Section I

10 Marks

Attempt all Questions

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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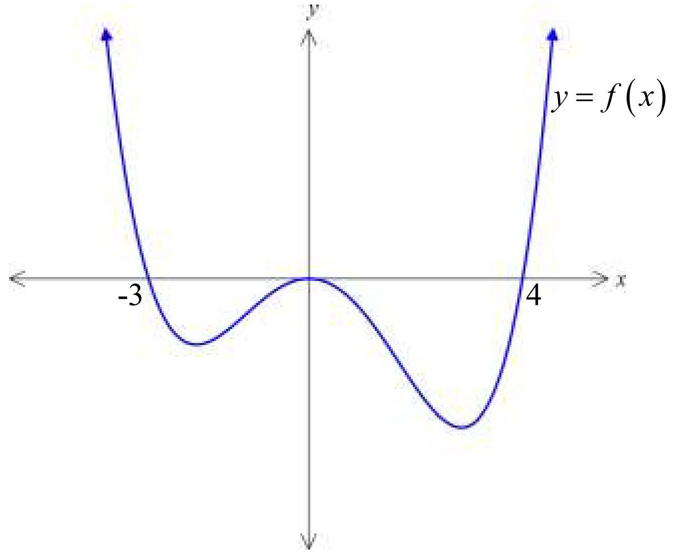
1. The graph of $y = f(x)$ is shown in the diagram.



The values of x for which $f(x) \geq 0$ are:

(A) $x < -3$ and $x > 4$

(B) $x \leq -3$ and $x \geq 4$

(C) $x \leq -3$, $x = 0$ and $x \geq 4$

(D) $-3 \leq x \leq 4$

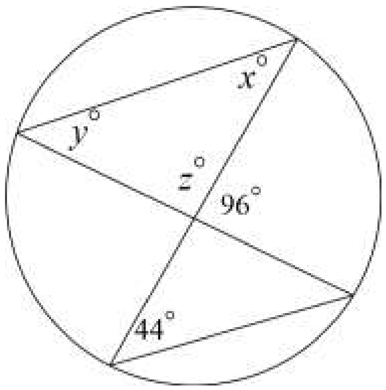
2 Chords are drawn in a circle. The value of x is:

(A) 44

(B) 48

(C) 52

(D) 84



3 The end points of the diameter of a circle are (2, 3) and (– 4, 3). The equation of the circle is:

(A) $(x-1)^2+(y+3)^2=9$

(B) $(x-1)^2+(y+3)^2=3$

(C) $(x+1)^2+(y-3)^2=9$

(D) $(x+1)^2+(y-3)^2=3$

4 The values of x where $\dfrac{dy}{dx}\geq 0$ for the graph of $y=\dfrac{1}{2}(x-4)(1-x)$ are :

(A) $x\leq 2\dfrac{1}{2}$

(B) $1\leq x\leq 4$

(C) $x<2\dfrac{1}{2}$

(D) $x\geq 2\dfrac{1}{2}$

5 Find the acute angle between the lines $y = 2x - 3$ and $3x + 5y - 1 = 0$.

(A) 32°

(B) 50°

(C) 82°

(D) 86°

6 The interval AB, where A is (1,2) and B is (7,5), is divided internally in the ratio 2:1 by the point P(x,y). What are the coordinates of point P?

(A) $(3,3)$

(B) $(3,4)$

(C) $(5,3)$

(D) $(5,4)$

7 Let α and β be roots of the equation $x^2 - 8x + 5 = 0$. What is the value of $\alpha^2+\beta^2$?

(A) 44

(B) 54

(C) 64

(D) 74

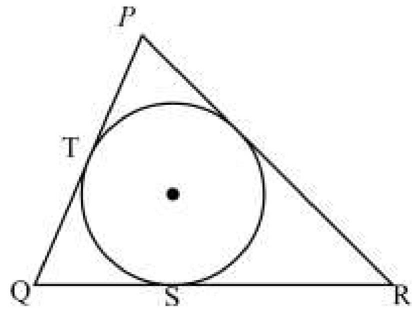
8 A circle is inscribed inside the triangle PQR.
SR = 7cm, QS = TP = 4 cm.
The perimeter of the triangle PQR, in centimetres, is:

(A) 30

(B) 50

(C) 15π

(D) 11π



9 A parabola has the parametric equations $x = \frac{2}{t}$ and $y = 2t^2$.
What is the Cartesian equation of this curve?

(A) $y = \frac{4}{x}$

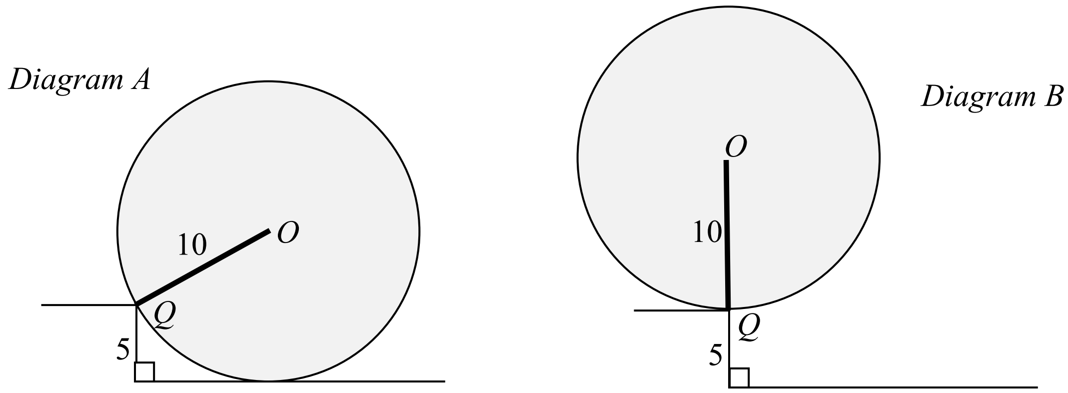
(B) $y = \frac{8}{x}$

(C) $y = \frac{4}{x^2}$

(D) $y = \frac{8}{x^2}$

10 A wheel of radius 10 cm rests against a step 5 cm high as shown in diagram A

Not to Scale



The wheel is rotated about Q by pushing it until its centre O is directly above Q as shown in diagram B. The spoke OQ has now been rotated through an angle equal to:

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Section II

60 Marks

Attempt Questions 11-15

Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 11 (12 Marks). Use a **SEPARATE** Writing Booklet. **Marks**

(a) Solve $4^x - 9\left(2^x\right) + 8 = 0$. **3**

(b) Solve $\dfrac{5}{x+1} \le 1$. **3**

(c) Express $2x^2 - 7x - 4$ in the form $a(x+2)^2 + b(x+2) + c$. **3**

(d) The sum of the second and fifth terms of an arithmetic progression is 32, whilst the sum of the third and eighth term is 48. **3**
Find the first term and the common difference.

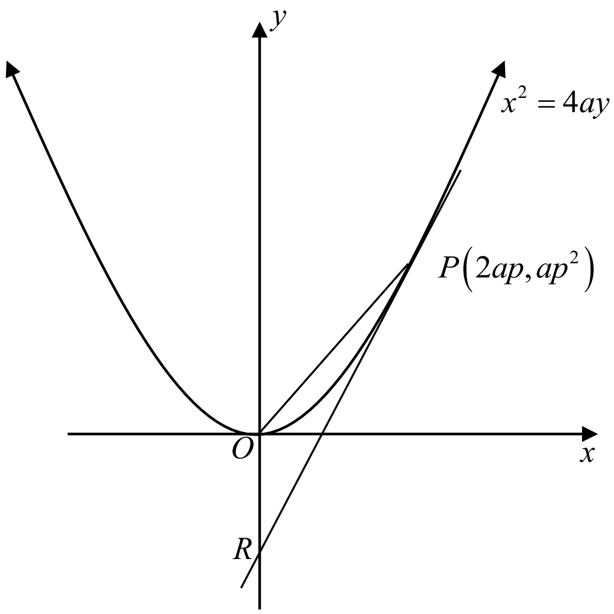
Question 12 (12 Marks). Use a **SEPARATE** Writing Booklet. **Marks**

(a) Given that one root of the equation $x^2 + mx + n = 0$ is twice the other root, prove that $2m^2 = 9n$. **3**

(b) The equation of a parabola is given by $(y-2)^2 = 8(x+1)$. Find the coordinates of the vertex and focus. **2**

(c) Prove that $cosec\,\theta\sqrt{1-sin^2\theta} = cot\,\theta$. **2**

(d)



(i) Derive the equation of the tangent to the point $P\left(2ap,\, ap^2\right)$ on the parabola $x^2 = 4ay$. **2**

(ii) R is the point where the tangent crosses the y-axis. Show that the coordinates of R are $(0, -ap^2)$. **1**

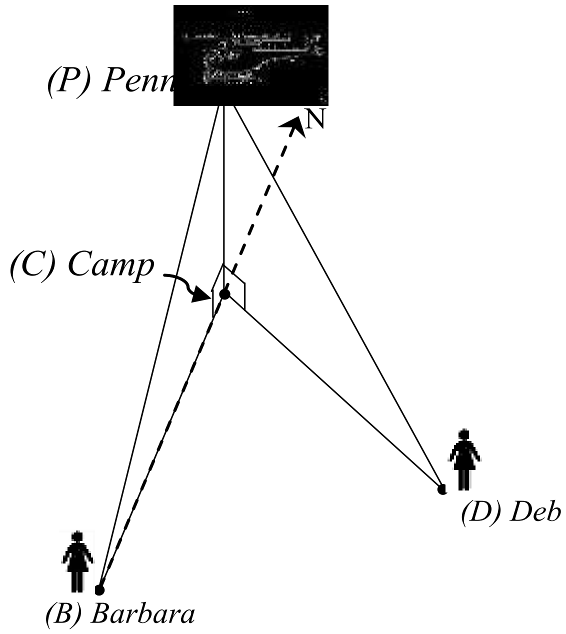
(iii) Hence show that the area of ΔOPR is a^2p^3 square units. **2**

Question 13 (12 Marks). Use a **SEPARATE** Writing Booklet. **Marks**

(a) Sketch the region $xy > 4$ on a number plane. **2**

(b) Solve $2\sin^2\theta - \cos\theta - 1 = 0$ for θ given $0° \le \theta \le 360°$. **3**

(c) Deb, Penny and Barbara were attending a maths camp. Deb and Barbara have a fight and have stormed off from camp in different directions. Penny hops into a helicopter, which then hovers 500 m directly above camp, from where she sights Deb on a bearing of 125° from camp, at an angle of depression of 23°. Meanwhile, Barbara has walked due south of the camp until the angle of elevation to Penny is 16°.



(i) Draw a neat diagram in your answer booklet, showing all the information from the question. **1**

(ii) Show that $\angle BCD$ is 55°. **1**

(iii) Find the distance between Barbara and Deb to nearest metre. **3**

(iv) Calculate the bearing of Deb from Barbara's position. **2**

Question 14 (12 Marks). Use a **SEPARATE** Writing Booklet. **Marks**

(a) Differentiate $\dfrac{x}{x^2+1}$ with respect to x, expressing your answer in simplest form. **2**

(b) Show that the point (2,0) is a horizontal point of inflexion on $y=(x-2)^3$. **3**

(c) Consider the function $f(x)=\dfrac{x+4}{x+1}$.

(i) Show that $f(x)=1+\dfrac{3}{x+1}$. **1**

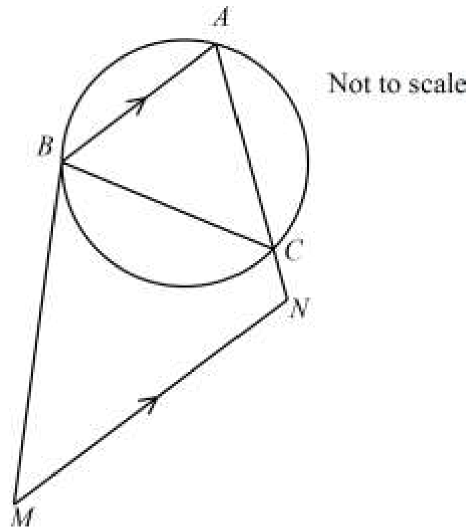
(ii) Find the coordinates of the x and y intercepts of the graph $y=f(x)$. **2**

(iii) Find the equations of all vertical and/or horizontal asymptotes, if they exist. **2**

(iv) Sketch the graph $y=f(x)$. Showing the features from parts (ii) and (iii) above. **2**

Question 15 (12 Marks). Use a **SEPARATE** Writing Booklet. **Marks**

(a) ABC is a triangle inscribed in a circle. The point M lies on the tangent to the circle at B and N is a point on AC produced so that MN is parallel to BA.



(i) State why $\angle MBC = \angle BAC$. **1**

(ii) Prove that $MNCB$ is a cyclic quadrilateral. **2**

(b) M is the midpoint of the chord PQ, where P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ and PQ passes through (0, 3a).

The equation of chord PQ is $y = \frac{1}{2}(p+q)x - apq$. **(Do NOT prove this)**

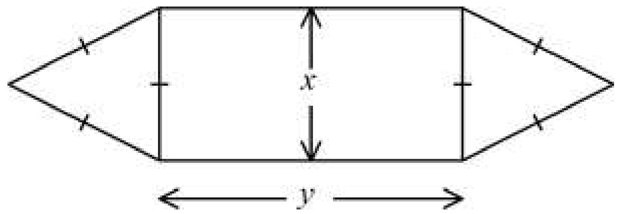
(i) Use the fact that PQ passes through $(0, 3a)$ to show that $pq = -3$. **1**

(ii) Determine the coordinates of M, use the midpoint of the chord PQ. **1**

(iii) Show that the Cartesian equation of the locus of M is $x^2 = 2a(y - 3a)$. **2**

Question 15 continues on page 9.

(c)



The figure shown above consists of a rectangle, x cm by y cm with two equilateral triangles on each side of length x cm. The perimeter of the figure is 26 cm.

(i) Show that $A = 13x - \dfrac{\left(4-\sqrt{3}\right)x^2}{2}$ where A is the area of the figure. **2**

(ii) Find the exact value of x that makes this area a maximum. **3**

End of Paper.

SOLUTIONS

1. C	5. D	9. D
2. C	6. D	10. C
3. C	7. B	
4. A	8. A	

Question 11

(a) $4^x - 9(2^x) + 8 = 0$

$(2^x)^2 - 9(2^x) + 8 = 0$ (1)

$u^2 - 9u + 8 = 0$

$(u-8)(u-1) = 0$

(3 marks) $u = 8, 1$ (1)

$\therefore 2^x = 8, \quad 2^x = 1$

$x = 3 \quad x = 0$ (1)

$\therefore x = 3, 0$

(b) $\dfrac{5}{x+1} \leq 1 \qquad x \neq -1$

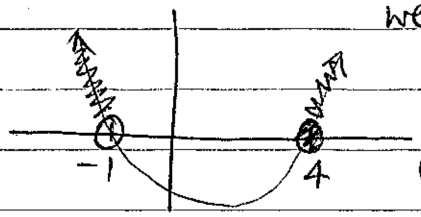
$5(x+1) \leq 1(x+1)^2$

$0 \leq (x+1)^2 - 5(x+1)$

$0 \leq (x+1)(x+1-5)$

$0 \leq (x+1)(x-4)$

(3 marks)



we want curve ABOVE axis (1)

$\therefore x < -1$ (1), $x \geq 4$ (1)

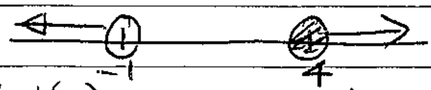
Note:
* one soln correct only (1)
must check soln.

$x \neq -1$

$\dfrac{5}{x+1} = 1$

$5 = x+1$

$4 = x$



test (-2) $\quad$ test (5)

$\dfrac{5}{-2+1} = -5 \qquad \dfrac{5}{5+1} = \dfrac{5}{6}$

$-5 \leq 1$ ✓ $\qquad \dfrac{5}{6} \leq 1$ ✓

$\therefore x < -1, \; x \geq 4$

c) $2x^2-7x-4 \equiv a(x+2)^2+b(x+2)+c$

$= ax^2+4xa+4a+bx+2b+c$

$= (ax^2)+(4ax+bx)$ 1 $+(4a+2b+c)$

$a=2$, $4a+b=-7$, $4a+2b+c=-4$

(3 marks)

$8+b=-7$ $8+-30+c=-4$

$b=-15$ 1 -22 $c=18$

$\therefore 2x^2-7x-4 \equiv 2(x+2)^2-15(x+2)+18$

1

d) $T_2 = a+(2-1)d$ $T_3 = a+2d$

$= a+d$

$T_5 = a+(5-1)d$ $T_8 = a+7d$

$= a+4d$

(3 marks)

1 equation

$S_{(2+5)} = 2a+5d = 32$ $S_{(3+8)} = 2a+9d = 48$

$2a+9d=48 \quad \ldots (1)$

$2a+5d=32 \quad \ldots (2)$

(1)−(2) $4d=16$

$d=4$ 1

sub $d=4$ in $T_2 = a+4$

$a=6$ 1

QUESTION 12:

a) let $\beta = 2\alpha$ so roots are $\alpha, 2\alpha$ $\qquad x^2 + mx + n = 0$

$\alpha + \beta = \alpha + 2\alpha$ $\qquad \alpha\beta = \alpha \times 2\alpha$

$= 3\alpha$ $\qquad = 2\alpha^2$

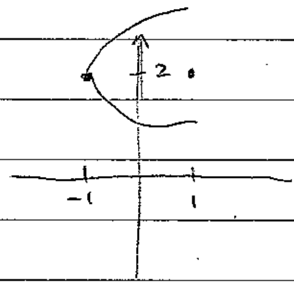
$\alpha + \beta = -\frac{m}{1}$ 1 $\qquad \alpha\beta = n$ 1

$\therefore 3\alpha = -m$ $\qquad \therefore 2\alpha^2 = n$

(3 marks) $\alpha = -\frac{m}{3}$ $\qquad 2 \times \left(-\frac{m}{3}\right)^2 = n$

$\frac{2m^2}{9} = n$

$\therefore \underline{2m^2 = 9n}$ 1



b) $(y-2)^2 = 8(x+1)$

vertex $= (-1, 2)$ 1 mark

$a = 2$

focus $= (1, 2)$ 1 mark

c) $\operatorname{cosec}\theta\sqrt{1-\sin^2\theta} = \cot\theta$

LHS $= \operatorname{cosec}\theta \times \sqrt{\cos^2\theta}$ 1

$= \frac{1}{\sin\theta} \times \cos\theta$

(2 marks) $= \frac{\cos\theta}{\sin\theta}$ 1

$= \cot\theta$

$=$ RHS

(d) (i) $x^2 = 4ay$

$y = \frac{x^2}{4a}$

$y' = \frac{2x}{4a}$

$= \frac{x}{2a}$

when $x = 2ap$ $\quad y' = \frac{2ap}{2a}$

$= p$ (1)

(2 marks)

$\therefore \quad y - y_1 = m(x - x_1)$

$y - ap^2 = p(x - 2ap)$

$y = xp - 2ap^2 + ap^2$

$y = xp - ap^2$ (1) in any form

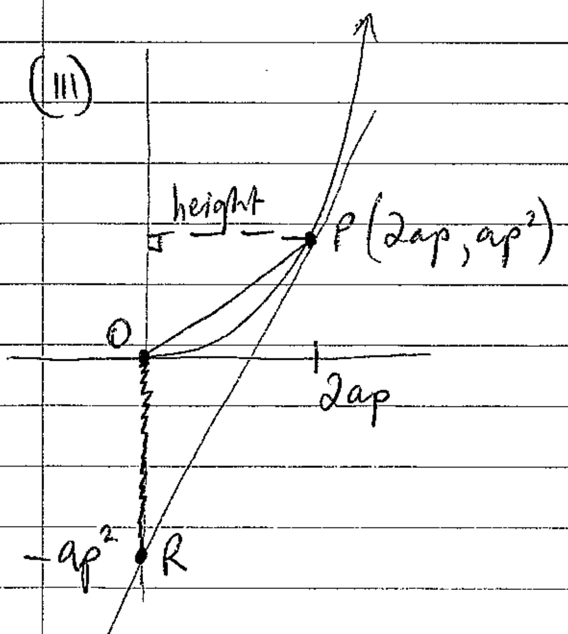
(ii) R crosses y axis when $x=0$

(1 mark)

$x = 0 \quad y = 0p - ap^2$

$= -ap^2 \qquad \therefore R(0, -ap^2)$

(iii)



Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times ap^2 \times 2ap$

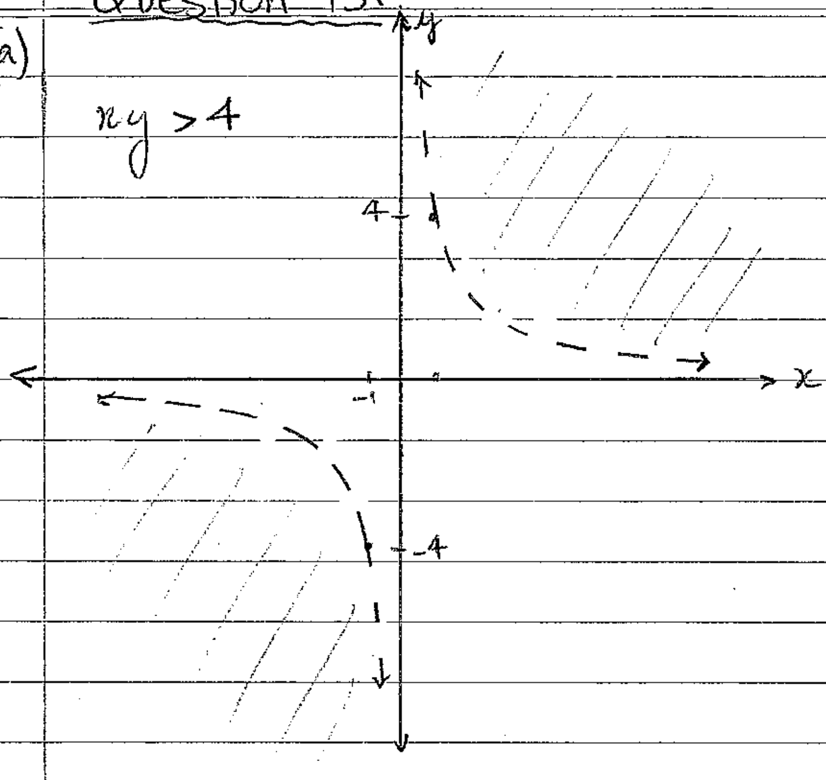
$= \frac{1}{\cancel{2}} \, a^2p^3$

(2 marks)

Question 13:

(a)

$xy > 4$



Do not penalise for no x y

(2 marks)

-1 for wrong shading

-1 for not dotted

-1 for not labelling pt on each arm.

0 – wrong shape.

b) $2\sin^2\theta - \cos\theta - 1 = 0 \qquad 0 \le \theta \le 360^\circ$

$2(1-\cos^2\theta) - \cos\theta - 1 = 0$

$2 - 2\cos^2\theta - \cos\theta - 1 = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$ (1)

(3 marks)

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta = 1 \qquad \cos\theta = -1$

$\cos\theta = \frac{1}{2}$

S	A ✓ θ
T	C ✓ $360-\theta$

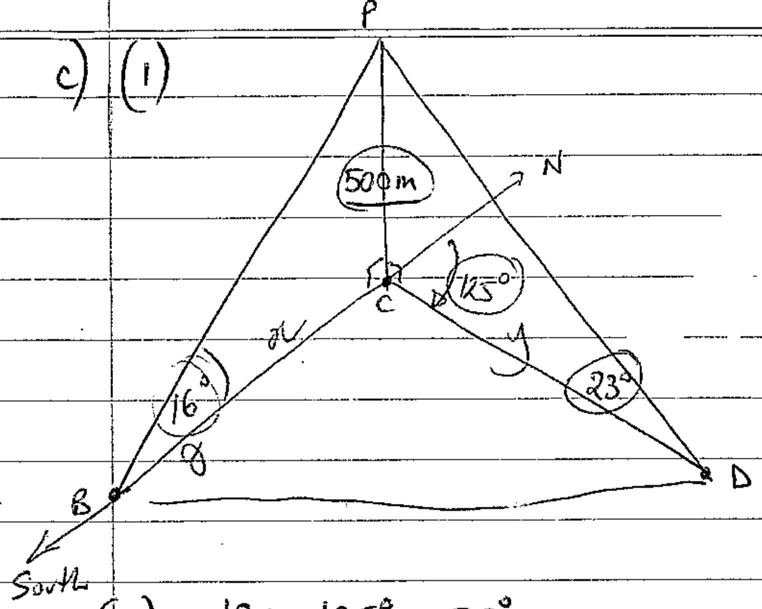
$\theta = 60^\circ, 360-60^\circ \qquad \therefore \theta = 180^\circ$ (1)

$= 60^\circ, 300^\circ$

(1)

$\therefore \theta = 60^\circ, 180^\circ, 300^\circ$

c) (i)

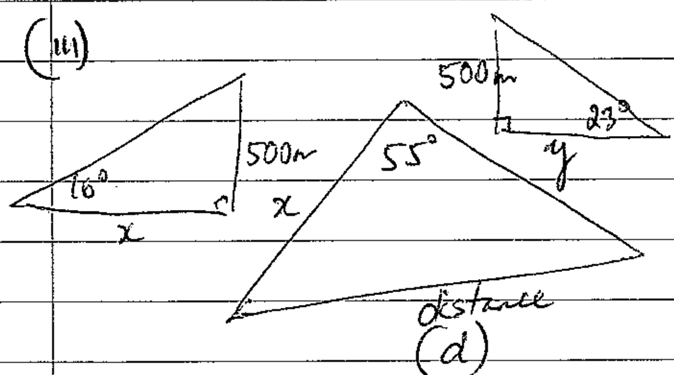


(1 mark)

all 4 things needed

(ii) $180 - 125^\circ = 55^\circ$ (1 mark)

(iii)



$\tan 16^\circ = \dfrac{500}{x}$ (1)

$x = \dfrac{500}{\tan 16^\circ}$

$\tan 23^\circ = \dfrac{500}{x}$

$y = \dfrac{500}{\tan 23^\circ}$

(3 marks)

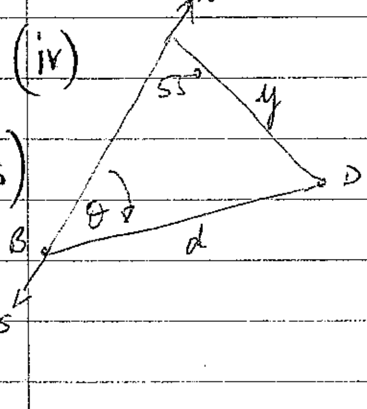
$d^2 = x^2 + y^2 - 2xy\cos 55^\circ$ (1)

$= \dfrac{500^2}{(\tan 16^\circ)^2} + \dfrac{500^2}{(\tan 23^\circ)^2} - 2 \times \dfrac{500}{\tan 16} \times \dfrac{500}{\tan 23} \times \cos 55^\circ$

$= 2071820.698$

$d = 1439.382054$ ∴ distance is (1)

(iv)



(2 marks)

$\dfrac{\sin\theta}{y} = \dfrac{\sin 55^\circ}{d}$ (1)

$\sin\theta = \dfrac{\sin 55}{d} \times \left(\dfrac{500}{\tan 23^\circ}\right)$

$\theta = 42^\circ 5' 40.8''$

∴ bearing is 042° (1)
or $N42^\circ E$

QUESTION 14:

a)

$$\frac{d}{dx}\left(\frac{x}{x^2+1}\right) = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2}$$

$u = x \quad v = x^2+1$

$u' = 1 \quad v' = 2x$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2} \quad 1$$

(2 marks)

$$= \frac{1-x^2}{(x^2+1)^2} \quad 1$$

b)

$y = (x-2)^3$

$y' = 3(x-2)^2$

stationary pt when $y' = 0$

$y' = 0 = 3(x-2)^2 \quad 1$

$\therefore x = 2$

(3 marks)

x	1	2	3
y'	$3\times(-1)^2$	0	$3\times1^2=3$
	/	—	/
y''	$6\times-1=-6$	0	$6\times1=6$
	↓		up

1

$\therefore$ point not a turning pt as change of concavity

$y'' = 6(x-2)$

when $x=2$ $\quad y'' = 6(2-2)$

$1 \quad = 0$

$\therefore$ horizontal pt inflexion

(c) (i) $f(x) = 1 + \frac{3}{x+1}$

$= \frac{x+1}{x+1} + \frac{3}{x+1}$ 1 mark here

(1 mark)

$= \frac{x+1+3}{x+1}$

$= \frac{x+4}{x+1}$

(ii) $x=0 \quad y = \frac{0+4}{0+1}$

$= 4$

(2 marks)

$\therefore (0,4)$ 1 mark

$y=0 \quad 0 = \frac{x+4}{x+1}$

$0 = x+4$

$x = -4$

$(-4,0)$ 1 mark

(iii) $y = f(x) = \frac{x+4}{x+1}$

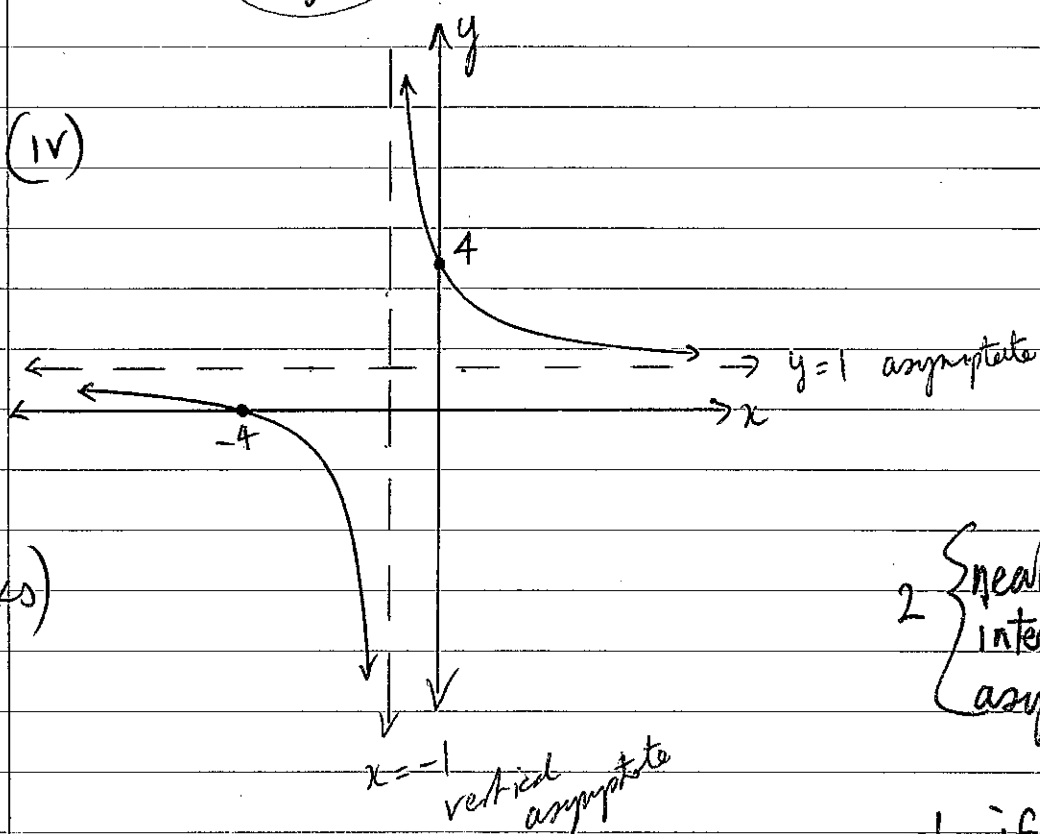
$x \neq -1 \quad \therefore \; x = -1$ vertical asymptote (1)

(2 marks)

$y = f(x) = 1 + \frac{3}{x+1}$

$y = 1$ ↑ horizontal asymptote (1)

(iv)

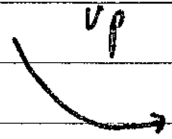


(2 marks)

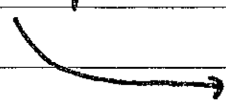
2 { neat curve
intercepts correct
asymptotes correct

-1 if not smooth curve

OR curve bends up

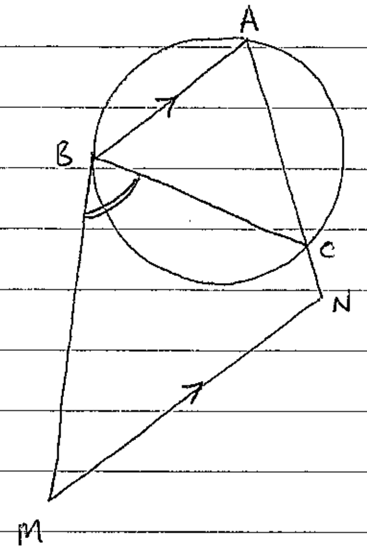


OR parallel



QUESTION 15

(a) (i) $\angle MBC = \angle BAC$



angle between tangent MB and chord BC at point of contact is equal to angle in alternate segment at circumference

(1 mark) 1

let $\angle BAC = x$ $\therefore \angle MBC = x$ (stated in (i) above)

(ii) $\angle MNC = 180 - \angle BAC$
$= 180 - x$ (cointerior angles supplementary as $AB \parallel MN$) 1

now $\angle MBC + \angle MNC = x^\circ + (180^\circ - x^\circ)$
$= 180^\circ$

(2 marks) $\therefore$ as opposite angles in quadrilateral MNCB are supplementary 1

$\therefore$ MNCB is a cyclic quadrilateral

2

Q15

b) (i) $y = \frac{1}{2}(p+q)x - apq$

$\overset{x}{(0}, \overset{y}{3a)}$

(1 mark)

$\therefore 3a = \frac{1}{2}(p+q) \times 0 - apq$

$3a = -apq$

$\frac{3a}{-a} = pq$ ①

$\therefore pq = -3$

(ii) $P(2ap, ap^2) \quad Q(2aq, aq^2)$

(1 mark)

$M = \left\{ \frac{2ap + 2aq}{2}, \frac{ap^2 + aq^2}{2} \right\}$ ①

$= \left\{ a(p+q), \frac{1}{2}a(p^2+q^2) \right\}$

(iii) $x = a(p+q)$

$\therefore p+q = \frac{x}{a}$ ....① also $pq = -3$ .....②

$\left[y = \frac{1}{2}a(p^2+q^2) \Rightarrow \frac{2y}{a} = p^2+q^2 \right.$

$x = a(p+q)$

$x^2 = a^2(p^2+q^2+2pq)$

$= a^2(p^2+q^2-6)$ 1

$= a^2\left(\frac{2y}{a} - 6\right)$] 1

$= 2ay - 6a^2$

$= 2a(y-3a)$

②

$y = \frac{1}{2}a(p^2+q^2)$

$2y = a[(p+q)^2 - 2pq]$

(2 marks)

$2y = a\left[\left(\frac{x}{a}\right)^2 - 2 \times (-3)\right]$ 1

$2y = \frac{x^2 a}{a^2} + 6a$

$2ay = x^2 + 6a^2$ 1

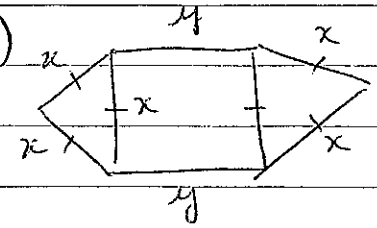
$2ay - 6a^2 = x^2$

②

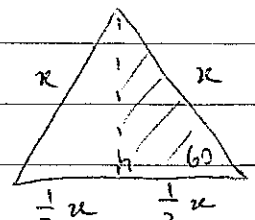
$\therefore x^2 = 2a(y-3a)$ as req'd.

Q 15

c) (i)



$$P = 26 = 2y + 4x$$
$$2y = 26 - 4x$$
$$\therefore y = 13 - 2x$$



$$\text{height} = \frac{\sqrt{3}}{2}x$$

$$\text{Area } \Delta = \frac{1}{2} \times x \times \frac{\sqrt{3}}{2}x$$
$$= \frac{\sqrt{3}\,x^2}{4}$$

(2 marks)

$$\text{Total Area} = xy + 2 \times \frac{\sqrt{3}x^2}{4}$$
$$= x(13 - 2x) + \frac{\sqrt{3}x^2}{2}$$
$$= 13x - 2x^2 + \frac{\sqrt{3}\,x^2}{2}$$
$$= 13x - \frac{4x^2}{2} + \frac{\sqrt{3}x^2}{2}$$
$$= 13x - \frac{(4 - \sqrt{3})\,x^2}{2}$$

(ii)

$$A = 13x - \left(\frac{4 - \sqrt{3}}{2}\right) x^2$$
$$A' = 13 - (4 - \sqrt{3})\,x$$

for stationary point $A' = 0$

$$13 - (4 - \sqrt{3})\,x = 0$$

(3 marks)

$$(4 - \sqrt{3})\,x = 13$$
$$x = \frac{13}{4 - \sqrt{3}}$$
$$= (4 + \sqrt{3})\text{ cm}$$

$$A'' = -(4 - \sqrt{3})$$

as $A'' < 0$ $x = (4 + \sqrt{3})$ will produce max. area

c) $2x + 2y + 2x = 26$

$$4x + 2y = 26$$

$$2x + y = 13$$

$$y = 13 - 2x$$ 1

Area of figure, A

$= 2\left(\frac{1}{2} x \cdot x \sin 60^\circ\right) + xy$

$= x^2 \cdot \frac{\sqrt{3}}{2} + xy$ 1

$= \frac{\sqrt{3}}{2} x^2 + x(13 - 2x)$

$= \frac{\sqrt{3}}{2} x^2 + 13x - 2x^2$

$= 13x - \left(2 - \frac{\sqrt{3}}{2}\right)x^2$

$= 13x - \frac{(4 - \sqrt{3})x^2}{2}$ 2

$\frac{dA}{dx} = 13 - (4 - \sqrt{3})x = 0$

$$x = \frac{13}{4 - \sqrt{3}}$$ 1

$\frac{d^2A}{dx^2} = -(4 - \sqrt{3})$

$= \sqrt{3} - 4$ 1

$= -2.26\ldots < 0$

$\therefore$ Max. area when $x = \frac{13}{4 - \sqrt{3}}$

$= \frac{13(4 + \sqrt{3})}{16 - 3}$

$= (4 + \sqrt{3})$ cm 1

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