Mr Antonio
Mr Dudley
Mrs Israel
Mrs Kerr
Mrs Soutar

Name: ....

Teacher: ....

2010
YEAR 11
PRELIMINARY EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Start each question in a new booklet
- Marks may be deducted for careless or untidy work

Total Marks – 72

- Attempt Questions 1–6
- All questions are of equal value

Mark	/72
Rank	/
Highest mark	/72

Question 1. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Express in simplest form $6\times 3^{n}+3^{n+1}$. **2**

(b) Solve the inequality $\dfrac{1}{1-x}>3$. **3**

(c) Evaluate $\lim\limits_{x\to 2}\dfrac{2x^2-7x+6}{x-2}$. **2**

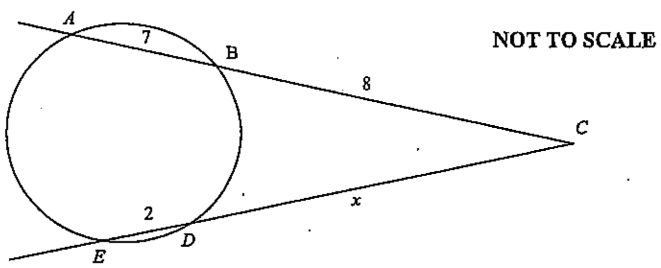
(d) Differentiate $f(x)=2x+5$ from first principles. **2**

(e) A curve has parametric equations $x=\dfrac{t}{2}$ and $y=3t^2$. **1**
Find the Cartesian equation for this curve.

(f) Find the acute angle between the line $y=3x+1$ and $2y=1+x$. **2**
(Leave your answer correct to the nearest degree.)

Question 2. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) In the diagram, not drawn to scale, ABC and EDC are straight lines. $AB = 7$ cm, $BC = 8$ cm, $DE = 2$ cm. Find CD. **2**



(b) Find the value(s) of m for which the equation $4x^2 - mx + 9 = 0$ has exactly one real root. **2**

(c) Given that $g(x) = x^4 - kx^3 - 2x + 33$ has a factor of $(x-3)$, find the value of k. **2**

(d) The point $P(5, -10)$ divides the interval AB, where A is the point $(a, 2b)$ and B point $(3b, -a)$, externally in the ratio 3 : 2. **3**

Find the values of a and b.

(e) Find the equation of the locus of a point P (x, y) which moves such that its distance from the point A(1, 4) is four times its distance from the point B (-2, 1). i.e. PA = 4 PB. **3**

Question 3. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Solve the equation $x^2 - 12 = |x|$. **3**

(ii) Sketch the graphs of $y = x^2 - 12$ and $y = |x|$ on a number plane, showing clearly any points of intersection. (Do not find the x-intercepts.) **2**

(iii) Hence solve the inequation $x^2 > |x| + 12$. **1**

(b) (i) Show that $\sin\theta\cos\theta\,\text{cosec}^2\theta = \cot\theta$. **1**

(ii) Hence solve $\sin^2\theta\cos^2\theta\,\text{cosec}^4\theta = \frac{1}{4}$ for $0 \le \theta \le 360^\circ$. **2**

(Leave your answer correct to the nearest degree.)

(c) Find the maximum area a triangle can have if the sum of its base and height is 10cm. **3**

Question 4. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the equation of the tangent to the curve $y=x\sqrt{x^2+3}$ at the point where $x=1$. **3**

(b) Consider the function $f(x)=\dfrac{x}{x-1}$.

(i) Show that $f(x)$ has no stationary points and no inflexion points. **3**

(ii) Sketch the graph, showing asymptotes and intercepts on the axes. **2**

Question 4 continues on page 6

Question 4. (continued) **Marks**

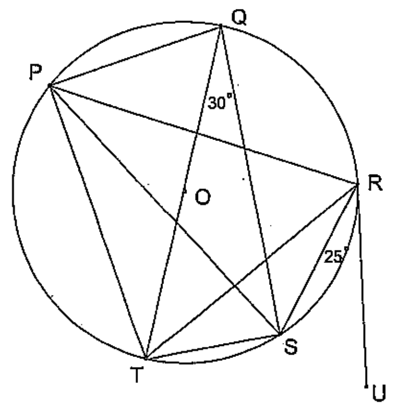
(c) RU is a tangent and QT is a diameter of the circle, centre O.

$\angle SRU = 25°$ and $\angle TQS = 30°$.

Showing all reasons:

(i) Prove that $\angle TPR = 55°$. **2**

(ii) Find $\angle QSR$. **2**



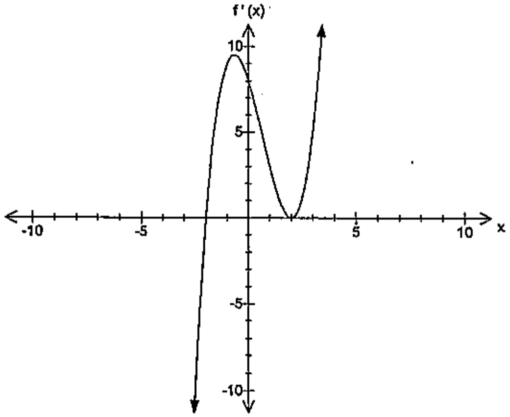
End of Question 4

Question 5. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\left(x+\frac{1}{x}\right)^2-5\left(x+\frac{1}{x}\right)+6=0$. **3**

(b) The diagram below represents a sketch of $y=f'(x)$.

Given that $f(0)=10$, in your writing book, sketch neatly a graph of $y=f(x)$. **2**



(c) Consider the polynomial $P(x)=x^3-2x^2-9x-6$.

(i) Show that $P(-1)=0$. **1**

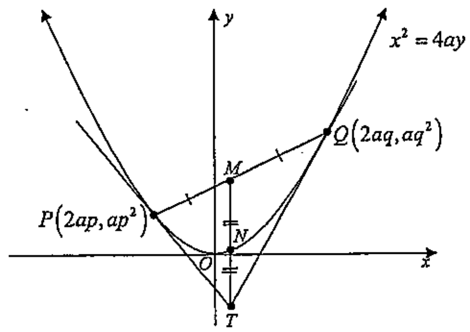
(ii) Hence express the polynomial as a product of two factors. **2**

(iii) Find the exact values of the other zeroes of the polynomial. **1**

(d) Prove that $\frac{\sin A}{1+\cos A}=\operatorname{cosec} A(1-\cos A)$. **3**

Question 6. (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)



In the diagram, $P(2ap,ap^2)$ and $Q(2aq,aq^2)$ are distinct variable points on the parabola $x^2=4ay$.

(i) Show that the tangent at P has equation $y=px-ap^2$. **2**

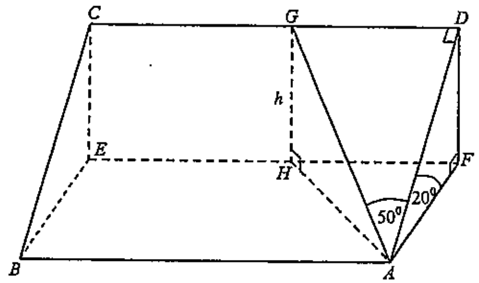
(ii) The tangents at P and Q meet at T. **2**
Show that T is the point $(a(p+q),apq)$.

(iii) N is the midpoint of MT. Show that as P and Q vary on the parabola $x^2=4ay$, N also varies on the parabola $x^2=4ay$. **3**

Question 6 continues on page 9

Question 6. (continued) **Marks**

(b) A plane hillside $ABCD$ makes an angle of 20^0 with the horizontal.
A path AG makes an angle of 50^0 with a line of greatest slope.
If $DF = GH = h$:



(i) Show that $AD = \dfrac{h}{\sin 20^0}$. **1**

(ii) Show that $AG = \dfrac{h}{\sin 20^0 \cos 50^0}$. **2**

(iii) Hence, find the angle of inclination of the path AG to the horizontal. **2**
(Leave your answer correct to the nearest degree.)

End of paper

Mathematics Ext 1 2010 Yearly

1a) 6×3^n

$= 6 \times 3^n + 3^n \times 3$

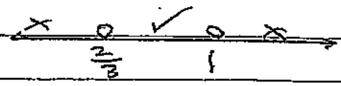
$= 3^n (6+3)$ (1 mark)

$= 3^n \times 9$

$= 3^n \times 3^2$

$= 3^{n+2}$ (1 mark)

b) $\frac{1}{1-x} > 3$



Consider $x \neq 1$

$\frac{1}{1-x} = 3$

$3 - 3x = 1$

$-3x = -2$

$x = \frac{2}{3}$

(1 mark)

Test

$x = 0$ $\quad \frac{1}{1-0} > 3$ False

$x = \frac{9}{10}$ $\quad \frac{1}{1-\frac{9}{10}} > 3$ True

$x = 2$ $\quad \frac{1}{1-2} > 3$ False

(1 mark)

$\therefore$ Solution $\{x : \frac{2}{3} < x < 1\}$

(1 mark)

c) $\lim_{x \to 2} \frac{2x^2 - 7x + 6}{x-2}$

$= \lim_{x \to 2} \frac{(2x-3)(x-2)}{x-2}$ (1 mark)

$= \lim_{x \to 2} 2x - 3$

$= 2 \times 2 - 3$

$= 1$ (1 mark)

1d) $f(x) = 2x + 5$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h) + 5 - (2x+5)}{h}$ (1 mark

$= \lim_{h \to 0} \frac{2x + 2h + 5 - 2x - 5}{h}$

$= \lim_{h \to 0} \frac{2h}{h}$

$= \lim_{h \to 0} 2$

$= 2$ (1 mark)

e) $x = \frac{t}{2}$ (1) $\quad y = 3t^2$ (2)

From (1) $t = 2x$ (3)

Sub (3) into (2)

$y = 3(2x)^2$

$y = 3 \times 4x^2$

$y = 12x^2$ (1 mark)

f) $y = 3x + 1$ $\quad 2y = 1 + x$

$y = \frac{x}{2} + \frac{1}{2}$

$m_1 = 3$ $\quad m_2 = \frac{1}{2}$

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$= \left| \frac{3 - \frac{1}{2}}{1 + 3 \times \frac{1}{2}} \right|$ (1 mark)

$= \left| \frac{2\frac{1}{2}}{2\frac{1}{2}} \right|$

$= 1$ (1 mark)

$\therefore \tan\theta = 1$

$\theta = 45°$

a) $8x(8+7) = x(x+2)$ (1 mark)

$120 = x^2+2x$

$x^2+2x-120=0$

$(x-10)(x+12)=0$

$\therefore x = 10$ or $x = -12$

Since $x =$ CD which is a length, CD $= 10$. (1 mark)

b) For one real root $\Delta = 0$

$4x^2 - mx + 9 = 0$ $a=4$ $b=-m$ $c=9$

$\Delta = m^2 - 4\times4\times9$

$\therefore m^2 - 144 = 0$ For one real root. (1 mark)

$m^2 = 144$

$m = \pm 12$ (1 mark)

c) If $(x-3)$ is a factor of $g(x)$ then

$g(3) = 0$

$\therefore (3)^4 - k(3)^3 - 2(3) + 33 = 0$ (1 mark)

$81 - 27k - 6 + 33 = 0$

$108 - 27k = 0$

$27k = 108$

$k = 4$ (1 mark)

2d) $\quad x_1 \; y_1 \qquad x_2 \; y_2$

$A(a, 2b)$ $B(3b, -a)$ $P(5, -10)$

External

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right) \qquad \begin{array}{cc} 3 & : -2 \\ m & n \end{array}$$

$5 = \dfrac{-2a+9b}{1}$ $\quad -10 = \dfrac{-4b-3a}{1}$ (1 mark)

$5 = -2a+9b$ $\quad -10 = -4b-3a$

$2a-9b=-5$ ① $\quad 3a+4b=10$ ②

$2a-9b=-5$ ①

$3a+4b=10$ ②

① $\times 4$ and ② $\times 9$

$8a-36b=-20$ ③

$27a+36b=90$ ④ (1 mark for both equations)

③ + ④

$35a = 70$

$a=2$

Sub $a=2$ into ①

$4-9b=-5$

$-9b=-9$

$b=1$

$\therefore a=2$, $b=1$ (1 mark for solving)

2e) $P(x,y)$ $A(1,4)$ $B(-2,1)$

$d_{PA} = \sqrt{(x-1)^2 + (y-4)^2}$

$d_{PB} = \sqrt{(x+2)^2 + (y-1)^2}$

Now $d_{PA} = 4\, d_{PB}$

$\sqrt{(x-1)^2 + (y-4)^2} = 4\sqrt{(x+2)^2 + (y-1)^2}$ (1 mark)

$(x-1)^2 + (y-4)^2 = 16\{(x+2)^2 + (y-1)^2\}$

$x^2 - 2x + 1 + y^2 - 8y + 16 = 16\{x^2 + 4x + 4 + y^2 - 2y + 1\}$

(1 mark) $x^2 - 2x + 1 + y^2 - 8y + 16 = 16x^2 + 64x + 64 + 16y^2 - 32y + 16$

$15x^2 + 66x + 15y^2 - 24y + 63 = 0$

$5x^2 + 5y^2 + 22x - 8y + 21 = 0$ (1 mark)

Question 3

a i) Consider $x^2 - 12 = x$ and $x^2 - 12 = -x$

$x^2 - x - 12 = 0$ $\quad$ $x^2 + x - 12 = 0$

$(x-4)(x+3) = 0$ $\quad$ $(x+4)(x-3) = 0$

$\therefore x = 4$ or $x = -3$ $\quad$ $\therefore x = -4$ or $x = 3$

Check solutions

When $x = 4$ LHS $= 16 - 12 = 4$ ✓

$x = -4$ LHS $= 16 - 12 = 4$ ✓

$x = 3$ LHS $= 9 - 12 = -3$ ✗

$x = -3$ LHS $= 9 - 12 = -3$ ✗

$\therefore$ Soln $x = 4$ and $x = -4$

a i) Consider $x^2 - 12 = x$ and $x^2 - 12 = -x$

$x^2 - x - 12 = 0$ $\quad$ $x^2 + x - 12 = 0$

$(x-4)(x+3) = 0$ $\quad$ $(x+4)(x-3) = 0$

1 mark $\therefore x = 4$ or $x = -3$ 1 $\quad$ $\therefore x = -4$ or $x = 3$ 1

Check solutions

When $x = 4$ LHS $= 16 - 12 = 4$ ✓

$x = -4$ LHS $= 16 - 12 = 4$ ✓

$x = 3$ LHS $= 9 - 12 = -3$ ✗

$x = -3$ LHS $= 9 - 12 = -3$ ✗

1 mark for checking solutions 1

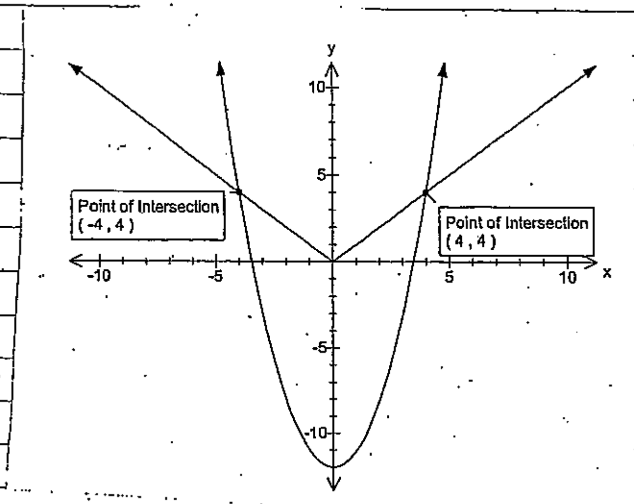
$\therefore$ Soln $x = 4$ and $x = -4$

1 mark for solutions

3

Question 3

a ii)



1 mark for each graph if all labels, scale and ruled axis are correct.

Max 1 mark if labels or scale missing or if axis not drawn with a ruler.

iii) Soln $\{x: x < -4 \text{ and } x > 4\}$ 1 mark

b) i) $\sin\theta\cos\theta \cdot \operatorname{cosec}^2\theta = \cot\theta$

$LHS = \sin\theta\cos\theta \times \frac{1}{\sin^2\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$ 1 mark

ii) $\sin^2\theta\cos^2\theta\operatorname{cosec}^4\theta = \frac{1}{4}$ Now $\sin\theta\cos\theta\operatorname{cosec}^2\theta = \cot$

$\therefore \cot^2\theta = \frac{1}{4}$

$\cot\theta = \pm\frac{1}{2}$

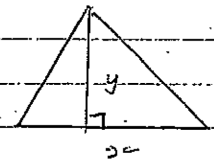
1 mark $\tan\theta = \pm 2$ for $0 \le x < 360°$

1 mark $\theta = 63°26', 116°34', 243°26', 296°34'$

$= 63°, 117°, 243°, 297°$

Question 3

c)



base $= x$ height $= y$

$A = \frac{1}{2}xy$ (1)

$x + y = 10$ (2)

$y = 10 - x$ (3)

Sub (3) into (1)

$A = \frac{1}{2}(x)(10 - x)$

$A = \frac{1}{2}(10x - x^2) = 5x - \frac{1}{2}x^2$ 1 mark

$\frac{dA}{dx} = \frac{1}{2}(10 - 2x)$

$\frac{dA}{dx} = 5 - x$

When $\frac{dA}{dx} = 0$ for S.P.

$5 - x = 0$

$x = 5$

Now $\frac{d^2A}{dx^2} = -1$

< 0 $\therefore$ $x = 5$ will produce a max area. 1 mark

$\therefore$ When $x = 5$ cm $y = 5$ cm.

Max Area $= \frac{1}{2} \times 5 \times 5$

$= \frac{25}{2}$ cm² 1 mark

Question 4

a) $y = x\sqrt{x^2+3}$

$y = x(x^2+3)^{\frac{1}{2}}$

$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

$u = x$

$\frac{du}{dx} = 1$

$v = (x^2+3)^{\frac{1}{2}}$

$\frac{dv}{dx} = x(x^2+3)^{-\frac{1}{2}}$

(1 mark) $= (x^2+3)^{\frac{1}{2}} \times 1 + x \cdot x(x^2+3)^{-\frac{1}{2}}$

$= (x^2+3)^{\frac{1}{2}} + \frac{x^2}{\sqrt{x^2+3}}$

or

$\frac{(x^2+3) + x^2}{(x^2+3)^{\frac{1}{2}}}$ ISE.

$= \frac{2x^2+3}{(x^2+3)^{\frac{1}{2}}}$

Now when $x=1$ $y=2$

(1 Mark for for correct ~~gradient~~ and point) $y'(1) = \frac{5}{2}$

Equation of tangent $y - y_1 = m(x - x_1)$

$y - 2 = \frac{5}{2}(x-1)$

$2y - 4 = 5x - 5$

(1 mark) $5x - 2y - 1 = 0$

Question 4

b i) $f(x) = \frac{x}{x-1}$

$f'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$u = x$

$\frac{du}{dx} = 1$

$v = x-1$

$\frac{dv}{dx} = 1$

$= \frac{(x-1)\times 1 - x \times 1}{(x-1)^2}$

$= \frac{-1}{(x-1)^2}$

(1 mark for $f'(x)$ and $f''(x)$ being correct.)

$f''(x) = 2(x-1)^{-3}$

$= \frac{2}{(x-1)^3}$

For S.P to occur $f'(x) = 0$

$\therefore \frac{-1}{(x-1)^2} \neq 0$ No S.P (1 mark)

and $f''(x) = \frac{2}{(x-1)^3}$

$f''(x) \neq 0$

$\therefore$ No point of inflexion (1 mark)

$\therefore$ There are no S.Ps or points of inflexion

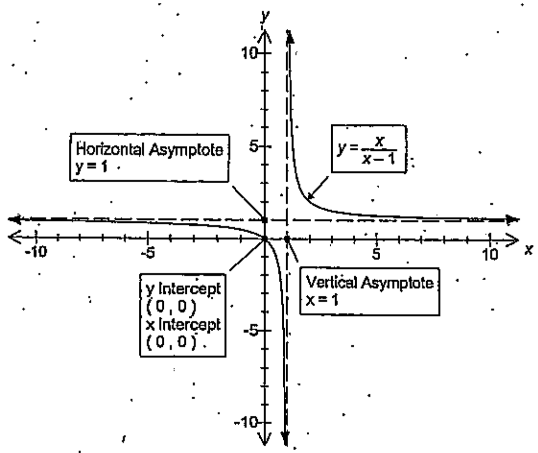
Question 4b

ii)

2 marks as per graph.

1 mark only if asymptotes, labels, and scale missing.

Must use a ruler.



c i) $\angle TPS = 30°$ (Angles at circumference standing on arc TS are equal).

1 mark for both reasons.

$\angle SPR = 25°$ (Angle between a tangent and a chord is equal to the angle in the alternate segment)

$\therefore \angle TPR = 30° + 25°$
$= 55°$ 1 mark

ii) $\angle TPQ = 90°$ (Angle in a semicircle is a right angle) 1 mark

$\angle QPR = 90° - 55°$
$= 35°$

1 mark $\angle QSR = 35°$ (Angles at circumference are equal standing on the same arc QR)

Question 5

a) Let $M = x + \frac{1}{x}$

$$M^2 - 5M + 6 = 0$$
$$(M-3)(M-2) = 0$$

$\therefore M = 3$ or $M = 2$ (1 mark)

$\therefore x + \frac{1}{x} = 3$ or $x + \frac{1}{x} = 2$

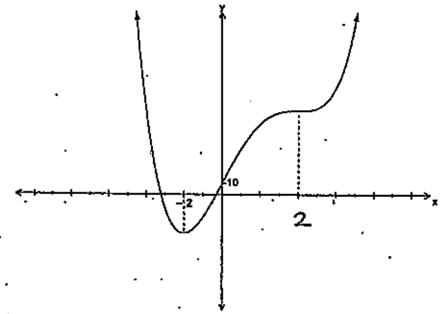
$x^2 + 1 = 3x$ or $x^2 + 1 = 2x$

$x^2 - 3x + 1 = 0$ $\quad x^2 - 2x + 1 = 0$ (1 mark)

$x = \frac{3 \pm \sqrt{9-4}}{2}$ or $(x-1)^2 = 0$

$\therefore x = \frac{3 \pm \sqrt{5}}{2}$ $\quad \therefore x = 1$ (1 mark for both solutions)

b)



2 marks for correct solution.
1 mark if 1 condition below is omitted.
(Maximum 1 mark if poorly labelled, scale missing or not ruled)

The graph passes through (0, 10)
The graph shows a turning point at $x = -2$, anywhere below $y = 10$
The graph shows a point of inflexion at $x = 4$ anywhere above $y > 10$.

Question 5

c) i) $P(x) = x^3 - 2x^2 - 9x - 6$

$P(-1) = (-1)^3 - 2(-1)^2 - 9(-1) - 6$

$= -1 - 2 + 9 - 6$

$= 0$ (1 mark)

ii) $\therefore x+1$ is a factor of $P(x)$

$$
\begin{array}{r|l}
 & x^2 - 3x - 6 \\
x+1 & x^3 - 2x^2 - 9x - 6 \\
 & \underline{x^3 + x^2} \\
 & -3x^2 - 9x \\
 & \underline{-3x^2 - 3x} \\
 & -6x - 6 \\
 & \underline{-6x - 6} \\
 & 0
\end{array}
$$

(1 for division)

$\therefore P(x) = (x+1)(x^2 - 3x - 6)$ (1 mark for solution)

iii) $x^2 - 3x - 6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{9 - 4 \times 1 \times -6}}{2}$

$x = \frac{3 \pm \sqrt{33}}{2}$ (1 for solution)

d) $\frac{\sin A}{1 + \cos A} = \operatorname{cosec} A(1 - \cos A)$ (Method 1)

$LHS = \frac{\sin A}{1 + \cos A}$

$= \frac{\sin A}{1 + \cos A} \times \frac{(1 - \cos A)}{(1 - \cos A)}$ (1 mark)

$= \frac{\sin A - \cos A \sin A}{1 - \cos^2 A}$

(13)

Question 5

d) con't'd

$= \frac{\sin A - \sin A \cos A}{\sin^2 A}$

$= \frac{\sin A(1 - \cos A)}{\sin^2 A}$ (1 mark)

$= \frac{1 - \cos A}{\sin A}$

$= \operatorname{cosec} A(1 - \cos A)$

$= RHS.$ (1 mark)

$\therefore LHS = RHS$ proven.

Question 6

a) i) $x^2 = 4ay$

$y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$\frac{dy}{dx} = \frac{x}{2a}$

Now When $x = 2ap$

$\frac{dy}{dx} = \frac{2ap}{2a}$

$\frac{dy}{dx} = p$

(1 mark for both differentiations)

Equation of tangent at P

$y - y_1 = m(x - x_1)$

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$ (1)

(1 mark for solution)

(14)

Question 6

ii) Equation of tangent at P

$y = px - ap^2$ (1)

Equation of tangent at Q

$y = qx - aq^2$ (2)

(1) = (2)

$px - ap^2 = qx - aq^2$ (1 mark)

$px - qx = ap^2 - aq^2$

$(p-q)x = a(p-q)(p+q)$

$x = a(p+q)$ Sub into (1)

$y = p(a(p+q)) - ap^2$

$= ap^2 + apq - ap^2$

$y = apq$.

(1 mark for solution)

$\therefore T$ is the point $(a(p+q), apq)$

iii) $M\left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$

$T(a(p+q), apq)$

$\therefore N\left(\frac{2a(p+q)}{2}, \frac{\frac{a(p^2+q^2)}{2} + apq}{2}\right)$

$\therefore x = a(p+q)$ $y = \frac{a(p^2+q^2)}{4} + \frac{apq}{2}$

(1 mark for x)

Question 6 (iii)

cont'd

$y = \frac{a(p^2+q^2) + 2apq}{4}$

$= \frac{a((p+q)^2 - 2pq) + 2apq}{4}$

$= \frac{a(p+q)^2 - 2apq + 2apq}{4}$

(1 mark for y)

$y = \frac{a(p+q)^2}{4}$ But $x = a(p+q)$

$\therefore p+q = \frac{x}{a}$

$\therefore y = \frac{a}{4} \times \left(\frac{x}{a}\right)^2$

$= \frac{a}{4} \times \frac{x^2}{a^2}$

$y = \frac{x^2}{4a}$

$x^2 = 4ay$ is the locus of N.

(1 mark for solution with correct working)

Question 6

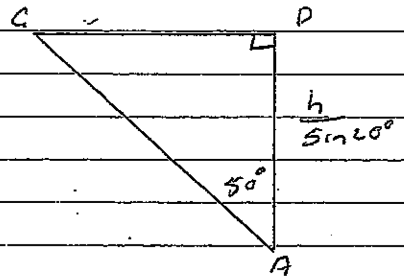
b) i) In $\triangle ADF$

$$\sin 20^\circ = \frac{DF}{AD}$$

$$\sin 20^\circ = \frac{h}{AD}$$

$$AD = \frac{h}{\sin 20^\circ} \qquad (1 \text{ mark})$$

ii) In $\triangle AGD$



$$\cos 50^\circ = \frac{\frac{h}{\sin 20^\circ}}{AG} \qquad (1 \text{ mark})$$

$$AG = \frac{\frac{h}{\sin 20^\circ}}{\cos 50^\circ}$$

$$AG = \frac{h}{\sin 20^\circ \cos 50^\circ} \qquad (1 \text{ mark})$$

Question 6

iii)
$$\sin\theta = \frac{h}{AG}$$

$$= \frac{h}{\frac{h}{\sin 20^\circ \cos 50^\circ}} \qquad (1 \text{ mark})$$

$$\sin\theta = \sin 20^\circ \cos 50^\circ$$

$$\sin\theta = 0.21989631\ldots$$

$$\theta = 12^\circ 42'$$

$$= 13^\circ \qquad (1 \text{ mark})$$