Question 1. (12 Marks) Use a **SEPARATE** Writing Booklet. **Marks**

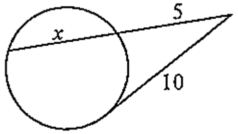
(a) Simplify $\frac{8x^3-27}{2x^2-x-3}$. **2**

(b) Solve $\tan 2\theta = \frac{1}{\sqrt{3}}$, $0^\circ \le \theta \le 360^\circ$. **2**

(c) Find the point P, which divides the interval joining $A(1,-2)$ and $B(-4,0)$ externally in the ratio 3:5. **2**

(d) State the locus of a point which moves so that it is always 2 units from the line $y=-1$. **1**

(e) Find x in the diagram below. **1**



(f) Find the values of A, B and C if $2x^2+8x-6 \equiv A(x+1)^2+Bx+C$. **2**

(g) If α and β are roots of the equation $2x^2+3x-4=0$, find the value of:

(i) $\alpha+\beta$ and $\alpha\beta$ **1**

(ii) $\alpha^2+\alpha\beta+\beta^2$ **1**

Question 2. (12 Marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Find the co-ordinates of the focus of the parabola with equation $y^2=-12x$. **1**

(b) Find the equation of the directrix of the parabola $y=x^2+3x$. **2**

(c) Find and describe the locus of a point which moves so that it is equidistant from the lines $y=5$ and $3x-4y+6=0$. **3**

(d) Solve $\frac{2x+3}{x-4} > 1$. **3**

(e) Prove that $\frac{\tan\theta+\cot\theta}{\sec\theta\,\mathrm{cosec}\,\theta}=1$. **3**

Question 3. (12 Marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Find the values of k for which the equation $x^2+(k+3)x+k(k+3)=0$ has 2 real and distinct roots. **3**

(b) (i) Sketch $y=|2x-5|$ showing x and y intercepts. **1**

(ii) Graphically or otherwise, solve $|2x-5|=x-1$. **2**

(iii) Find all values of x such that $|2x-5|< x-1$. **1**

(c) Graph the region $y \le \sqrt{4-x^2}$. **2**

(d) Solve $2^{2x+3}+6.2^x-5=0$. **3**

Question 4. (12 Marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) (i) Sketch the curves $y=(x-1)(x+2)$ and $y=(x-1)(x-3)$ and hence find their point of intersection. **1**

(ii) Find the acute angle between the tangents to each curve at this point. **3**

(b) (i) Prove that $x^2+x+1=0$ has no real roots. **2**

(ii) Hence show that the curve with equation $y=2x^3+3x^2+6x$ has no stationary points. **2**

(iii) For what values of x is this curve concave up? **2**

(iv) Sketch $y=2x^3+3x^2+6x$. **2**

Question 5. (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) The variable point $\left(\frac{t}{4}, 6t^2\right)$ lies on a parabola. **1**

Find the Cartesian equation for the parabola.

(b) The points P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) PQ is a focal chord. Show that $pq = -1$. **2**

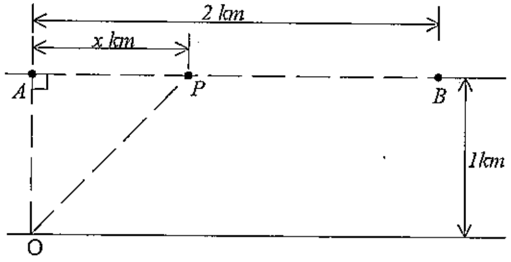
(ii) Show that the coordinates of the midpoint, M, of the chord PQ are $\left[a(p+q), \frac{a}{2}(p^2+q^2)\right]$. **1**

(iii) Find the equation of the locus of M and describe the locus of M geometrically. **3**

Question 5 continued on page 6.

Question 5. (continued)

(c)



The diagram shows a straight section of river, one kilometre wide. Alice is at a point O on one bank and she wishes to reach a point B on the opposite bank.
The point A is directly opposite O and the distance from A to B is two kilometres.

Alice can row at 6 km/h and jog at 10 km/h. She intends to row in a straight line to a point P on the opposite bank and then jog directly from P to B.

Let the distance AP be x kilometres.

(i) Show that the time T, in hours, that Alice takes to reach B is given by **1**

$$T = \frac{\sqrt{x^2+1}}{6} + \frac{2-x}{10}.$$

(ii) Show that if Alice wishes to minimise the time taken to complete the journey then she should row to a point P, $\frac{3}{4}$ kilometre from A. **4**

Question 6 is on the next page.

Question 6. (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) (i) Show that $\frac{2x^2}{x^2-1}=2+\frac{2}{(x-1)(x+1)}$. **1**

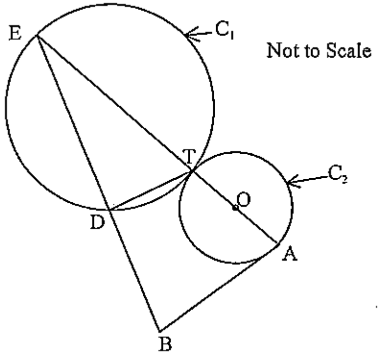
(ii) Show that $f(x)=\frac{2x^2}{x^2-1}$ is an even function. **1**

(iii) Find the equations of any vertical or horizontal asymptotes. Justify your answers. **2**

(iv) Sketch $y=f(x)$. **2**

(v) What is the range of the function? **1**

(b)



Two circles C_1 and C_2 touch at T. The line AE passes through O, the centre of C_2, and through T.
The point A lies on C_2 and E lies on C_1.
The line AB is a tangent to C_2 at A. D lies on C_1 and BE passes through D.
The radius of C_1 is R and the radius of C_2 is r.

(i) Prove that $\angle EDT = 90°$. **2**

(ii) If DE = 2r, find an expression for the length of EB in terms of r and R. **3**

End of Paper

Extension 1 Yearly 2011.

Question 1.

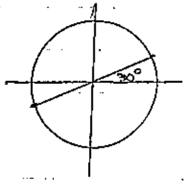
a) $\dfrac{8x^3-27}{2x^2-x-3} = \dfrac{(2x-3)(4x^2+6x+9)}{(2x-3)(x+1)}$ ← (1)

$= \dfrac{4x^2+6x+9}{x+1}$ (1)

b) $\tan 2\theta = 1/\sqrt{3}$ $\quad 0° \le \theta \le 360°$

$2\theta = 30°, 210°, 390°, 570°$ $\quad 0° \le 2\theta \le 720°$

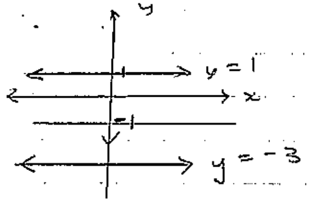
$\underline{\theta = 15°, 105°, 195°, 285°}$



c) $\left(\dfrac{5\times1 + (-3)\times(-4)}{-3+5} , \dfrac{5\times -2 + -3\times0}{-3+5} \right)$ (1)

$\left(\dfrac{17}{2} , -5 \right)$ (1)

d) $\underline{y = 1 \text{ or } y = -3}$ R/W. (1)



e) $(x+5)\times5 = 10^2$

$5x + 25 = 100$

$5x = 75$

$\underline{x = 1.5}$ R/W (1)

f) $2x^2 + 8x - 6 \equiv Ax^2 + 2Ax + A + Bx + C$

$\therefore A = 2$ //

$2A + B = 8$ (1) 2 correct answers.

$B = 8 - 2\times2 = 4$ // (1) 3rd correct answer

$A + C = -6$

$C = -6 - 2 = -8$ //

g) i) $\alpha + \beta = -3/2$ // (1)

$\alpha\beta = -4/2 = -2$ //

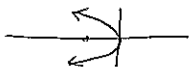
ii) $\alpha^2 + \alpha\beta + \beta^2$ R/W

$= (\alpha+\beta)^2 - \alpha\beta$ (1)

$= (-3/2)^2 - -2$

Question 2.

a) $y^2 = -12x$



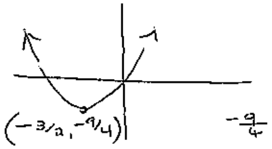
$4a = 12$

$\therefore a = 3$ Focus is $(-3, 0)$ // (1) R/W

b) $y = x^2 + 3x$

$= (x + 3/2)^2 - 9/4$

$\therefore (x + 3/2)^2 = y + 9/4$ (1)



$-\frac{9}{4} - 1/4 = -10/4$

$4a = 1$

$a = 1/4$ $\therefore$ Directrix is $y = -5/2$ // (1)

c) Let the point be $P(x, y)$

then

$\dfrac{|3x - 4y + 6|}{\sqrt{3^2+4^2}} = \dfrac{|y-5|}{\sqrt{1^2}}$ (1)

$|3x - 4y + 6| = 5|y - 5|$

$\therefore$ Either $3x - 4y + 6 = 5y - 25$ or $3x - 4y + 6 = 25 - 5y$

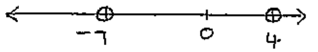
(1) Lines $\underline{3x - 9y + 31 = 0}$ or $\underline{3x + y - 19 = 0}$ (1)

d) $\dfrac{2x+3}{x-4} > 1$ $\quad$ cp : $\underline{x = 4}$

Consider $\dfrac{2x+3}{x-4} = 1$

$2x + 3 = x - 4$ (1)

cp : $\underline{x = -7}$



Test $x = -8$ $\quad \dfrac{2\times-8+3}{-8-4} = \dfrac{-13}{-12} > 1 \therefore$ ✓ (1)

$x = 0$ $\quad \dfrac{2\times0+3}{0-4} = -3/4 < 1 \therefore$ ×

$x = 5$ $\quad \dfrac{2\times5+3}{5-4} = 13 > 1 \therefore$ ✓

(1) $\therefore \underline{x < -7 \text{ or } x > 4}$

e) LHS $= \dfrac{\tan\theta + \cot\theta}{\sec\theta \operatorname{cosec}\theta}$

$= \dfrac{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\cos\theta} \times \dfrac{1}{\sin\theta}}$ (1) $\dfrac{\times \cos\theta\sin\theta}{\times \cos\theta\sin\theta}$ (1)

$= \dfrac{\sin^2\theta + \cos^2\theta}{1}$ (1)

$= 1$

$=$ RHS

Question 3

a) $x^2 + (k+3)x + k(k+3) = 0$

For real and distinct roots $\Delta > 0$

$\therefore (k+3)^2 - 4.1.k(k+3) > 0$

① calculating Δ correctly and stating $\Delta > 0$

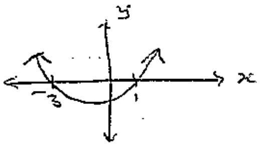
$k^2 + 6k + 9 - 4k^2 - 12k > 0$

$3k^2 + 6k - 9 < 0$

$k^2 + 2k - 3 < 0$

① Setting up a correct inequality

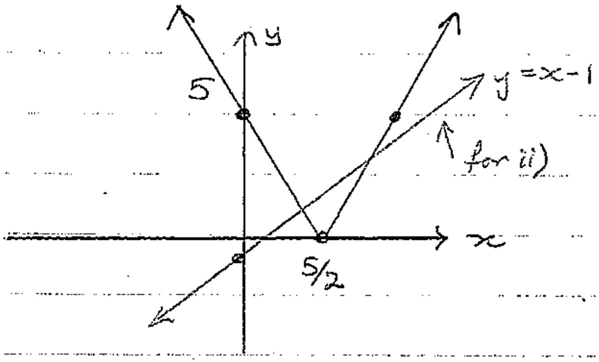
$(k+3)(k-1) < 0$ (1)



$\underline{-3} < k < 1$ (1)

① Sol^n

b) i) $y = |2x-5|$



(1) r/w

ii) $2x - 5 = x - 1$ or $-(2x-5) = x-1$

$x = 4$ // (1)

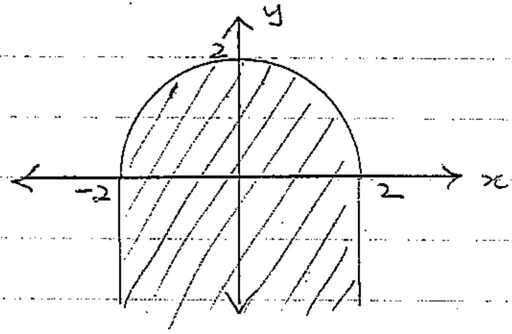
$2x - 5 = 1 - x$

$3x = 6$

$x = 2$ // (1)

iii) $2 < x < 4$ (1) r/w

c)



(1) correct shape and intercep

(1) correct shading and boundary lines

d) $2^{2x+3} + 6.2^x - 5 = 0$

$2^3.2^{2x} + 6.2^x - 5 = 0$

Let $a = 2^x$

$8a^2 + 6a - 5 = 0$

$(4a+5)(2a-1) = 0$

$a = {}^{-}5/4$ or $a = 1/2$

$\therefore 2^x = {}^{-}5/4$ or $2^x = 1/2$

no solution $x = -1$

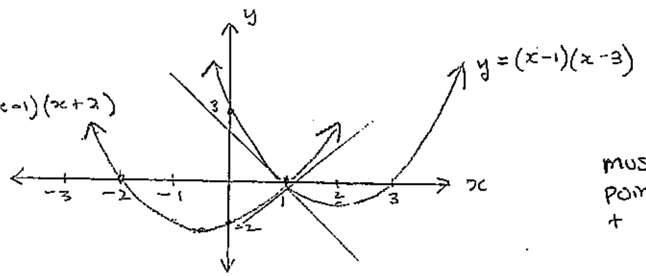
① For creation of quadratic

① For sol^n and resubstd

① disqualification of 1 sol^n

Question 4.

a) i)



must state point of intersection + include sketch (1)

Point of intersection (1,0)

ii) $y = (x-1)(x+2)$

$y = x^2 + x - 2$

$dy/dx = 2x + 1$

At (1,0) $dy/dx = 2\times1 + 1 = 3$

$y = (x-1)(x-3)$

$y = x^2 - x - 3x + 3$

$y = x^2 - 4x + 3$

$dy/dx = 2x - 4$

At (1,0) $dy/dx = 2\times1 - 4 = -2$

Acute Angle between tangents, α, is :

m - all correct

m - correct derivs + grad + incorrect use of formula or incorrect derivs with corresp correct grads + correct use of formula or correct derivs, incorrect grads + correct use of formula.

$$\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad \text{where } m_1 = 3 \text{ and } m_2 = -2$$

$$= \left|\frac{3 - -2}{1 + 3 \times -2}\right|$$

$$= \left|\frac{5}{-5}\right|$$

$$= 1$$

$$\therefore \alpha = 45^\circ \; //$$

m - correct derivs. or correct use of formula (derivs + grad incorrect)

b) i) $x^2 + x + 1 = 0$

$\Delta = 1^2 - 4.1.1 = -3$ (1)

Since $\Delta < 0$, $x^2 + x + 1 = 0$ has no real roots . (1)

ii) For stationary points $dy/dx = 0$ (1)

$dy/dx = 6x^2 + 6x + 6$

$= 6(x^2 + x + 1)$ (1)

Since $x^2 + x + 1$ has no real roots, there are no stat pts

Question 4 continued.

iii) For concave up $d^2y/dx^2 > 0$

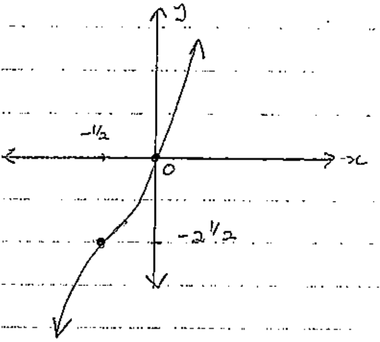
$d^2y/dx^2 = 12x + 6$

$12x + 6 > 0$ (1)

$x > -6/12$

$x > -1/2 \; //$ (1)

iv)



When $x = -1/2$,

$y = 2(-1/2)^3 + 3(-\frac{1}{2})^2 + 6($

$= -1/4 + 3/4 - 3$

$= -2\frac{1}{2}$

1 - Correctly placed cubic passing through origin

1 - labelling point of inflexion.

Question 5

a) $(t/4, 6t^2)$ $x = t/4$ $y = 6t^2$

$\therefore t = 4x$

$y = 6(4x)^2$

$= 96x^2$ // (1)

b) $x^2 = 4ay$

Gradient of PQ is $\frac{aq^2 - ap^2}{2aq - 2ap} =$

$= \frac{\cancel{a}(q-\cancel{p})(q+p)}{2\cancel{a}(q-\cancel{p})}$

$= \frac{q+p}{2}$

$\therefore$ Equation of PQ is

$y - ap^2 = \frac{q+p}{2}(x - 2ap)$ ← (1)

Since PQ is a focal chord, (0,a) lies on this line

$a - ap^2 = \frac{q+p}{2}(0 - 2ap)$

$2a - 2ap^2 = -2apq - 2a\cancel{p}^2$

$2a = -2apq$

$1 = -pq$

$pq = -1$ // (1)

ii) Midpoint of PQ is $\left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$ (1)

$= \left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$

iii) M : $x = a(p+q)$ $y = \frac{a}{2}(p^2+q^2)$

$\Rightarrow p+q = \frac{x}{a}$ $= \frac{a}{2}[(p+q)^2 - 2pq]$ (1)

But $p+q = x/a$ and $pq = -1$

$\therefore y = \frac{a}{2}\left[\left(\frac{x}{a}\right)^2 - 2(-1)\right]$

5biii) continued

$y - a = x^2/2a$

$x^2 = 2a(y-a)$

Hence the locus of M is a parabola with Vertex (0,a) and focal length $\frac{a}{2}$ } (1)

5c) i) $OP = \sqrt{x^2+1}$ By pythagoras

$\therefore$ Rowing time $= \frac{\sqrt{x^2+1}}{6}$ $s = D/T$, $T = D/S$

Jogging time $= \frac{2-x}{10}$ (1)

$\therefore$ Time taken to reach B is $T = \frac{\sqrt{x^2+1}}{6} + \frac{2-x}{10}$

ii) To minimise journey, let $dT/dx = 0$ and $d^2T/dx^2 > 0$ (or change in sign of dT/dx from − to +)

$\frac{dT}{dx} = \frac{1}{6} \cdot \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x - \frac{1}{10}$ (1)

$= \frac{x}{6\sqrt{x^2+1}} - \frac{1}{10}$

$= 0$ when $\frac{x}{6\sqrt{x^2+1}} = \frac{1}{10}$

$10x = 6\sqrt{x^2+1}$

$100x^2 = 36(x^2+1)$

$64x^2 = 36$

$x^2 = 36/64$ (1) $x > 0$ (1) It's a distance

$x = 6/8 = 3/4$

Check for minimum time (1)

x	$1/2$	$3/4$	1
dT/dx	$\frac{\frac{1}{2}}{6\sqrt{5/4}} - \frac{1}{10}$	0	$\frac{1}{6\sqrt{2}} - \frac{1}{10}$

$\therefore$ dT/dx changes in sign from negative to positive either side

Question 6

a) i) $$\text{RHS} = 2 + \frac{2}{(x-1)(x+1)}$$
$$= \frac{2(x-1)(x+1)+2}{(x-1)(x+1)}$$
$$= \frac{2(x^2-1)+2}{(x-1)(x+1)}$$
$$= \frac{2x^2-2+2}{(x-1)(x+1)}$$
$$= \frac{2x^2}{(x-1)(x+1)}$$
$$= \text{LHS}$$
$$\therefore \frac{2x^2}{x^2-1} = 2 + \frac{2}{(x-1)(x+1)}$$ ① r/w

OR

$$\begin{array}{r} 2 \\ x^2-1\,\overline{)\,2x^2\quad} \\ -(2x^2-2) \\ \hline 2 \end{array}$$

$$\therefore \frac{2x^2}{x^2-1} = 2 + \frac{2}{x^2-1}$$
$$= 2 + \frac{2}{(x-1)(x+1)}$$

ii) $$f(x) = \frac{2x^2}{x^2-1}$$
$$f(-x) = \frac{2(-x)^2}{(-x)^2-1} = \frac{2x^2}{x^2-1}$$ ① r/w

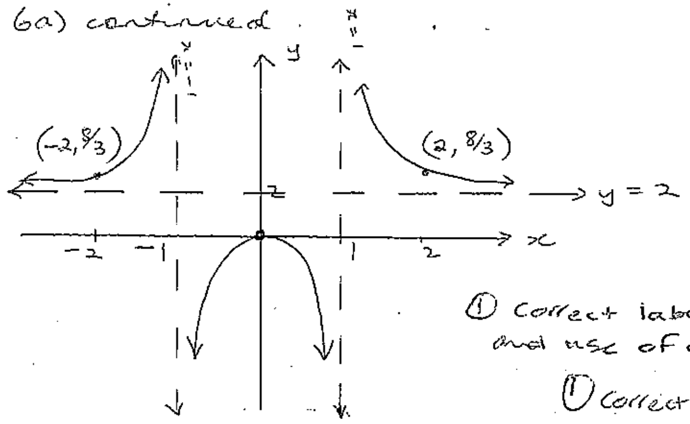
Since $f(-x) = f(x)$, $f(x)$ is an even function

iii) Vertical Asymptotes : $x = \pm 1$ since $x^2-1 \neq 0$

Horizontal Asymptote $y = 2$ since
$$\frac{2}{(x-1)(x+1)} \rightarrow 0 \text{ as } x \rightarrow \pm\infty \text{ so } 2 + \frac{2}{(x-1)(x+1)} \rightarrow 2$$

Question 6a) continued

iv)



① Correct labelling and use of asymptotes

① Correct graph

Correctly positioned

v) Range : $y \leq 0$; $y > 2$ ① r/w

6b) i) When 2 circles touch, the line of centres passes through the point of contact.) ①

$\therefore$ ET is a diameter of C_1

$\therefore \angle EDT = 90°$ Angles on a semi circle is a right ∠ ①

ii) In $\triangle DET$ and $\triangle AEB$

If there is a justification of why $\triangle DET$ is Right angled

$\angle E$ is common

① Similarity Proof $\angle EDT = 90°$ from (i)

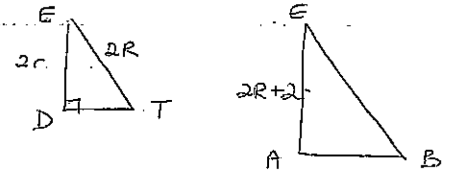
$\angle EAB = 90°$ (angle between a tangent and radius at point of contact is 90°)

① Corresponding Sides Comparison

$\therefore \angle EDT =$ ~~∠EAB~~

① Expression $\therefore \triangle DET \;|||\; \triangle AEB$ (equiangular)

Here :



(Note $ED = 2r$ is given)

$$\therefore \frac{EB}{ET} = \frac{EA}{ED}$$ (ratio of corresponding sides in similar △'s)

$EB = 2(R+r)$