (1) \\ 7 - 2y + 9 &= 0 \\ 2y &= 16 \\ y &= 8 \\ \therefore B&(7, 8) \end{aligned}$$

(b) (2 marks)

$$\begin{aligned} m_{AC} &= \frac{\frac{9}{2}-0}{0-5} = -\frac{9}{10} \\ \frac{y-\frac{9}{2}}{x-0} &= -\frac{9}{10} \\ \underset{\times 10}{y} &= -\underset{\times 10}{\frac{9}{10}} + \underset{\times 10}{\frac{9}{2}} \\ 10y &= -9x+45 \\ 9x+10y-45 &= 0 \end{aligned}$$

(c) (2 marks)

$$\begin{aligned} d_{AC} &= \sqrt{5^2+\left(\frac{9}{2}\right)^2} \\ &= \sqrt{\frac{181}{4}} \\ &= \frac{\sqrt{181}}{2} \end{aligned}$$

(d) (2 marks)

$$\begin{aligned} m_{BC} &= 4 \\ y &= -\frac{1}{4}x + \frac{9}{2} \end{aligned}$$

(e) (3 marks)

Perpendicular distance from $(7,8)$ to $9x+10y-45=0$ is

$$\begin{aligned} d_\perp &= \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \\ &= \frac{|9(7)+10(8)-45|}{\sqrt{9^2+10^2}} \\ &= \frac{|63+80-45|}{\sqrt{181}} \\ &= \frac{98}{\sqrt{181}} \\ A &= \frac{1}{2}bh \\ &= \frac{1}{2}\times\frac{\cancel{\sqrt{181}}}{2}\times\frac{98}{\cancel{\sqrt{181}}} \\ &= \frac{98}{4} = \frac{49}{2} \\ &= 24.5\text{ units}^2 \end{aligned}$$

Question 5 (Trenwith)

(a) (3 marks)

- $\angle TAB = 25°$ (angle between a chord and a tangent is equal to the angle in the alternate segment).
- $\angle OAT = 90°$ (angle between a tangent and a radius.
- $\therefore \angle OAB = 65°$ (by subtraction).
- $\angle AEB = 40° + 25° = 65°$.
  (exterior angle of $\triangle ADE$ equals the sum of the opposite interior angles).

$\therefore \triangle AEB$ isosceles as it has 2 equal angles.

(b) (2 marks)

- $\angle ABE = 50°$ (angle sum of $\triangle ABE$)
- $\therefore \angle ABC = 50° + 80° = 130°$.

$$\therefore \text{ reflex } \angle AOC = 260°$$

  (Angle at the centre is twice the angle at the circumference subtended by the same arc AC.)
- Hence $\angle AOC = 100°$ (angle of revolution at the centre O).
- $\therefore OEBC$ is a cyclic quadrilateral as its opposite angles $\angle COE$ & $\angle EBC$ are supplementary.

Question 6 (Barrett)

(a) (2 marks)

$$\begin{aligned} a = 5 \qquad \ell &= a+(n-1)d = 165 \\ S_n &= 1\,785 \\ 1\,785 &= \frac{n}{2}(a+\ell) \\ 3\,570 &= n(5+165) \\ n &= \frac{3\,570}{170} = 21 \end{aligned}$$

(b) i. (2 marks)

$$\begin{aligned} a = 1 \qquad r &= x^2 \\ S_n &= \frac{a(r^n-1)}{r-1} \\ &= \frac{\left(x^2\right)^n-1}{x^2-1} \\ &= \frac{x^{2n}-1}{x^2-1} \end{aligned}$$

ii. (1 mark)

$$\begin{aligned}|r| &< 1\\ \left|x^2\right| &< 1\\ \therefore 0 \le x^2 &< 1\\ \therefore -1 < x &< 1\end{aligned}$$

iii. (2 marks)

$$\begin{aligned}S &= \frac{a}{1-r} = \frac{a}{1-x^2}\\ &= \frac{1}{1-\left(\frac{1}{3}\right)^2} = \frac{1}{1-\frac{1}{9}} = \frac{9}{8}\end{aligned}$$

Question 7 (Barrett)

(a) i. (2 marks)

$$\begin{aligned}y &= 4x^8 - x^{-\frac{1}{2}}\\ \frac{dy}{dx} &= 32x^7 + \frac{1}{2}x^{-\frac{3}{2}}\end{aligned}$$

ii. (2 marks)

$$\begin{aligned}y &= \left(3x^2+7\right)^8\\ \frac{dy}{dx} &= 8 \times 6x\left(3x^2+7\right)^7\\ &= 48x\left(3x^2+7\right)^7\end{aligned}$$

iii. (2 marks)

$$y = \frac{x^3}{x^2+1}$$

$$u = x^3 \qquad v = x^2+1$$

$$u' = 3x^2 \qquad v' = 2x$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{vu'-uv'}{v^2}\\ &= \frac{\left(3x^2\right)\left(x^2+1\right)-(2x)\left(x^3\right)}{\left(x^2+1\right)^2}\\ &= \frac{3x^4+3x^2-2x^4}{\left(x^2+1\right)^2}\\ &= \frac{x^4+3x^2}{\left(x^2+1\right)^2}\end{aligned}$$

(b) i. (3 marks)

$$\begin{aligned}x(t) &= t^3+3t^2-9t\\ v(t) &= 3t^2+6t-9\\ a(t) &= 6t+6\end{aligned}$$

$$x(0) = 0 \qquad v(0) = -9 \qquad a(0) = 6$$

ii. (2 marks) Particle is stationary when $v(t) = 0$.

$$\begin{aligned}3t^2+6t-9 &= 0\\ t^2+2t-3 &= 0\\ (t+3)(t-1) &= 0\end{aligned}$$

As $t > 0$,

$$\begin{aligned}t &= 1\\ x(1) &= 1+3-9 = -5\end{aligned}$$

The particle is 5 units to the left of the origin when it is stationary.

iii. (2 marks)

$$\begin{aligned}x(0) &= 0\\ x(1) &= 1+3-9 = -5\\ x(2) &= 8+12-18 = 2\end{aligned}$$

The particle travels -5 units between $t = 0$ and $t = 1$, then from $t = 1$ to $t = 2$ travels from -5 to 2 (7 units). Total distance travelled: $5 + 7 = 12$ units.

Question 8 (Trenwith)

(a) i. (3 marks)

$$\begin{aligned}y &= x^3+3x^2-24x+2\\ y' &= 3x^2+6x-24\\ y'' &= 6x+6\end{aligned}$$

Stationary pts occur when $y' = 0$,

$$\begin{aligned}3x^2+6x-24 &= 0\\ x^2+2x-8 &= 0\\ (x+4)(x-2) &= 0\\ \therefore x &= -4, 2\end{aligned}$$

$$\begin{aligned}y'' &= 6x+6\Big|_{x=-4}\\ &= 6(-4)+6 < 0\end{aligned} \qquad \Bigg| \qquad \begin{aligned}y'' &= 6x+6\Big|_{x=2}\\ &= 6(2)+6 > 0\end{aligned}$$

Hence $x = -4$ is a local maximum and $x = 2$ is a local minimum.

x		-4		2	
y'	$+$	0	$-$	0	$+$
y	↗	82	↘	-26	↗

ii. (2 marks)
Points of inflexion occur when $y'' = 0$ with a change in concavity:

$$\begin{aligned} y'' = 6x + 6 &= 0 \\ 6x + 6 &= 0 \\ 6x &= -6 \\ x &= -1 \end{aligned}$$

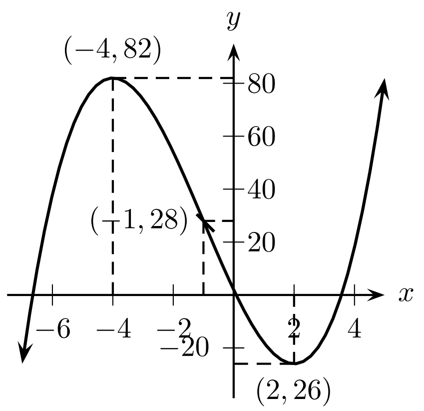
$$y'' = 6x + 6\Big|_{x=-2} < 0 \qquad \Big| \qquad y'' = 6x + 6\Big|_{x=0} > 0$$

Concavity changes $x = -1$.

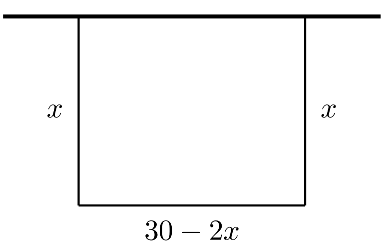
$$\begin{aligned} y &= x^3 + 3x^2 - 24x + 2\Big|_{x=-1} \\ &= -1 + 3 + 24 + 2 = 28 \end{aligned}$$

$(-1, 28)$ is a point of inflexion.

iii. (2 marks)



(b) (4 marks)



$$\begin{aligned} A &= x(30 - 2x) \\ &= 30x - 2x^2 \\ \frac{dA}{dx} &= 30 - 4x \\ \frac{d^2A}{dx^2} &= -4 \end{aligned}$$

Maximum area occurs when $\frac{dA}{dx} = 0$

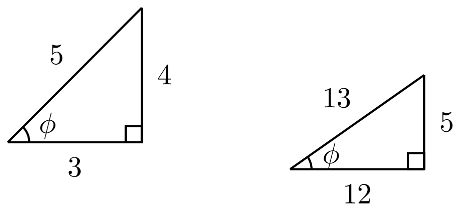
$$\begin{aligned} 30 - 4x &= 0 \\ \underset{\div 4}{4x} &= \underset{\div 4}{30} \\ x &= \frac{15}{2} \end{aligned}$$

$\therefore$ maximum area is

$$\begin{aligned} A &= x(30 - 2x)\Big|_{x=\frac{15}{2}} \\ &= \left(\frac{15}{2}\right)(30 - 15) \\ &= \frac{225}{2}\ \text{m}^2 \end{aligned}$$

Question 9 (Lowe)

(a) (3 marks)



$$\sin\phi = \frac{4}{5} \qquad \sin\alpha = \frac{5}{13}$$

$$\Rightarrow \cos\phi = \frac{3}{5} \qquad \cos\alpha = \frac{12}{13}$$

$$\begin{aligned} \cos(\phi - \alpha) &= \cos\phi\cos\alpha + \sin\phi\sin\alpha \\ &= \left(\frac{3}{5} \times \frac{12}{13}\right) + \left(\frac{4}{5} \times \frac{5}{13}\right) \\ &= \frac{36 + 20}{65} = \frac{56}{65} \end{aligned}$$

(b) (3 marks)

$$\begin{aligned} \sin 105^\circ &= \sin(60^\circ + 45^\circ) \\ &= \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ \\ &= \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

(c) (3 marks)

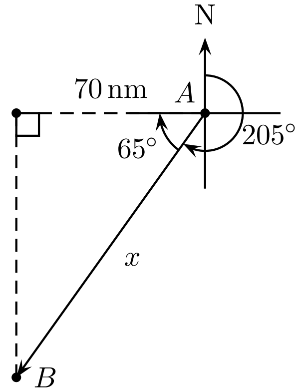
$$\cos\left(2x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$
$$2x - \frac{\pi}{6} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

As $x \in [0, 2\pi]$, then $2x \in [0, 4\pi]$.

$$2x - \frac{\pi}{6} = \frac{\pi}{6}, \left(2\pi - \frac{\pi}{6}\right)$$
$$\left(2\pi + \frac{\pi}{6}\right), \left(4\pi - \frac{\pi}{6}\right)$$
$$2x = \frac{\pi}{3}, 2\pi, \frac{14\pi}{6}, 4\pi$$
$$x = 0, \frac{\pi}{6}, \pi, \frac{7\pi}{6}, 2\pi$$

$x = 0$ is inserted as $x = 2\pi$ is also a solution and $x \in [0, 2\pi]$.

(d) (3 marks)



$$\cos 65^\circ = \frac{70}{x}$$
$$x = \frac{70}{\cos 65^\circ} = 165.6\,\text{nm}$$

Question 10 (Weiss)

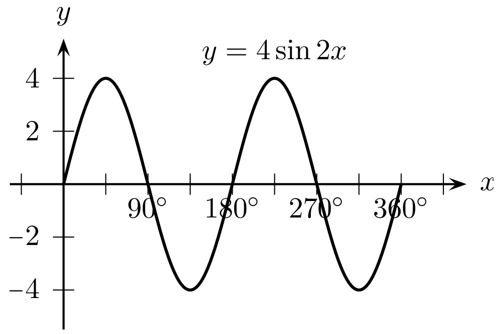
(a) (1 mark)

$$f'(4) = 0$$

(b) (2 marks)

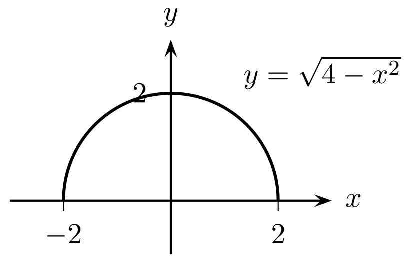
$$f'(3) = \frac{6-10}{3-0} = -\frac{4}{3}$$

(c) i. (3 marks)



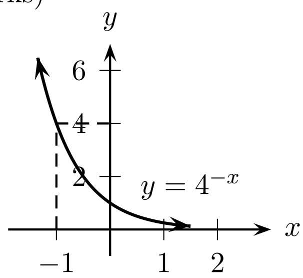
$$D = \{x : 0^\circ \le x \le 360^\circ\}$$
$$R = \{y : -4 \le y \le 4\}$$

ii. (3 marks)



$$D = \{x : -2 \le x \le 2\}$$
$$R = \{y : 0 \le y \le 2\}$$

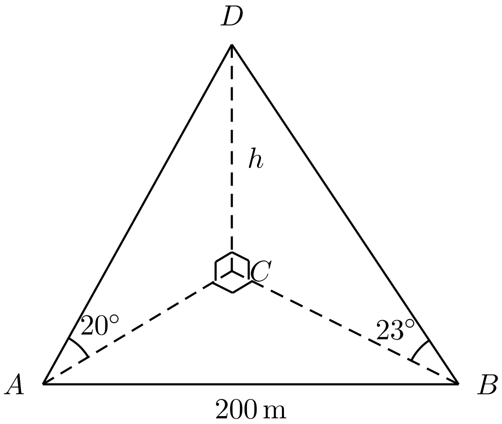
iii. (3 marks)



$$D = \{x : x \in \mathbb{R}\}$$
$$R = \{y : y > 0\}$$

Question 11 (Ireland)

(a) (4 marks)



$$\tan 20° = \frac{h}{AC} \qquad \tan 23° = \frac{h}{BC}$$

$$AC = \frac{h}{\tan 20°} \qquad BC = \frac{h}{\tan 23°}$$

Using Pythagoras' Theorem in $\triangle ABC$,

$$AC^2 + BC^2 = 200^2$$

$$\frac{h^2}{\tan^2 20°} + \frac{h^2}{\tan^2 23°} = 200^2$$

$$\underset{\times \tan^2 20° \tan^2 23°}{\frac{h^2\left(\tan^2 20° + \tan^2 23°\right)}{\tan^2 20° \tan^2 23°}} = \underset{\times \tan^2 20° \tan^2 23°}{200^2}$$

$$h^2\left(\tan^2 20° + \tan^2 23°\right) = 200^2\left(\tan^2 20° \tan^2 23°\right)$$

$$h^2 = \frac{200^2\left(\tan^2 20° \tan^2 23°\right)}{\tan^2 20° + \tan^2 23°}$$

$$h = \frac{200\left(\tan 20° \tan 23°\right)}{\sqrt{\tan^2 20° + \tan^2 23°}}$$

$$= 55.26$$

$$= 55\,\text{m (2 s.f.)}$$

(b) (3 marks)

$$\log_2 x - \log_2(x-2) = \frac{2}{3}\log_2 27$$

$$\log_2\left(\frac{x}{x-2}\right) = \log_2 27^{\frac{2}{3}}$$

$$\left(\frac{x}{x-2}\right) = 9$$

$$x = 9x - 18$$

$$\underset{\div 8}{8x} = \underset{\div 8}{18}$$

$$x = \frac{9}{4}$$

(c) (3 marks)

$$y = x^{\frac{1}{2}} \qquad x = 1$$

$$y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$y'' = -\frac{1}{4}x^{-\frac{3}{2}}$$

$$R = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

$$= \frac{\left(1 + \left(\frac{1}{2\sqrt{x}}\right)^2\right)^{\frac{3}{2}}}{\left|-\frac{1}{4}x^{-\frac{3}{2}}\right|}$$

$$= \left.\frac{\left(1 + \frac{1}{4x}\right)^{\frac{3}{2}}}{\left|-\frac{1}{4}x^{-\frac{3}{2}}\right|}\right|_{x=1}$$

$$= \frac{\left(\frac{5}{4}\right)^{\frac{3}{2}}}{\frac{1}{4}} = \frac{\frac{5\sqrt{5}}{8}}{\frac{1}{4}}$$

$$= \frac{5\sqrt{5}}{2}$$

(d) (3 marks)

$$f(x) = \sqrt{3 - \sqrt{x}}$$

As the expression within the radical is ≥ 0

$$3 - \sqrt{x} \geq 0$$

$$3 \geq \sqrt{x}$$

$$0 \leq x \leq 9$$

$$\therefore D = \{x : x \in [0, 9]\}$$

$$f(0) = \sqrt{3} \qquad f(9) = 0$$

$$\therefore R = \left\{y : y \in \left[0, \sqrt{3}\right]\right\}$$