Section I (10 marks)

Use the multiple-choice answer sheet for Questions 1-10.

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1. Given that $\cos\alpha = \frac{1}{3}$, where α is an acute angle, find the value of cos 2α.

(A) $-\frac{7}{9}$ (B) $-\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{2}{9}$

2. In how many ways can a committee of 4 be selected from a group of 5 men and 6 women if it must contain an equal number of men and women?

(A) 75 (B) 150 (C) 330 (D) 660

4. What is the size of the angle between $3x - y = 0$ and $y = 1$ correct to the nearest minute?

(A) $18° \, 26'$ (B) $18° \, 27'$ (C) $71° \, 33'$ (D) $71° \, 34'$

5. Which expression is the equivalent to $2\cos^2\frac{3x}{2}$?

(A) $-1+\cos 3x$ (B) $1-\cos 3x$ (C) $1+\cos 3x$ (D) $1+\cos\frac{3x}{4}$

7. Consider the function $f(x) = \frac{x}{(4x+1)^3}$.

Which of the following is the correct expression for $f'(x)$?

(A) $\frac{x+1}{(4x+1)^4}$ (B) $\frac{1-8x}{(4x+1)^4}$ (C) $\frac{x+1}{(4x+1)^2}$ (D) $\frac{1-8x}{(4x+1)^2}$

9. Which of the following is an expression for $\frac{1}{1-\tan x} - \frac{1}{1+\tan x}$

(A) $\frac{2\tan x}{\sec^2 x}$ (B) $\frac{\tan 2x}{\tan x}$ (C) $\tan 2x$ (D) $\tan x \tan 2x$

10. Solve $\left|\frac{1}{4x}\right| > x$.

(A) $x < 0,\ x > \frac{1}{2}$ (B) $-\frac{1}{2} < x < \frac{1}{2}$ (C) $x < -\frac{1}{2},\ x > \frac{1}{2}$ (D) $x < 0,\ 0 < x < \frac{1}{2}$

End of Section 1

Section 2 (45 marks)

Question 11 (15 marks) Start a new Answer Booklet

Marks

a) Consider the polynomial $Q(x) = 2x^3 - 3x^2 + px + r$ where p and r are constants. **3**

This polynomial is divisible by x and gives a remainder of 2 when divided by $2x + 1$.

Find the values of p and r.

c) Show that $\frac{\tan^3 \theta + 1}{\tan \theta + 1} = \frac{1}{2}\sec^2 \theta(2 - \sin 2\theta)$ **3**

d) Four teams with 3 members each are to sit around a circular table. The members of each team are to sit in a group next to each other such that the team leader is in the middle between the other 2 members. **2**

In how many ways can this be done?

e) Solve the inequality $\frac{3x}{x+1} \le 2$. **2**

Question 12 (15 marks) Start a new Answer Booklet **Marks**

a) The polynomial $P(x) = (x + a)(x + 2a)Q(x)$ has $x - 1$ as a factor and Q (1) does not equal zero. **2**

What are the values of a?

c) Find the exact value of $\cos 15° \sin 75° - \sin^2 15°$ **2**

d) Jared has 20 birds and 20 cages. He is to place one bird into every cage. These twenty cages are hung up around his shop. **2**

Four of the cages are next to each other on the wall in front of his shop and the rest are next to each other on the wall inside the shop.

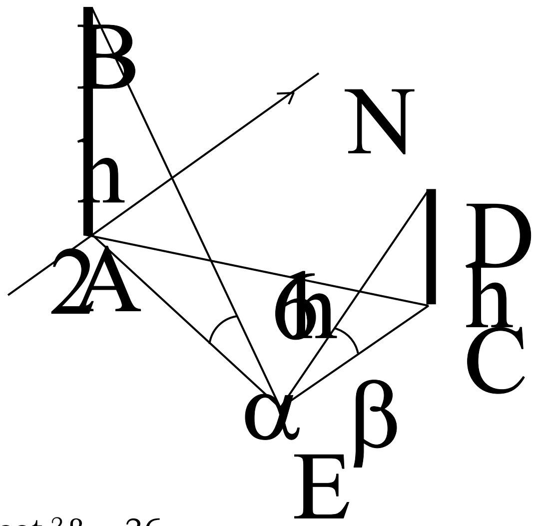
He wants his favourite four birds in any of the cages in front of the shop and the remaining birds inside the shop.

In how many ways can Jared put his birds in these cages?

e) A and C are the feet of two buildings AB and CD with heights $2h$ metres and h metres respectively. **3**

CD is due east of AB and at a distance of $6h$ metres from AB.

From a point E south of CD, the angles of elevations of AB and CD are α and β respectively.



Show that $4\cot^2\alpha - \cot^2\beta = 36$.

Question 13 (15 marks) Start a new Answer Booklet **Marks**

a) The equation $3x^3 - 6x - 1 = 0$ has roots α, β and γ. **3**

What is the value of

$$\alpha^2 + \beta^2 + \gamma^2 .$$

b) Consider the polynomial $P(x) = x^3 + 2x^2 + kx - 8$, where k is a real number. **3**

Let the roots of the polynomial be α, $-\alpha$ and β.

Find the roots of $P(x)$.

c) Solve $\cos\theta + 3\sin\dfrac{\theta}{2} - 2 = 0$ for $0° \le \theta \le 180°$ **3**

e) Melissa divides a rectangle into six squares and she wants to colour these squares with different colours. She has nine different colours to choose from.

In how many ways can she colour these squares:

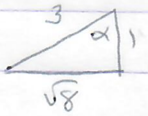
i) if she can use any of the nine colours? **1**

ii) if she wants to colour three consecutive squares using her favourite three colours and the remaining squares with three of the other six colours? **2**

End of Paper

2017 Extension 1

1) $\cos\alpha = \frac{1}{3}$



$\therefore \sin\alpha = \frac{\sqrt{8}}{3}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \frac{1}{9} - \frac{8}{9}$

$= -\frac{7}{9}$

OR

$\cos 2\alpha = 2\cos^2\alpha - 1$

$= 2 \times \frac{1}{9} - 1$

$= -\frac{7}{9}$ (A)

2) ${}^5C_2 \times {}^6C_2 =$

4) application of $\tan(\alpha - \beta)$ which technically we don't do

5) $2\cos^2\frac{3x}{2} = \cos\left(\frac{3x}{2} \times 2\right) + 1$

$= \cos 3x + 1$ (C)

As $\cos 2\theta = 2\cos^2\theta - 1$

7. $y = \frac{x}{(4x+1)^3}$

$u = x$ $u' = 1$

$v = (4x+1)^3$ $v' = 12(4x+1)^2$

$y' = \frac{(4x+1)^3 \times 1 - x \times 12(4x+1)^2}{(4x+1)^6}$

$= \frac{(4x+1)^2[4x+1-12x]}{(4x+1)^6}$

$= \frac{1-8x}{(4x+1)^4}$ (B)

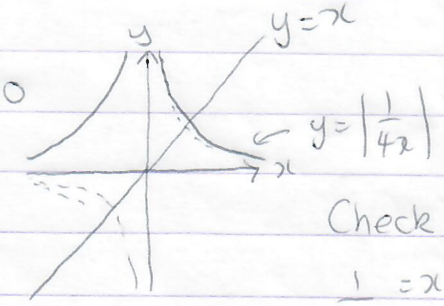
9. $\frac{1}{1-\tan x} - \frac{1}{1+\tan x}$

$= \frac{(1+\tan x)-(1-\tan x)}{(1-\tan x)(1+\tan x)}$

$= \frac{2\tan x}{1-\tan^2 x}$ ($=\sec^2 x$ Pyth)

$= \frac{2\tan x}{\sec^2 x}$ (A)

10



Check equality

$\frac{1}{4x} = x$ $x \neq 0$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$ but $x > 0$

$\therefore x = \frac{1}{2}$

$\therefore \left|\frac{1}{4x}\right| > x$ when

$x < 0,\ 0 < x < \frac{1}{2}$ (D)

QUESTION 11

a) $Q(x) = 2x^3 - 3x^2 + px + r$

divisible by $x \Rightarrow Q(0) = 0$

$\div (2x+1)$ rem 2 $\Rightarrow Q(\frac{-1}{2}) = 2$

$Q(0) = 0 - 0 + 0 + r$

$\therefore r = 0$

$Q(\frac{-1}{2}) = 2 \times \frac{-1}{8} - 3 \times \frac{1}{4} + \frac{1}{2} p + r$

$= -1 - \frac{1}{2} p + r$

$\therefore -1 - \frac{p}{2} = 2 \quad (r=0)$

$\frac{-p}{2} = 3$

$p = -6$

$\therefore p = -6 \quad r = 0$

b) RTP

$\frac{\tan^3\theta + 1}{\tan\theta + 1} = \frac{1}{2} \sec^2\theta (2 - \sin 2\theta)$

RHS $= \frac{1}{2} \sec^2\theta (2 - \sin 2\theta)$

$= \sec^2\theta (1 - \frac{1}{2} \sin 2\theta)$

$= \sec^2\theta (1 - \sin\theta \cos\theta)$

$= \sec^2\theta - \frac{1}{\cos^2\theta} \sin\theta \cos\theta$

$= \sec^2\theta - \tan\theta$

LHS $= \frac{\tan^3\theta + 1}{\tan\theta + 1}$

$= \frac{(\tan\theta + 1)(\tan^2\theta - \tan\theta + 1)}{\tan\theta + 1}$

$= \tan^2\theta - \tan\theta + 1$

$= \sec^2\theta - \tan\theta$

$=$ RHS

d) * teams sit together

$\therefore$ 3! ways to arrange 4 things in a circle

* for each team there is 2! ways to seat the 2 players either side of the leader

$\therefore$ # ways $= 3! \times (2!)^4$

$= 960$

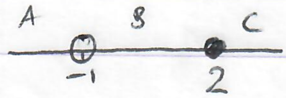
e) $\frac{3x}{x+1} \leq 2$

(a) $x \neq -1$

(b) equality

$3x = 2x + 2$

$x = 2$



A: If $x = -2$ LHS $= \frac{-6}{-1} = 6 \nleq 2$ ✗

B: If $x = 0$ LHS $= 0 \leq 2$ ✓

C: If $x = 3$ LHS $= \frac{9}{4} \nleq 2$ ✗

$\therefore -1 < x \leq 2$

QUESTION 12

a) $P(x) = (x+a)(x+2a)Q(x)$

$(x-1)$ factor $\Rightarrow P(1)=0$

but given $Q(1) \neq 0$!

Sub $x=1$:

$P(1) = (1+a)(1+2a)Q(x)$

$\therefore$ for $P(1)=0$

$(1+a)(1+2a) = 0$

$\therefore a = -1, \frac{-1}{2}$

b) $\cos 15 \sin 75 - \sin^2 15$

$= \cos 15 \, \sin(90-15) - \sin^2 15$

$= \cos 15 . \cos 15 - \sin^2 15$

$= \cos^2 15 - \sin^2 15$

$= \cos 30$

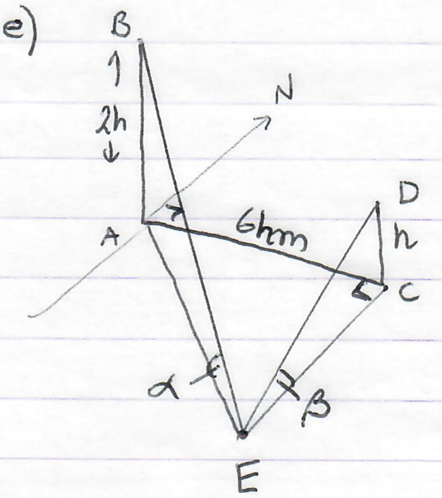
$= \frac{\sqrt{3}}{2}$

d) Using box method

4 faves other 16

$\therefore$ # ways $= 4! \times 16!$

e)



In ΔAEB

$\frac{2h}{AE} = \tan\alpha$

$\therefore AE = \frac{2h}{\tan\alpha}$

$= 2h \cot\alpha$

In ΔDCE

$\frac{h}{EC} = \tan\beta$

$EC = \frac{h}{\tan\beta}$

$= h \cot\beta$

In ΔACE $\angle ACE = 90°$

$\therefore AC^2 + CE^2 = AE^2$

$(6h)^2 + (h\cot\beta)^2 = (2h\cot\alpha)^2$

$36h^2 + h^2\cot^2\beta = 4h^2\cot^2\alpha$

$\therefore 36 = 4\cot^2\alpha - \cot^2\beta$

ie $4\cot^2\alpha - \cot^2\beta = 36$

as required

Draw the separate triangles if necessary (usually helpful)

QUESTION 13

a) $3x^3 - 6x - 1 = 0$

$\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-6}{3} = -2$

$\alpha\beta\gamma = \frac{-(-1)}{3} = \frac{1}{3}$

$\alpha^2 + \beta^2 + \gamma^2$

$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= 0^2 - 2 \times (-2)$

$= 4$

b) $P(x) = x^3 + 2x^2 + kx - 8$

Roots $\alpha, -\alpha, \beta$

Sum of the roots:

$$\alpha + (-\alpha) + \beta = -2 \qquad \left(\frac{-b}{a}\right)$$

$$\therefore \beta = -2$$

Product of roots

$$\alpha(-\alpha)\beta = 8 \qquad \left(\frac{-d}{a}\right)$$

$$-\alpha^2\beta = 8 \quad \text{but } \beta = -2$$

$$-\alpha^2 = -4$$

$$\alpha^2 = 4$$

$$\alpha = \pm 2$$

$\therefore$ the roots are $2, -2, -2$

c) $\cos\theta + 3\sin\frac{\theta}{2} - 2 = 0$ $\qquad 0 \leq \theta \leq 180^\circ$

$\qquad \therefore 0 \leq \frac{\theta}{2} \leq 90^\circ$

double angle $\qquad$ half angle

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\therefore \cos\theta = 1 - 2\sin^2\frac{\theta}{2}$$

$$1 - 2\sin^2\frac{\theta}{2} + 3\sin\frac{\theta}{2} - 2 = 0$$

$$2\sin^2\frac{\theta}{2} - 3\sin\frac{\theta}{2} + 1 = 0$$

let $x = \sin\frac{\theta}{2}$

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - 2x - x + 1 = 0$$

$$2x(x-1) - (x-1) = 0$$

$$(x-1)(2x-1) = 0$$

$$x = 1, \frac{1}{2}$$

So $\sin\frac{\theta}{2} = 1$ & $\sin\frac{\theta}{2} = \frac{1}{2}$

$\frac{\theta}{2} = 90^\circ$ & $\frac{\theta}{2} = 30^\circ$

$\therefore \theta = 180^\circ, 60^\circ$

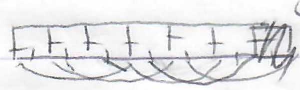
e) 6 squares 9 colours

i) ${}^9C_6$

ii) dodgy question

What shape is the rectangle?

Assume it's long:



$\therefore$ 4 ways to put her favourites together

3! ways to arrange them together

$6 \times 5 \times 4$ ways to choose the rest

$\therefore 4 \times 3! \times 6 \times 5 \times 4$

$= 2880$