Section I 2018 Ext 1 Prelim Final Adjusted

10 marks
Attempt Questions 1–10
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following expressions is equal to $\cos 5x \cos 3x + \sin 5x \sin 3x$?

(A) $\sin 2x$

(B) $\cos 2x$

(C) $\sin 8x$

(D) $\cos 8x$

2 What is the Cartesian equation of the parabola $x = 4t, y = 4t^2$?

(A) $x^2 = -4y$

(B) $x^2 = -8y$

(C) $x^2 = 4y$

(D) $x^2 = 8y$

3 Seven people are to be seated around a circular table. If two particular people must be seated together, how many seating arrangements are possible?

(A) $7!$

(B) $5! \times 2$

(C) $6!$

(D) $6! \times 2$

4 Find the values of a and b such that the graph of $y = (ax - 7)(x - b)^2$ cuts the x-axis at $x = 3.5$ and touches the x-axis at $x = 5$.

(A) $a = 2, b = 5$

(B) $a = 5, b = 2$

(C) $a = 3.5, b = 5$

(D) $a = 5, b = 3.5$

6 Find the derivative of $f(x)=\dfrac{x}{(4x+1)^3}$ with respect to x.

(A) $\dfrac{x+1}{(4x+1)^4}$

(B) $\dfrac{1-8x}{(4x+1)^4}$

(C) $\dfrac{x+1}{(4x+1)^2}$

(D) $\dfrac{1-8x}{(4x+1)^2}$

7 Simplify $\dfrac{(n-1)!n!}{(n!)^2}$

(A) n

(B) $\dfrac{n-1}{n}$

(C) $\dfrac{1}{n^2}$

(D) $\dfrac{1}{n}$

8 If θ is an acute angle, where $\sin\theta=\dfrac{1}{\sqrt{5}}$, find the exact value of $\sin 2\theta$.

(A) $\dfrac{4}{5}$

(B) $\dfrac{2}{\sqrt{5}}$

(C) $\dfrac{1}{2\sqrt{5}}$

(D) $\dfrac{2}{5}$

9 The remainder when the polynomial $P(x)=x^4-6x^3-5x^2+7$ is divided by x^2+1 is $ax+13$.
What is the value of a?

(A) -6

(B) 6

(C) -2

(D) 2

Section II

45 marks
Attempt Questions 11–13
Allow about 1 hour and 20 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–13, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) In how many ways can a committee of 3 boys and 3 girls be formed from a group of 6 boys and 8 girls? **1**

(b) By making the substitution $t = \tan\frac{\theta}{2}$, prove that $\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}$ **2**

(c) Solve $\frac{4}{x+1} < 3$. **2**

(d) If the roots of the equation $x^3 - 5x^2 + 3x - 2 = 0$ are α, β and γ, find the value of:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha$ **1**

(iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(f) (i) Prove that $\frac{\sin(2\theta)}{1+\cos(2\theta)} = \tan\theta$. **2**

(ii) Hence, find the exact value of $\tan 22.5°$, giving your answer with a rational denominator. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the function $y = \dfrac{x}{2-x}$.

(i) State the equation of the vertical and horizontal asymptote. **2**

(ii) Sketch the graph of the function, including all asymptotes and intercepts where necessary. **2**

(b) Find the number of ways the letters of the word ANGLE can be arranged in a straight line so that: **2**

(i) No two consonants are next to each other

(ii) The three consonants are side by side **1**

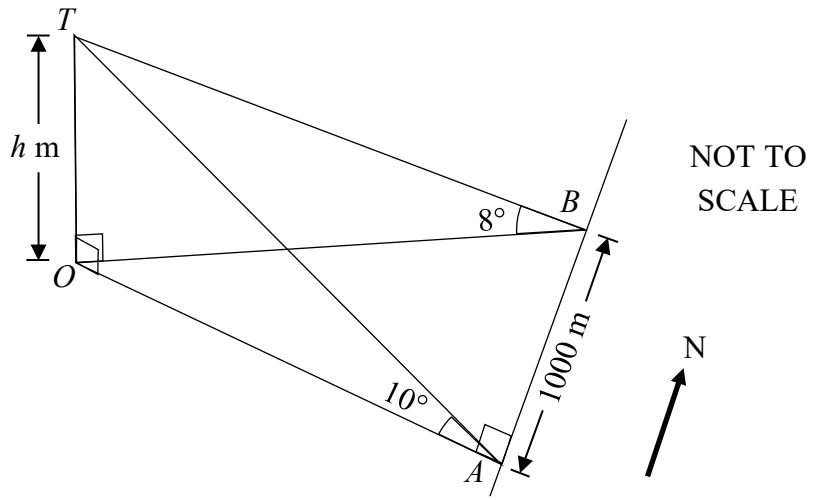
(iii) Exactly 2 of the 3 consonants are side by side. **1**

(e) A man is walking on a straight, level road with his GPS and inclinometer.

At point A on the road, due east of a distant tower, he measures the angle of elevation to the top of the tower, T, to be 10°.

After walking 1000 metres on the road to point B, he measures the angle of elevation to T to be 8°.

Let O represent the base of the tower, h be the height of the tower and $\angle OAB = 90°$.



(i) Show that $OA = \dfrac{h}{\tan 10°}$ and find a similar expression for OB. **2**

(ii) Hence, find the value of h , to the nearest metre. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(c) Consider the polynomial $P(x) = x^3 + cx^2 - 9x + d$

(i) Find the values of c and d given that $(x - 3)$ is a factor of $P(x)$ and the remainder is 42 when $P(x)$ is divided by $(x - 4)$. **2**

(ii) Fully factorise the polynomial $P(x)$ as a product of linear factors. **2**

(iii) Hence, sketch the graph of $P(x)$, showing any x and y-intercepts. **2**

2018 Ext1 Adjusted

1. $\cos 5x \cos 3x + \sin 5x \sin 3x$

$= \cos(a-b)$ where

$a = 5x \quad b = 3x$

$= \cos(5x-3x)$

$= \cos 2x$ (B)

Check by substituting values.

2. $x = 4t \quad y = 4t^2$

$\cdots ① \quad \cdots ②$

① $\Rightarrow t = \frac{x}{4}$

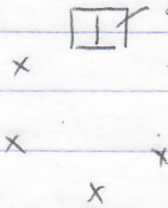
sub into ②

$y = 4\left(\frac{x}{4}\right)^2$

$= \frac{x^2}{4}$

$\therefore x^2 = 4y$ (C)

3.



Seat the 2 people = 2 ways

Remaining = 5! ways

$\therefore 2 \times 5!$ (B)

4. $y = (ax-7)(x-b)^2$

Given: cut x at $x = 3.5$ ie $(3.5, 0)$

touches at $x = 5$ ie $(5, 0)$

Could a) substitute points, and solve simultaneously

OR

b) think about the factors

$(ax-7)$ linear factor $\therefore$

$\frac{7}{a}$ is where it cuts

$\therefore a = 2$

$(x-b)^2$ quadratic factor

$\therefore b$ is where it touches

$\therefore b = 5$

(A)

6. $f(x) = \frac{x}{(4x+1)^3}$

$u = x \quad u' = 1$

$v = (4x+1)^3 \quad v' = 3 \times 4 \times (4x+1)^2$

$= 12(4x+1)^2$

$\frac{dy}{dx} = \frac{(4x+1)^3 \times 1 - x.12(4x+1)^2}{(4x+1)^4}$

$= \frac{(4x+1)^2[4x+1-12x]}{(4x+1)^4}$

$= \frac{1-8x}{(4x+1)^2}$ (D)

7. $\dfrac{(n-1)!\,n!}{(n!)^2}$

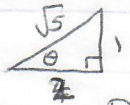
$= \dfrac{(n-1)! \times n!}{n! \times n!}$ split fractions

$= \dfrac{(n-1)!}{n!}$

$= \dfrac{(n-1)!}{n \times (n-1)!}$

$= \dfrac{1}{n}$ (D)

8 $\sin\theta = \dfrac{1}{\sqrt{5}}$



Draw the triangle

$\therefore \cos\theta = \dfrac{2}{\sqrt{5}}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \times \dfrac{1}{\sqrt{5}} \times \dfrac{2}{\sqrt{5}}$

$= \dfrac{4}{5}$ (A)

9. $P(x) = x^4 - 6x^3 - 5x^2 + 7$

$$\begin{array}{r} x^2 - 6x - 6 \\ x^2+1 \overline{)\, x^4 - 6x^3 - 5x^2 + 7} \\ x^4 \quad\quad + x^2 \\ \hline -6x^3 - 6x^2 + 7 \\ -6x^3 \quad\quad -6x \\ \hline -6x^2 + 6x + 7 \\ -6x^2 \quad\quad -6 \\ \hline 6x + 13 \end{array}$$

oops – keep spaces

$\therefore a = 6$ (B)

QUESTION 11

a) ${}^6C_3 \times {}^8C_3 =$

b) $t = \tan\dfrac{\theta}{2}$

LHS $= \operatorname{cosec}\theta + \cot\theta$

$= \dfrac{1}{\sin\theta} + \dfrac{1}{\tan\theta}$

$= \dfrac{1+t^2}{2t} + \dfrac{1-t^2}{2t}$

$= \dfrac{1+t^2+1-t^2}{2t}$

$= \dfrac{2}{2t}$

$= \dfrac{1}{t}$

$= \cot\dfrac{\theta}{2}$ as required

Reference sheet for t ratios.

c) $\dfrac{4}{x+1} < 3$

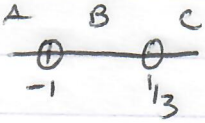
① $x \neq -1$

② equality

$\dfrac{4}{x+1} = 3$

$4 = 3x + 3$

$x = 1/3$



ctd...

test regions:

(A) $x = -2$ LHS $= \frac{4}{-1} = -4 < 3$ ✓

(B) $x = 0$ LHS $= \frac{4}{1} = 4 \not< 3$

(C) $x = 1$ LHS $= \frac{4}{2} = 2 < 3$ ✓

$\therefore x < -1$ or $x > \frac{1}{3}$

d) $x^3 - 5x^2 + 3x + 2 = 0$

i. $\alpha + \beta + \gamma = \frac{-b}{a} = -(-5)$
$= 5$

ii. $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = 3$

iii. $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$
$= 5^2 - 2 \times 3$
$= 19$

Reference sheet

f) i. RTP $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$

LHS $= \frac{2\sin\theta\cos\theta}{1 + \cos^2\theta - \sin^2\theta}$ {double angles

$= \frac{2\sin\theta\cos\theta}{1 + \cos^2\theta - (1 - \cos^2\theta)}$ Pythagoras

$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$ as required

ii. $\tan 22.5 = \frac{\sin 45}{1 + \cos 45}$

$= \frac{1/\sqrt{2}}{1 + 1/\sqrt{2}}$

$= \frac{1/\sqrt{2}}{\frac{\sqrt{2}+1}{\sqrt{2}}}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}+1}$

$= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{\sqrt{2}-1}{1}$

$= \sqrt{2} - 1$

Use the previous question and then exact triangle values

Question 12

a) $y = \frac{x}{2-x}$

i. ~~hori~~ $y = \frac{-(2-x)+2}{2-x}$

$= \frac{2}{2-x} - 1$

vertical asymptote

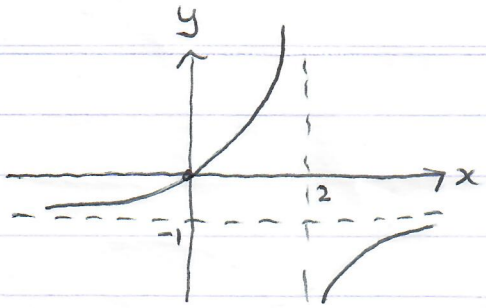
$2 - x \neq 0$

$\therefore x = 2$

horizontal asymptote

$y = -1$

ii)



b) i. ie each separated by a vowel

consonants = 3! ways

vowels = 2! ways

∴ $2 \times 3! = 12$ ways

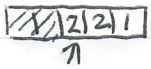
ii. 3 consonants side by side

∴ $3 \times 3! \times 2!$

$= 36$ ways

iii. 2 consonants side by side

- 4 possible positions for ${}^3C_2$

eg 

Need to consider all possible layouts

OR

Consonants can be separate, side by side (2's) or all together

∴ In 2's

= all possible arrangements (5!)

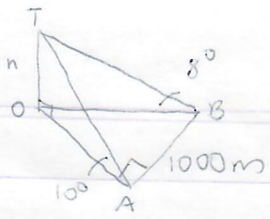
- ways separately (12)
- ways 3 together (36)

$= 5! - (12 + 36)$

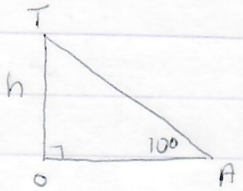
$= 120 - 48$

$= 72$ ways

e)



a) In ΔAOT



$\angle OAT = 90°$

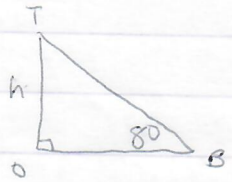
$\frac{h}{OA} = \tan 10°$

$\frac{OA}{h} = \frac{1}{\tan 10°}$

$\therefore OA = \frac{h}{\tan 10°}$

Always draw the simplified diagrams.

b) In ΔBOT

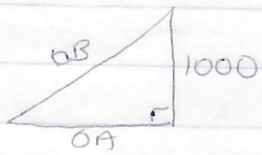


$\angle BOT = 90°$

$\frac{h}{OB} = \tan 8°$

$\therefore OB = \frac{h}{\tan 8°}$

In ΔOBA



$1000^2 = OB^2 - OA^2$ Pythagoras

$1000^2 = \frac{h^2}{\tan^2 8} + \frac{h^2}{\tan^2 10}$

$h^2 = 1000^2 \div \left(\frac{1}{\tan^2 8} - \frac{1}{\tan^2 10}\right)$

$= 1000^2 \div (50.628\ldots)$

$= 54156.3691\ldots$

$h = 232.7 \doteq 233$ m (nearest m)

QUESTION 13

c) $P(x) = x^3 + cx^2 - 9x + d$

i. $(x-3)$ a factor $\Rightarrow P(3) = 0$

rem 42 when $(x-4)$ $\therefore P(4) = 42$

Sub $x = 3$:

$27 + 9c - 27 + d = 0$

$9c + d = 0$

$d = -9c$ ... ①

Sub $x = 4$

$64 + 16c - 36 + d = 42$

$28 + 16c - 9c = 42$ sub ①

$7c = 14$

$c = 2$

$\therefore d = -18$

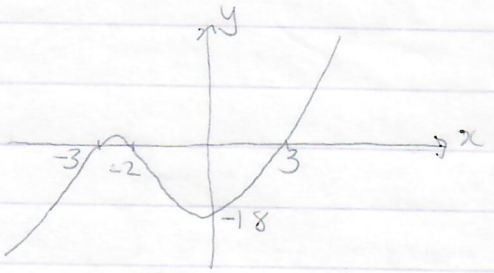
ii. $P(x) = x^3 + 2x^2 - 9x - 18$

$P(2) = 8 + 8 - 18 - 18 \neq 0$

$P(-2) = -8 + 8 + 18 - 18 = 0$

$\therefore P(x) = (x+3)(x+2)(x-3)$

iii.



At $x = 0$ $P(0) = -18$

At $y = 0$, $x = 3, -2, -3$