Penrith Selective High School

2016

Year 11 Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A separate reference sheet is provided for this examination paper
- In Questions 6-10, show relevant mathematical reasoning and/or calculations in answer booklets provided
- All diagrams are not to scale
- Multiple choice answer sheet is on page 10 of this paper

Total Marks – 55

Section I Pages 2–4

5 marks

- Attempt Questions 1–5

Section II Pages 5–9

50 marks

- Attempt Questions 6–10

Student Number: ____

Teacher’s Name: ____

Section I:

5 marks
Attempt Questions 1–5

Use the multiple choice answer sheet provided on page 10 for Questions 1–5.

Q1. The asymptotes of $y = \dfrac{5x}{x^2 + 4x - 12}$ are:

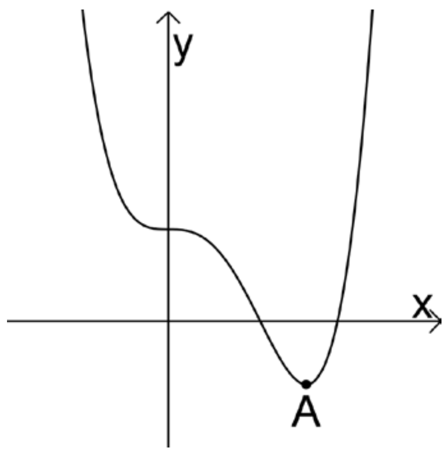
(A) $x = -2, x = 6, y = 0$

(B) $x = 2, x = -6, y = 0$

(C) $x = 2, x = -6, y = 5$

(D) $x = -2, x = 6, y = 5$

Q2. The diagram below shows $y = f(x)$.



At point A:

(A) $f'(x) < 0, f''(x) = 0$

(B) $f'(x) > 0, f''(x) = 0$

(C) $f'(x) = 0, f''(x) < 0$

(D) $f'(x) = 0, f''(x) > 0$

Q3. Simplifying the expression $\frac{{}^{n}C_{r}}{{}^{n}P_{r}}$ gives:

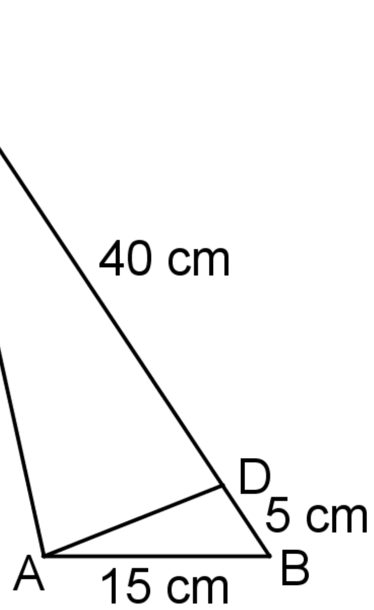
(A) $\frac{1}{r!}$

(B) $r!$

(C) $\frac{n!}{(n-r)!}$

(D) $\frac{n!}{r!}$

Q4. In the following figure, AB is $15\ cm$, BD is $5\ cm$ and CD is $40\ cm$. If ΔABC has area $612\ cm^2$, then the area of ΔABD is:



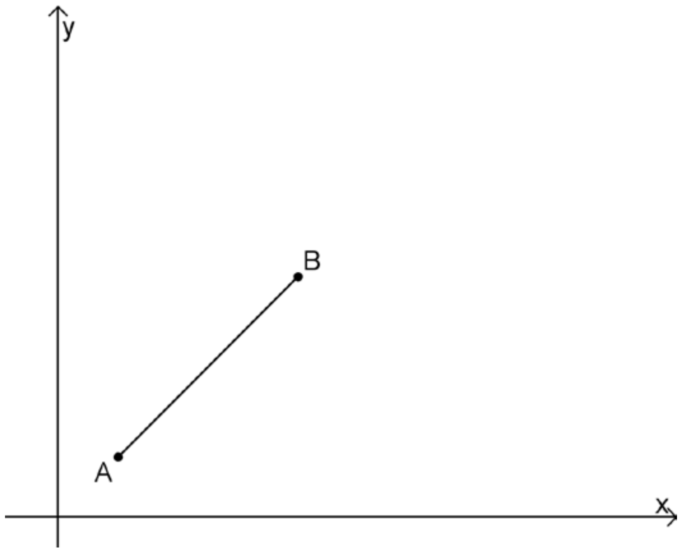
(A) $68\ cm^2$

(B) $65\ cm^2$

(C) $62\ cm^2$

(D) $59\ cm^2$

Q5. AB is an interval on the number plane as shown below.



C divides AB internally in the ratio $2:3$, D divides AB externally in the ratio $2:1$. B divides DC in the ratio:

(A) $1:3$

(B) $3:1$

(C) $3:5$

(D) $5:3$

End of Section I

Section II

50 Marks
Attempt Questions 6–10

Answer each question on a SEPARATE answer booklet.

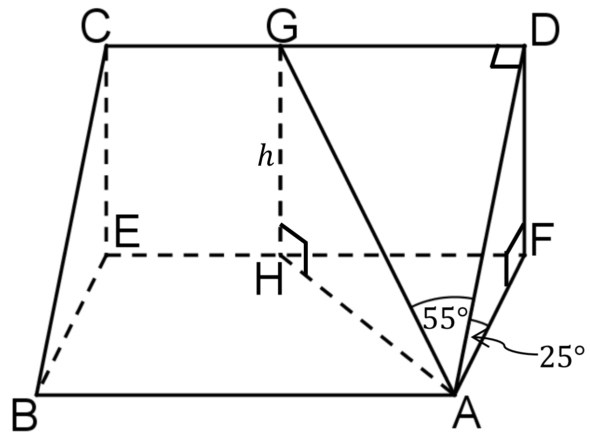
In Questions 6–10, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) **Start this question on a new answer booklet**

a) Solve $\dfrac{4x-1}{x+6} \geq 1$ **3**

b) Given that $\cos 2\theta = -2\sin^2\theta - \sin 2\theta$, find the value of $\tan\theta$. **3**

c) A plane hillside $ABCD$ makes an angle of $25°$ with the horizontal. A path AG makes an angle of $55°$ between AD and AG as shown below.



If $DF = GH = h$:

i) Show that $AD = \dfrac{h}{\sin 25°}$ **1**

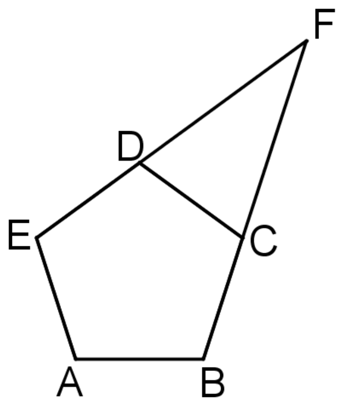
ii) Show that $AG = \dfrac{h}{\sin 25° \cos 55°}$ **1**

iii) Hence, find the inclination of the path AG to the horizontal. Leave your answer to the nearest minute. **2**

End of Question 6

Question 7 (10 marks) **Start this question on a new answer booklet**

a) $ABCDE$ is a regular pentagon. Sides BC and ED are produced to meet at F.

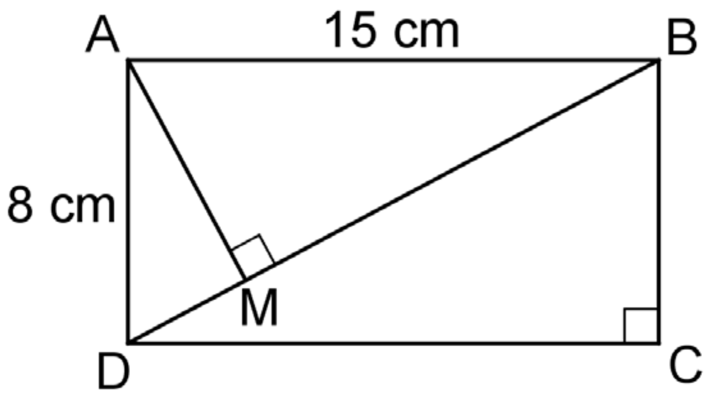


Copy the diagram onto your answer booklet.

i) Prove that $CF = DF$, giving reasons. **2**

ii) Prove that AF bisects $\angle BAE$, giving reasons. **3**

b) $ABCD$ is a rectangle with $AB = 15\ cm$, $AD = 8\ cm$ and AM is perpendicular to BD.



Copy the diagram onto your answer booklet.

i) Find the length of BD. **1**

ii) Prove that $\Delta ABM ||| \Delta DBA$. **2**

iii) Hence find the exact length of BM. **2**

End of Question 7

Question 8 (10 marks) **Start this question on a new answer booklet**

a) From five teachers, seven students and ten parents, a committee of six is formed. How many different committees can be chosen if it contains:

i) exactly three parents, two students and one teacher? **1**

ii) at least three teachers, at least one student and at least one parent are present on this committee? **2**

b) How many different arrangements of the letters of the word of **DISTINCTION**, are possible if:

i) all letters are used. **1**

ii) the two T's occupy the first and last place. **1**

iii) all three I's are not together. **2**

c) At a dinner party, five singers and five dancers sit at a round table.

i) If there are no restrictions, in how many ways can they be arranged? **1**

ii) If they are seated alternately at a round table, so that no two singers and no two dancers sit next to each other. In how many ways can this be done? **1**

d) If ${}^{2n+1}P_{n+1} = 11 \times {}^{2n}P_n$, find the value of n. **1**

End of Question 8

Question 9 (10 marks) **Start this question on a new answer booklet**

a) Differentiate $y = \dfrac{4}{9(2x^3 - x)}$ **1**

b) Use the definition of the derivative $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ to find $f'(x)$ where $f(x) = \sqrt{2x - 5}$. **2**

c) $\left(\dfrac{1}{2}, \dfrac{17}{4}\right)$ is the point of intersection of the curves $y = 2x^3 + 8x$ and $y = x^2 + 4$. Find the acute angle between the curves at this point. Correct your answer to the nearest degree. **2**

d) i) If $f(x) = \left(2 + 3\sqrt{x}\right)\left(2 - x\sqrt{x}\right)^{-1}$, show that: **3**

$$f'(x) = \frac{3\left(1 + x + x\sqrt{x}\right)}{\sqrt{x}\left(2 - x\sqrt{x}\right)^2}$$

ii) Find the equation of the normal to the curve **2**

$f(x) = \left(2 + 3\sqrt{x}\right)\left(2 - x\sqrt{x}\right)^{-1}$ when $x = 1$.

End of Question 9

Question 10 (10 marks) **Start this question on a new answer booklet**

a) Consider the function $f(x) = \dfrac{x+3}{x^2+7}$,

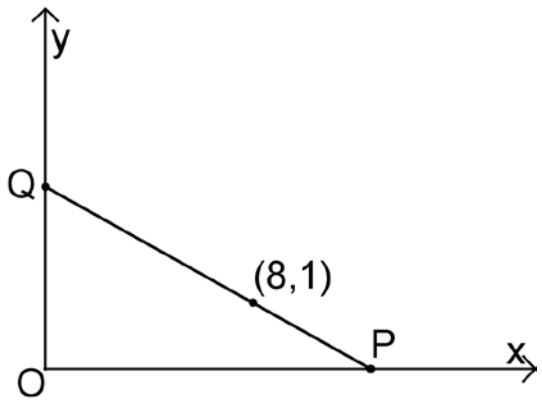
i) Show that the derivative of this function is given by: **1**

$$f'(x) = \frac{-x^2 - 6x + 7}{(x^2+7)^2}$$

ii) Find the stationary points and determine their nature. **2**

iii) Without finding any point of inflexion, sketch the above function, showing the stationary points, the y-intercept and other key features. **2**

b) A line is drawn through the point $(8, 1)$ to cut the positive x-axis at P and the positive y-axis at Q. The gradient of PQ is m.



i) Show that the coordinates of P are $\left(\dfrac{8m-1}{m}\ ,\ 0\right)$ **1**

ii) Show that the area of ΔOPQ is $\dfrac{1}{2}\left(16 - 64m - \dfrac{1}{m}\right)$ **1**

iii) Show that the minimum area is $16\ units^2$. **3**

End of Paper

Student Number:____

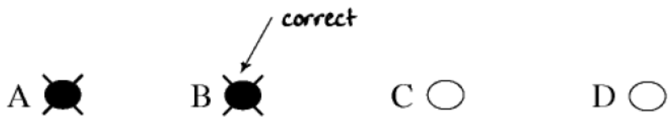
Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.



Start Here ➔	**1.**	A ○	B ○	C ○	D ○
	2.	A ○	B ○	C ○	D ○
	3.	A ○	B ○	C ○	D ○
	4.	A ○	B ○	C ○	D ○
	5.	A ○	B ○	C ○	D ○

2016 Yr11 Extension 1 Solution

Section I - MCQ

Q1. B

$$f(x) = \frac{5x}{x^2 + 4x - 12}$$
$$f(x) = \frac{5x}{(x+6)(x-2)}$$

Asymptotes: $x = 2, x = -6,\ y = 0$

Q2. D (Minimum turning point at point A)

Q3. A

$$\frac{^{n}C_r}{^{n}P_r} = \frac{n!}{(n-r)!\,r!} \div \frac{n!}{(n-r)!} = \frac{1}{r!}$$

Q4. A

In ΔABC and ΔABD

$\angle ACB = \angle BCD$ (Common angle)

$$\frac{BD}{AB} = \frac{5}{15} = \frac{1}{3} \qquad \frac{AB}{AC} = \frac{5}{15} = \frac{1}{3}$$

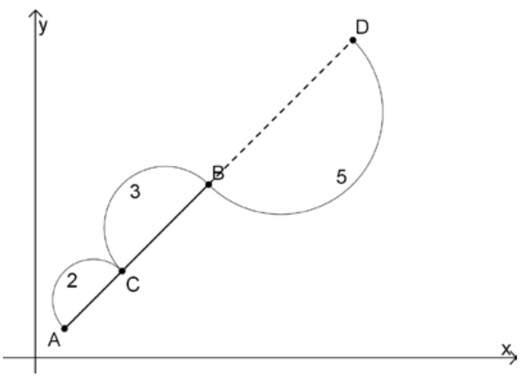
$$\frac{BD}{AB} = \frac{AB}{AC}$$

$\therefore \Delta ABC|||\Delta ABD$ (2 corresponding sides in proportion and their included angle equal)

The sides are in ratio $1:3$ so the areas are in ratio $1:9$.

If ΔABC has area 612 cm^2, then ΔABD has area 68 cm^2.

Q5. D



$DA:DB = 2:1 = 10:5 \qquad AB = BD$

$\therefore B$ divides DC internally $5:3$.

Section II

Q6.

a)

$$\frac{4x-1}{x+6} \geq 1 \qquad x \neq -6$$

$$(4x-1)(x+6) \geq (x+6)^2$$

$$(4x-1)(x+6) - (x+6)^2 \geq 0$$

$$(x+6)[(4x-1)-(x+6)] \geq 0$$

$$(x+6)(3x-7) \geq 0$$

$$x < -6, \qquad x \geq \frac{7}{3}$$

b) $\cos 2\theta = -2\sin^2\theta - \sin 2\theta$

$$\cos^2\theta - \sin^2\theta = -2\sin^2\theta - 2\sin\theta\cos\theta$$

$$\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 0$$

$$(\cos\theta + \sin\theta)^2 = 0$$

$$\cos\theta = -\sin\theta$$

$$\tan\theta = -1$$

c) i)

$$\sin 25° = \frac{DF}{AD}$$

$$\sin 25° = \frac{h}{AD}$$

$$AD = \frac{h}{\sin 25°}$$

ii)

$$\cos 55° = \frac{AD}{AG}$$

$$AG = \frac{AD}{\cos 55°}$$

$$AG = \frac{h}{\sin 25° \cos 55°}$$

iii)

Let $\angle HGA = \theta$

$$\sin\theta = \frac{GH}{AG}$$

$$\sin\theta = h \div \frac{h}{\sin 25° \cos 55°}$$

$$\sin\theta = \sin 25° \cos 55°$$

$$\theta \approx 14°1'.47''$$

$\theta = 14°2'$ (nearest minute)

Q7.

a) i) $\angle FDC = \angle FCD = 360° \div 5 = 72°$ (Exterior angle of a regular pentagon)
ΔFDC is an isosceles triangle (equal base angles)
$\therefore FD = FC$ (equal sides of isosceles ΔFDC)

ii) Construct FA
In ΔFEA and ΔFBA
$ED = CB$ (equal sides of regular pentagon)
$FD = FC$ (Proven above)
$ED + FD = CB + FC$
$EF = BF$
FA is common
$EA = AB$ (equal sides of regular pentagon)
$\therefore \Delta FEA \equiv \Delta FBA\ (SSS)$
$\angle EAF = \angle BAF$ (corresponding angles in congruent triangles)
$\therefore AF$ bisects $\angle BAE.$

b) i) $BD = \sqrt{8^2 + 15^2} = 17$ (Pythagoras' Theorem)

ii) In ΔABM and ΔDBA
$\angle ABM = \angle ABD$ (Common angle)
$\angle AMB = \angle DAB = 90°$
$\therefore \Delta ABM ||| \Delta DBA$ (Equiangular)

iii) $BM: AB = AB: BD$ (corresponding sides in similar triangles)

$$\frac{BM}{15} = \frac{15}{17}$$

$$BM = \frac{225}{17}$$

Q8.

a) i) ${}^{10}C_3 \times {}^{7}C_2 \times {}^{5}C_1 = 12600$

ii) 4 teachers, 1 parent and 1 student ${}^{5}C_4 \times {}^{10}C_1 \times {}^{7}C_1 = 350$
3 teachers, 2 parents and 1 student ${}^{5}C_3 \times {}^{10}C_2 \times {}^{7}C_1 = 3150$
3 teachers, 1 parent and 2 students ${}^{5}C_3 \times {}^{10}C_1 \times {}^{7}C_2 = 2100$
Total: $350 + 3150 + 2100 = 5600$

b) **DISTINCTION**
i) 11 letters, 3 I's, 2 T's, 2 N's
$$\frac{11!}{3!\,2!\,2!} = 1663200$$

ii) T _ _ _ _ _ _ _ _ _ T
$$\frac{9!}{3!\,2!} = 30240$$

iii) 3 I's together
$$\frac{9!}{2!\,2!} = 90720$$

3 I's not together
$1663200 - 90720 = 1572480$

c) i) $9! = 362880$

ii) $5! \times 4! = 2880$

d) ${}^{2n+1}P_{n+1} = 11 \times {}^{2n}P_n$
$$\frac{(2n+1)!}{\big(2n+1-(n+1)\big)!} = 11 \times \frac{(2n)!}{(2n-n)!}$$
$$\frac{(2n+1)!}{n!} = 11 \times \frac{(2n)!}{n!}$$
$(2n+1)! = 11 \times (2n)!$
$(2n+1) \times (2n)! = 11 \times (2n)!$
$2n + 1 = 11$
$n = 5$

Q9.

a)

$$y = \frac{4}{9(2x^3 - x)}$$
$$y = \frac{4}{9} \times (2x^3 - x)^{-1}$$
$$\frac{dy}{dx} = -\frac{4}{9} \times (6x^2 - 1)(2x^3 - x)^{-2}$$
$$\frac{dy}{dx} = -\frac{4(6x^2 - 1)}{9(2x^3 - x)^2}$$

b) $f(x) = \sqrt{2x - 5}$

$$f(x + h) = \sqrt{2(x + h) - 5}$$
$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{\sqrt{2(x + h) - 5} - \sqrt{2x - 5}}{h} \times \frac{\sqrt{2(x + h) - 5} + \sqrt{2x - 5}}{\sqrt{2(x + h) - 5} + \sqrt{2x - 5}}$$
$$f'(x) = \lim_{h \to 0} \frac{2(x + h) - 5 - (2x - 5)}{h\left(\sqrt{2(x + h) - 5} + \sqrt{2x - 5}\right)}$$
$$f'(x) = \lim_{h \to 0} \frac{2h}{h\left(\sqrt{2(x + h) - 5} + \sqrt{2x - 5}\right)}$$
$$f'(x) = \lim_{h \to 0} \frac{2}{\left(\sqrt{2(x + h) - 5} + \sqrt{2x - 5}\right)}$$
$$f'(x) = \frac{2}{\sqrt{2x - 5} + \sqrt{2x - 5}}$$
$$f'(x) = \frac{1}{\sqrt{2x - 5}}$$

c) $y = 2x^3 + 8x$

$$\frac{dy}{dx} = 6x^2 + 8$$
$$at\ x = \frac{1}{2} \qquad m_1 = \frac{19}{2}$$

$$y = x^2 + 4$$
$$\frac{dy}{dx} = 2x$$
$$at\ x = \frac{1}{2} \qquad m_2 = 1$$

$$\tan\theta = \left|\frac{\frac{19}{2} - 1}{1 + \frac{19}{2} \times 1}\right|$$
$$\tan\theta = \frac{17}{21}$$
$\theta = 39°$ (neares degree)

d) i)

$$f(x)=\frac{2+3\sqrt{x}}{2-x\sqrt{x}}=\frac{2+3x^{\frac{1}{2}}}{2-x^{\frac{3}{2}}}$$

$$f'(x)=\frac{\left(2-x^{\frac{3}{2}}\right)\times\frac{3}{2}x^{-\frac{1}{2}}-\left(2+3x^{\frac{1}{2}}\right)\times-\frac{3}{2}x^{\frac{1}{2}}}{\left(2-x^{\frac{3}{2}}\right)^2}$$

$$f'(x)=\frac{\frac{3}{2}x^{-\frac{1}{2}}\left(2-x^{\frac{3}{2}}+\left(2+3x^{\frac{1}{2}}\right)\times x\right)}{\left(2-x^{\frac{3}{2}}\right)^2}$$

$$f'(x)=\frac{3\left(2-x^{\frac{3}{2}}+2x+3x^{\frac{3}{2}}\right)}{2x^{\frac{1}{2}}\left(2-x^{\frac{3}{2}}\right)^2}$$

$$f'(x)=\frac{3\left(1+x+x^{\frac{3}{2}}\right)}{x^{\frac{1}{2}}\left(2-x^{\frac{3}{2}}\right)^2}$$

$$f'(x)=\frac{3\left(1+x+x\sqrt{x}\right)}{\sqrt{x}\left(2-x\sqrt{x}\right)^2}$$

ii)

$$f'(1)=\frac{3\left(1+1+1\times\sqrt{1}\right)}{\sqrt{1}\left(2-1\times\sqrt{1}\right)^2}$$

$$f'(1)=9$$

$$m_N=-\frac{1}{9}$$

$$f(1)=\frac{2+3\times\sqrt{1}}{2-1\times\sqrt{1}}=5$$

Equation of normal at $x=1$

$$y-5=-\frac{1}{9}(x-1)$$

$$x+9y-46=0$$

Q10.

a) i)

$$f(x) = \frac{x+3}{x^2+7}$$

$$f'(x) = \frac{(x^2+7) \times 1 - (x+3) \times 2x}{(x^2+7)^2}$$

$$f'(x) = \frac{x^2+7-2x^2-6x}{(x^2+7)^2}$$

$$f'(x) = \frac{-x^2-6x+7}{(x^2+7)^2}$$

ii) Stationary points when $f'(x) = 0$

$-x^2 - 6x + 7 = 0$

$x^2 + 6x - 7 = 0$

$(x+7)(x-1) = 0$

$x = -7\,, \qquad x = 1$

$y = -\frac{1}{14}\,, \qquad y = \frac{1}{2}$

x	-8	-7	0	1	2
$f'(x)$	$-\frac{9}{5041}$	0	$\frac{1}{7}$	0	$-\frac{9}{121}$

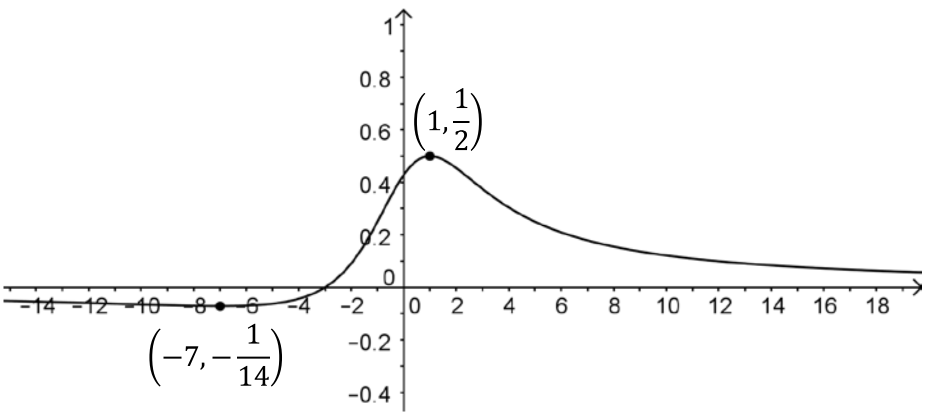
Minimum turning point at $\left(-7, -\frac{1}{14}\right)$

Maximum turning point at $\left(1, \frac{1}{2}\right)$

iii)

$y = \lim_{x\to\infty} \frac{x+3}{x^2+7}$

As $x \to \pm\infty, \;\; y \to 0$



b) i) Equation of PQ is:

$y - 1 = m(x - 8)$

at P, $y = 0$

$0 - 1 = mx - 8m$

$mx = 8m - 1$

$$x = \frac{8m - 1}{m}$$

$$\therefore P\left(\frac{8m - 1}{m}\ ,\ 0\right)$$

ii) at Q, $x = 0$

$y - 1 = m(0 - 8)$

$y = 1 - 8m$

$\therefore Q\ (0, 1 - 8m)$

Area of ΔOPQ

$$A = \frac{1}{2} \times (1 - 8m) \times \frac{8m - 1}{m}$$

$$A = \frac{1}{2} \times \left(\frac{8m - 1 - 64m^2 + 8m}{m}\right)$$

$$A = \frac{1}{2}\left(\frac{16m - 64m^2 - 1}{m}\right)$$

$$A = \frac{1}{2}\left(16 - 64m - \frac{1}{m}\right)$$

iii)

$$\frac{dA}{dm} = \frac{1}{2}\left(-64 + \frac{1}{m^2}\right)$$

$$\frac{1}{2}\left(-64 + \frac{1}{m^2}\right) = 0$$

$$\frac{1}{m^2} = 64$$

$$m = \pm\frac{1}{8}$$

$$m = -\frac{1}{8}$$

Gradient is negative due to interval cutting positive x and y axis.

$$\frac{d^2A}{dm^2} = \frac{1}{2}\left(-\frac{2}{m^3}\right)$$

$$\frac{d^2A}{dm^2} = -\frac{1}{m^3}$$

$$at\ m = -\frac{1}{8} \qquad \frac{d^2A}{dm^2} = -\frac{1}{\left(-\frac{1}{8}\right)^3} = 512 > 0 \quad \rightarrow minimum$$

Minimum area occurs when $m = -\frac{1}{8}$

$$A = \frac{1}{2}\left(16 - 64m - \frac{1}{m}\right)$$

$$A = \frac{1}{2}\left(16 - 64 \times \left(-\frac{1}{8}\right) - \frac{1}{\left(-\frac{1}{8}\right)}\right)$$

$$A = 16\ units^2$$