SCEGGS Darlinghurst

2006

Preliminary Course
Semester 2 Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1½ hours
- Write using blue or black pen
- Write your Centre Number and Student Number at the top of each page
- Attempt **all** questions and show all necessary working
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used
- **Start each question on a new page**
- **Do not** attach all question together in one bundle

Total marks – 60

- Attempt Questions 1–4

Question	Comm	Reason	Calc	Total
1	/2	/4		/15
2	/4	/3	/2	/15
3		/2		/15
4	/2	/9		/15
TOTAL	**/8**	**/18**	**/2**	**/60**

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Total marks – 60
Attempt Question 1 – 6

Attempt **all** questions on the pad paper provided
Write your Centre Number and Student Number at the top of each page
Show all necessary working
Marks may be deducted for careless or badly arranged work
Mathematical templates, geometrical equipment and scientific calculators may be used
Start each question on a new page

Marks

Question 1 (15 marks)

(a) Solve:

(i) $x^2 + x \leq 6$ **2**

(ii) $\left| \frac{1}{2}x + 3 \right| = x - 8$ **3**

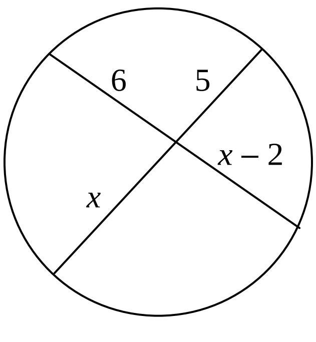
(b) When the polynomial $P(x) = x^3 - kx^2 + 3x - 4$ is divided by $x + 2$, the remainder is 3. **2**

Find the value of *k*.

(c) Solve the equation: **3**

$$2\sin^2\frac{\theta}{2} = 1 \text{ for } 0° \leq \theta \leq 360°$$

(d) **2**



Not to scale

Find the value of *x*, stating your reason.

Question 1 continues on the next page

Marks

Question 1 (continued)

(e) The point P divides the interval joining $A(2, -3)$ and $B(-1, 7)$ externally in the ratio $2:1$.

(i) Find the co-ordinates of P. **2**

(ii) Find the ratio in which B divides AP. **1**

Start a new page

Marks

Question 2 (15 marks)

(a) Solve $\dfrac{3}{x-1} \leq 4$ **3**

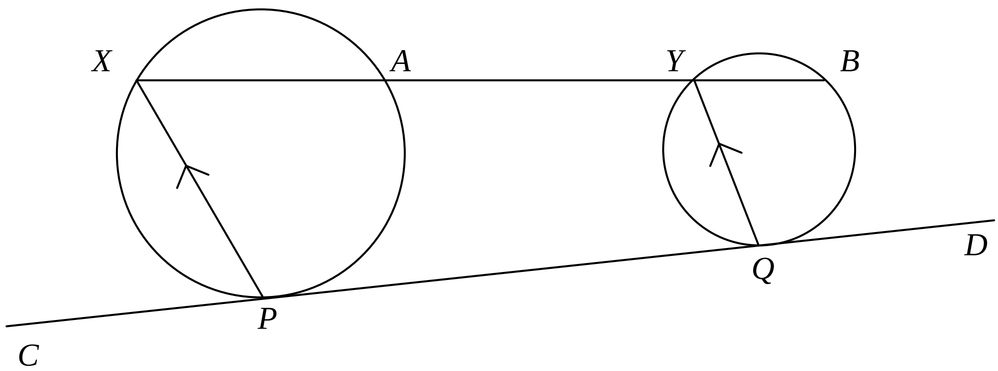
(b) (i) Sketch the curves $y = x^2$ and $y = 4x - x^2$ on the same set of axes, indicating that they intersect at the origin and also at the point P. **2**

(ii) Prove that the point P is $(2, 4)$. **2**

(iii) Find the gradient of each curve at the point P. **2**

(iv) Hence find the acute angle between the curves at the point P. **2**

(c) **4**



CD is a tangent to two circles of different radii at the points P and Q.
$PX \parallel QY$
XY is produced to cut the circles at A and B.
Let $\angle PQY = \alpha$.

Prove that $APQY$ is a cyclic quadrilateral.

Start a new page

Marks

Question 3 (15 marks)

(a) Express $2\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$ if $R > 0$ and α is acute. **3**

(b) (i) Prove that $x - 1$ is a factor of the polynomial. **1**

$$H(x) = 2x^3 + 3x^2 - 3x - 2$$

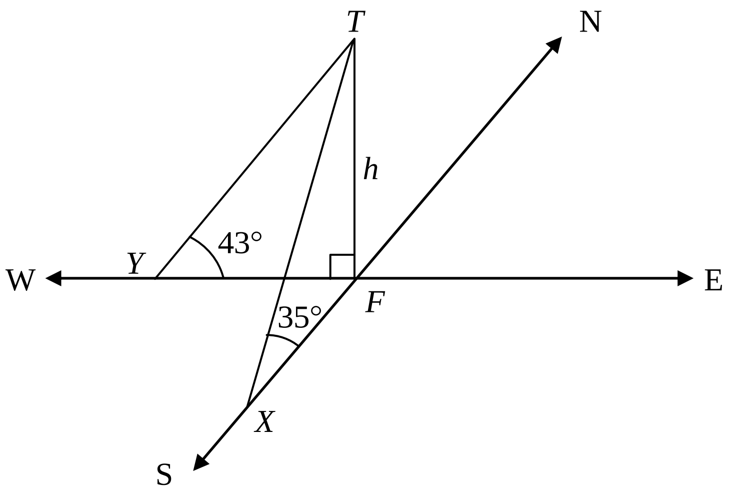
(ii) Hence find all the factors of $H(x)$. **2**

(iii) Graph $y = H(x)$. **1**

(iv) Hence solve $H(x) > 0$. **1**

(c) Prove that $\cos 105^\circ = \dfrac{\sqrt{2} - \sqrt{6}}{4}$ **3**

(d) Point X is due South and Point *Y* is due West of the foot *F* of the mountain *FT*, which has a height of *h* metres. From *X* and *Y*, the angles of elevation of *T* are 35° and 43° respectively. *X* and *Y* are 1200 metres apart.



(i) Prove that $XF = h\tan 55^\circ$ **1**

(ii) Prove that $h = \dfrac{1200}{\sqrt{\tan^2 55^\circ + \tan^2 47^\circ}}$ **3**

Start a new page

Marks

Question 4 (15 marks)

(a) α, β and γ are the roots of the polynomial equation $3x^3 + x^2 + 2x - 4 = 0$.
Find:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

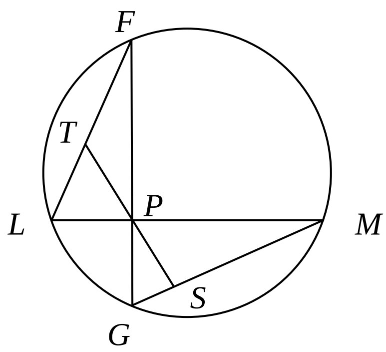
(iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(iv) Explain what the sign of $\alpha^2 + \beta^2 + \gamma^2$ indicates about the nature of the roots α, β and γ. **1**

(v) Without any further calculations, sketch a possible graph of **1**

$$y = 3x^3 + x^2 + 2x - 4$$

(b)



LM and *FG* are two chords of a circle that intersect at right angles at *P*.

PS is perpendicular to *GM* and *SP* is produced to meet *LF* at *T*.

Let $\angle LMG = \alpha$

Prove:

(i) $\angle FLP = \angle MPS$ **2**

(ii) ΔLTP is isosceles **2**

(iii) *T* is the midpoint of *LF* **2**

Question 4 continues on the next page

Marks

Question 4 (continued)

(c) In the triangle ABC, $\angle BAC = 60°$. **3**

Prove $a^2 - b^2 = c(c - b)$

where a, b and c are sides of the triangle.

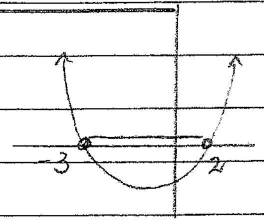
End of paper

Preliminary Extension 1 Semester 2 2006.

1) a) (i) $x^2 + x - 6 \le 0$

$(x+3)(x-2) \le 0$. ✓

$-3 \le x \le 2$ ✓



/2

(ii) $\frac{1}{2}x + 3 = x - 8$ or $\frac{1}{2}x + 3 = -x + 8$.

$x + 6 = 2x - 16$ $\quad$ $x + 6 = -2x + 16$

$-x = -22$ $\quad$ $3x = 10$

$x = 22$ ✓ $\quad$ $x = \frac{10}{3}$ ✓

Testing indicates only solution is $x = 22$. ✓ /3 Reasoning

b) $P(-2) = -8 - 4k - 6 - 4 = 3$ ✓

$-4k - 18 = 3$

$4k = -21$

$k = \frac{-21}{4}$ ✓ /2.

c) $\sin\frac{\theta}{2} = \pm\frac{1}{\sqrt{2}}$

Acute angle = 45°. All 4 quadrants. ✓

$\frac{\theta}{2} = 45°, 135°, 225°, 315°$. ✓ /3.

$\theta = 90°, 270°$. ✓

d) $6(x-2) = 5x$ (products of intercepts cut off on chords are =)

$6x - 12 = 5x$

$x = 12$. ✓ 2/ Communication.

e) (i) $(2, -3)$ $(-1, 7)$ ratio $2 : -1$

$x = \frac{2 \times -1 + -1 \times 2}{1}$

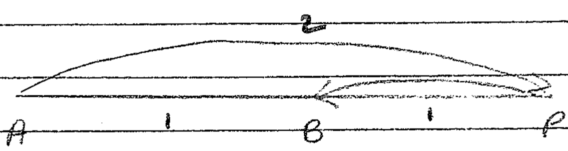
$= -4$ ✓

$y = \frac{-3 \times -1 + 7 \times 2}{1}$ /2

$= 17$. ✓

Point P is $(-4, 17)$

(ii)



1 : 1 /1 Reasoning

② a) $\frac{3}{x-1} \le 4$

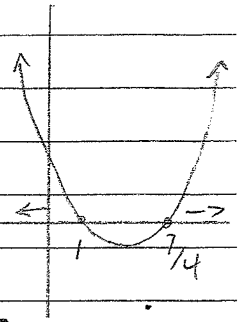
$\frac{3}{x-1}(x-1)^2 \le 4(x-1)^2$

$3(x-1) \le 4(x^2-2x+1)$

$3x-3 \le 4x^2-8x+4.$

$4x^2-11x+7 \ge 0.$ ✓

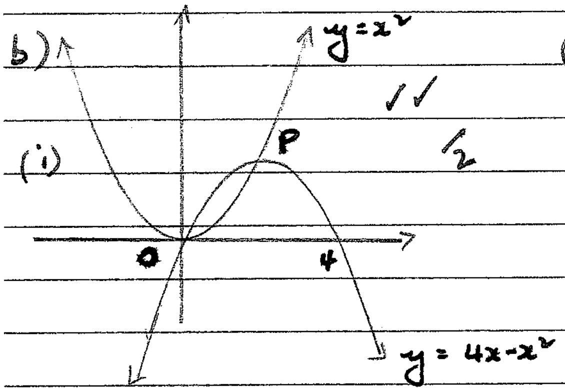
$(x-1)(4x-7) \ge 0$ ✓



$x<1,\ x \ge \frac{7}{4}.$ ✓

/3 Reasoning

b) (i)



✓✓ /2

(ii) $y = x^2$

$y = 4x - x^2$

$x^2 = 4x - x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0, 2.$ ✓

$y = 0, 4.$

∴ points of intersecti[on] $O(0,0)$ $P(2,4)$

∴ P is $(2,4)$ ✓ /2

(iii) $y = x^2$

$y' = 2x$

if $x = 2$, $y' = 4.$

$y = 4x - x^2$

$y' = 4 - 2x$

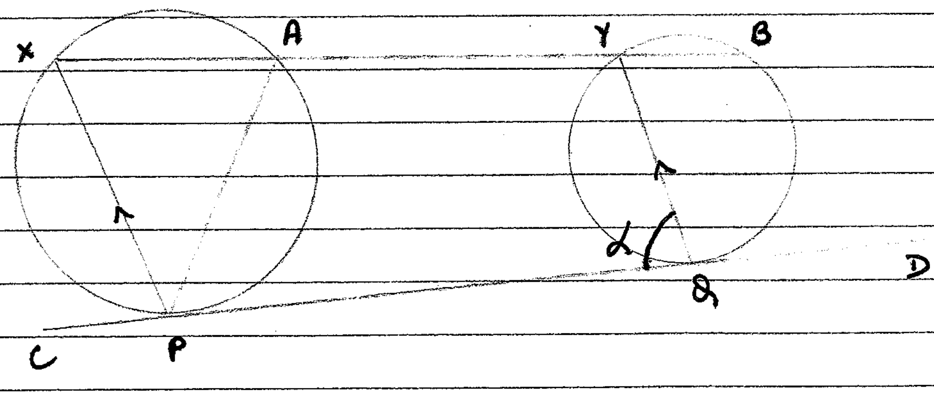
if $x = 2$ $y' = 4 - 4 = 0.$

/2 Calculus.

(iv) $\tan\theta = \left|\frac{4-0}{1+0}\right| = 4$ ✓

$\theta = 75°58'$ (nearest minute) ✓ /2

c)



$\alpha = \angle YQP = \angle XPC$ (corresp ∠s = XP ∥ YQ) ✓

$\angle XPC = \angle XAP = \alpha$ (angle between tangent and chord = angle in alternate segment) ✓

∴ $\angle PAY = 180° - \alpha$ (angle sum of straight angle is 180°)

∴ $\angle PAY$ and $\angle PQY$ are supplementary ✓

i.e. APQY is cyclic (one pair opp. angles supplementary) ✓

/4

Communication

(3) a) $R\sin(\theta - \alpha) = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$= 2\sin\theta - 3\cos\theta$

$\therefore R\cos\alpha = 2 \qquad R\sin\alpha = 3.$ ✓

$\frac{4}{R^2} + \frac{9}{R^2} = 1$

$R = \sqrt{13}$ ✓

$\cos\alpha = \frac{2}{\sqrt{13}}, \quad \alpha = 56°19'$ ✓

/3.

$\therefore 2\sin\theta - 3\cos\theta = \sqrt{13}\sin(\theta - 56°19')$

b) (i) $H(1) = 2+3-3-2 = 0$ ✓ /1

$\therefore x-1$ is a factor.

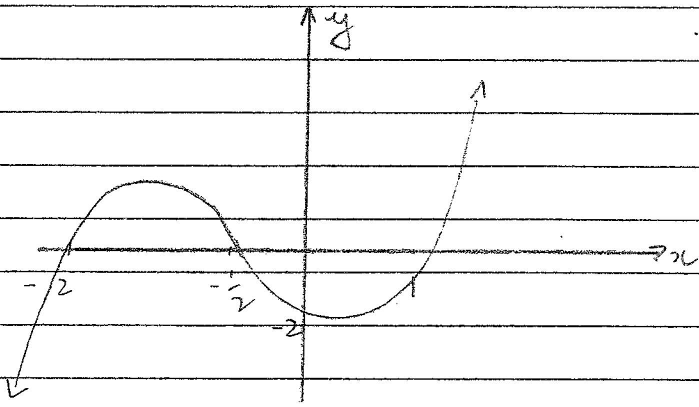
(ii)

$$\begin{array}{r} 2x^2 + 5x + 2 \\ x-1 \overline{) \, 2x^3 + 3x^2 - 3x - 2} \\ \underline{2x^3 - 2x^2} \\ 5x^2 - 3x \\ \underline{5x^2 - 5x} \\ 2x - 2 \\ \underline{2x - 2.} \end{array}$$

$\therefore H(x) = (x-1)(2x^2+5x+2)$ ✓

$= (x-1)(2x+1)(x+2)$ ✓ /2.

(iii)



✓1

(iv) $-2 < x < -\frac{1}{2}$ and $x > 1$ ✓1

2/ Reasoning (iii) and (iv)

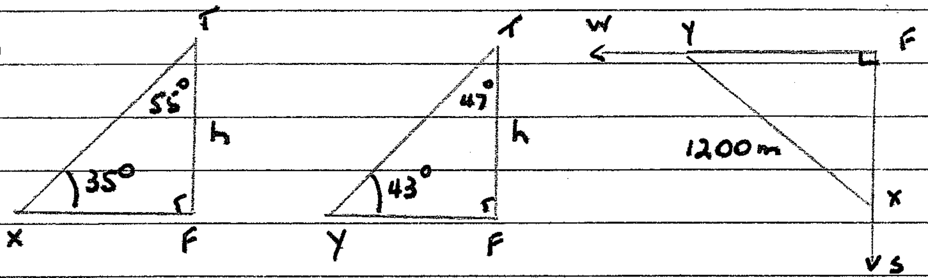
c) $\cos(60^\circ + 45^\circ) = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$. ✓

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$ ✓

$= \frac{1 - \sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2} - 6}{4}$ ✓ /3.

d)



(i) $\tan 55^\circ = \frac{XF}{h}$ (ii) $\tan 47^\circ = \frac{YF}{h}$

$\therefore XF = h \tan 55^\circ$ ✓ /1 $YF = h \tan 47^\circ$. ✓

$1200^2 = h^2 \tan^2 55^\circ + h^2 \tan^2 47^\circ$.

$= h^2 (\tan^2 55^\circ + \tan^2 47^\circ)$ ✓

$h^2 = \frac{1200^2}{\tan^2 55^\circ + \tan^2 47^\circ}$

$h = \frac{1200}{\sqrt{\tan^2 55^\circ + \tan^2 47^\circ}}$ ✓

/3.

(4) a) (i) $\alpha+\beta+\gamma = \frac{-1}{3}$ ✓ /1

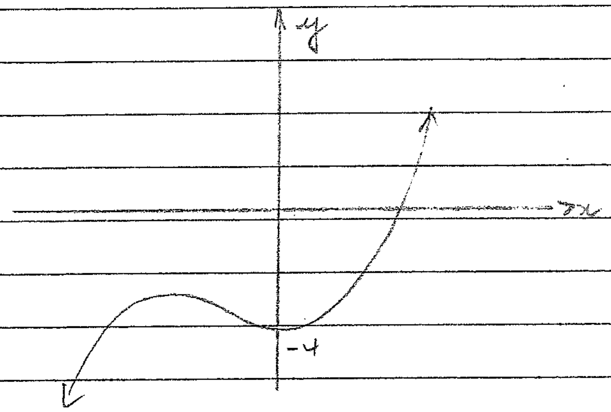
(ii) $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{2}{3}$ ✓ /1

(iii) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$ ✓

$= \frac{1}{9} - \frac{4}{3} = -1\frac{2}{9}$. ✓ /2.

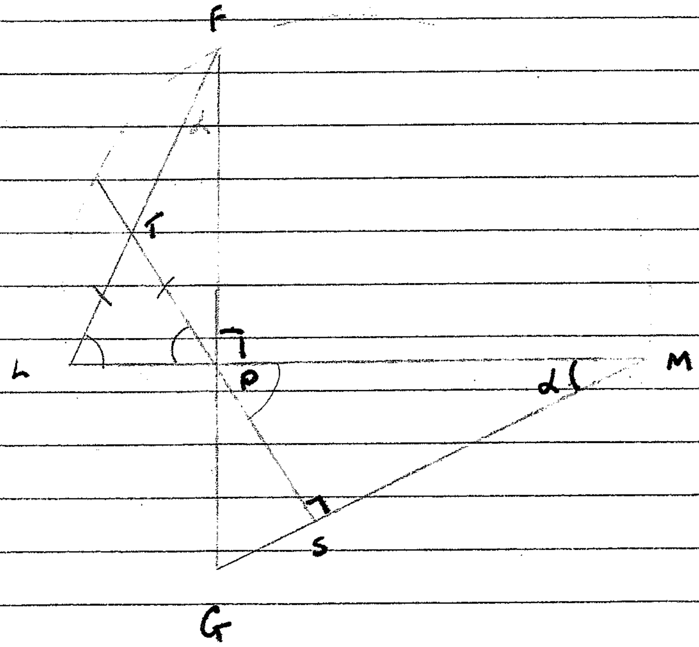
(iv) the negative sign means that 2 of the roots are complex. 1 communication

(v)



1 communication

b)



$\angle MPS = 90° - \alpha$

$\angle LFP = \angle PMG = \alpha$ (angles subtended by same arc are =)

$FG \perp LM$ (given)

$\therefore \angle FLP = 180^\circ - 90^\circ - \alpha$ (angle sum of Δ FLP $= 180^\circ$). ✓

$= 90^\circ - \alpha$

$\angle MPS = 180^\circ - 90^\circ - \alpha$ (angle sum of Δ MPS $= 180^\circ$). ✓

$= 90^\circ - \alpha$

$\therefore \angle FLP = \angle MPS$.

(ii) $\angle MPS = \angle TPL$ (vert opp. angles are equal). ✓

$\therefore \angle TPL = \angle TLP$

$\therefore$ ΔLTP is isosceles (1 pair of sides equal) ✓

(iii) $\angle TPL = 90^\circ - \alpha$.

$\therefore \angle TPF = \alpha$. (angle sum of right angle is 90°).

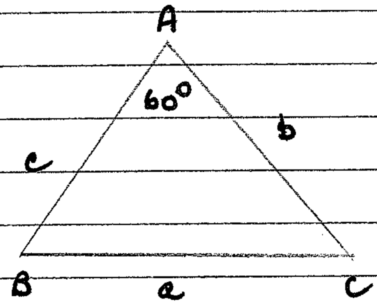
$\angle TPF = \angle TFP = \alpha$

$\therefore$ $TF = TP$ (opp equal angles in Δ TPF)

$\therefore$ $LT = TF$

i.e. T is midpoint LF. /6 Reasoning.

e)



By Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos 60^\circ$$
$$= b^2 + c^2 - 2bc \times \tfrac{1}{2}$$
$$= b^2 + c^2 - bc$$
$$\therefore a^2 - b^2 = c^2 - bc$$
$$= c(c - b)$$

/3 Reasoning