SCEGGS Darlinghurst

2008

Preliminary Course
Semester 2 Examination

Mathematics Extension 1

Outcomes Assessed: PE2 – PE6
Task Weighting: 40%

General Instructions

- Reading time – 5 minutes
- Working time – 1½ hours
- This paper has **five** questions
- Write using blue or black pen
- Answer all questions on the pad paper provided
- Write your Student Number at the top of each page
- Attempt **all** questions and show all necessary working
- **Start each question on a new page**
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used

Total marks – 60

- Attempt Questions 1 – 5

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Marks

Question 1 (12 marks)

(a) The point P divides the interval AB joining $A(-2, -3)$ and $B(1, 2)$ externally in the ratio 3 : 2. **2**

Find the co-ordinates of P.

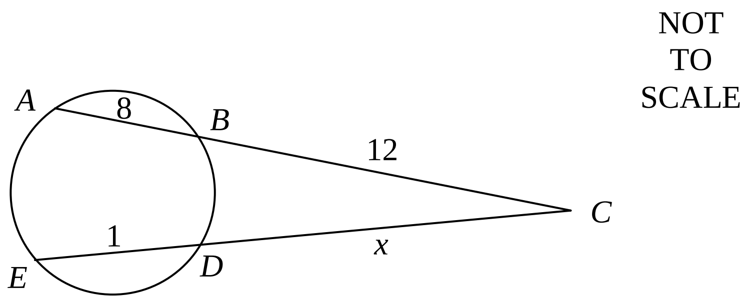
(b) The equation $2x^3 - 4x - 7 = 0$ has roots α, β and γ.
Find the value of:

(i) $\alpha\beta\gamma$ **1**

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ **1**

(iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(c) **3**



In the diagram ABC and EDC are straight lines.
AB = 8cm, BC = 12cm and DE = 1cm
Find x giving reasons.

(d) A polynomial is given by $P(x) = x^3 + ax^2 + bx + 6$. Find the values of a and b if $(x+3)$ is a factor and if 12 is the remaider when $P(x)$ is divided by $(x+1)$ **3**

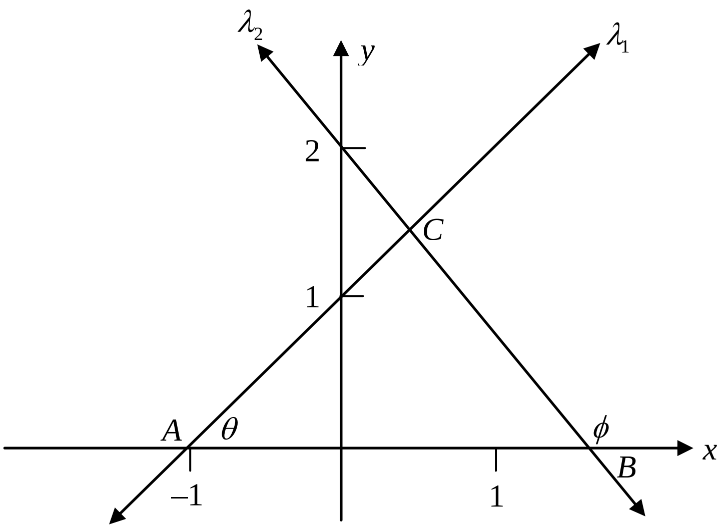
Marks

Question 2 (12 marks)

(a) (i) Express $\sqrt{3}\cos\theta + \sin\theta$ in the form $A\sin(\theta+\alpha)$ where $A > 0$. **2**

(ii) Hence solve the equation $\sqrt{3}\cos\theta + \sin\theta = -\sqrt{3}$ for $0^\circ \le \theta \le 360^\circ$. **2**

(b) The line λ_1 has the equation $x - y + 1 = 0$ and meets the *x*-axis at *A*. The line λ_2 has the equation $\sqrt{3}x + y - 2 = 0$ and meets the *x*-axis at *B*. λ_1 and λ_2 meet at *C*.



(i) Find the exact value for tan $\angle ACB$ ($\angle ACB$ is acute) in its simplest form. **2**

(ii) Find θ and ϕ and hence show $\angle ACB = 75^\circ$. **2**

(iii) Hence find the exact value of tan 75° **1**

Question 2 continues on the next page

Marks

Question 2 (continued)

(c) (i) How many words can be created from the letters of the word COONABARABRAN. **1**

(ii) What is the probability that a word chosen at random has all the "A"s together? **2**

Marks

Question 3 (12 marks)

(a) Let $P(x) = (x-2)(x-1)^2(x+2)^3$

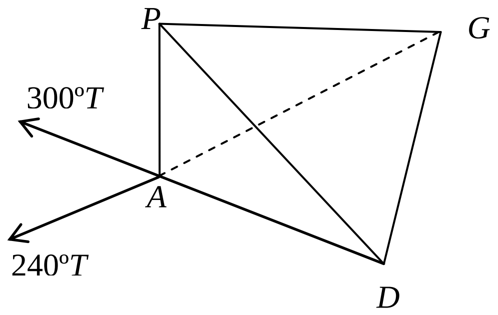
(i) Evaluate $P(0)$. **1**

(ii) Sketch $y = P(x)$ labelling all important features **3**

(b) (i) If there are 8 men and 6 women, how many committees of 5 people can be chosen? **1**

(ii) If a committee is chosen by random find the probability that it would have a majority of men. **2**

(c) The diagram below shows Donna standing at *D* on level ground, whilst Gemma is standing 2000m away at G on the same level ground. They both take the bearing and elevation of a place *P* at the same instant. Donna finds the bearing is 300°*T* and the angle of elevation 25°, whilst Gemma finds the bearing to be 240°*T* and the angle of elevation 17°.



(i) Copy the diagram onto your sheet, showing all the information given. **1**

ii) Show that if the height *PA* of the plane is *h* metres then **3**

$$h = \frac{2000}{\left(\tan^2 65^\circ + \tan^2 73^\circ - 2\tan 65^\circ \tan 73^\circ \cos 60^\circ\right)^{\frac{1}{2}}}$$

(iii) Find *h* to 3 significant figures. **1**

Marks

Question 4 (12 marks)

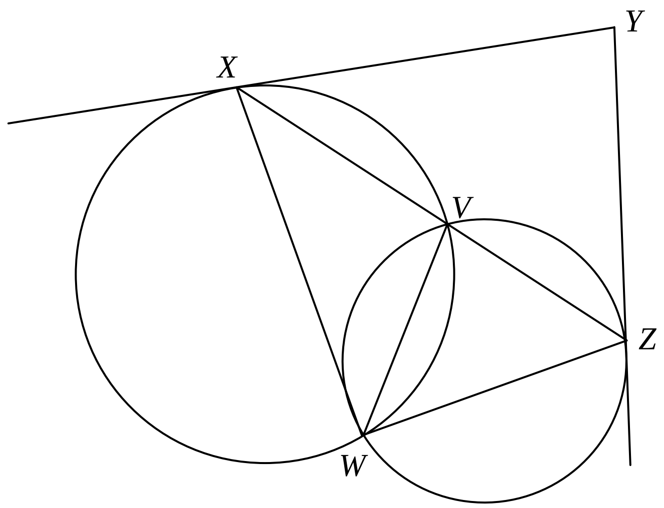
(a) Todd and Meaghan go to the cinema with three other couples. They sit together as a group in a single row.

(i) In how many ways can they be arranged? **1**

(ii) In how many ways can they sit so that each couple is together? **2**

(iii) Todd and Meaghan had an argument going into the cinema and decided they do not want to sit together. How many arrangements are possible if the other couples are still sitting with their partners? **2**

(b) Two circles intersect at V and W as shown. A line through V cuts the two circles at X and Z. The tangents at X and Z meet at Y. **3**



Prove $XYZW$ is a cyclic quadrilateral.

Question 4 continues on the next page

Marks

Question 4 (continued)

(c) (i) Sketch the graph of the polynomial $P(x) = x^3 - x^2 - 12x$ showing the intercepts on the x-axis. **2**

(ii) Hence, solve the inequality $x - 1 \geq \dfrac{12}{x}$. **2**

Marks

Question 5 (12 marks)

(a) If $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$ find the values of *a* and *b*. **2**

(b) Show that $\dfrac{\sin 5x}{\sin x} - \dfrac{\cos 5x}{\cos x} = 4\cos 2x$ **3**

(c) Let $f(x) = \dfrac{x^2}{x^2 - 1}$

(i) For what values of *x* is $f(x)$ undefined **1**

(ii) Evaluate $\lim\limits_{x \to \infty} \dfrac{x^2}{x^2 - 1}$ **1**

(iii) Find $f(0)$ and hence sketch the curve of $y = f(x)$ **3**

(iv) On the same axes sketch $y = x - 1$ **1**

(v) Hence find the number of solutions to $x^3 - 2x^2 - x + 1 = 0$ **1**
Explain your answer.

End of paper

Preliminary Course Extension 1 Semester 2 Examination 2008 – Solutions

Q1 a) $A(-2,-3)$ $B(1,2)$

$3 : -2$

$x = \frac{3\times 1 + -2\times -2}{3 + -2}$ $y = \frac{3\times 2 + -2\times -3}{3+-2}$ ✓

$= \frac{3+4}{1}$ $= \frac{6+6}{1}$

$= 7$ $= 12$

$\therefore P(7, 12)$ ✓

Several students have not learned the correct formula

b) i) $\alpha\beta\gamma = -\frac{d}{a}$

$= -\frac{(-7)}{2}$

$= \frac{7}{2}$ ✓

ii) $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$= \frac{-4}{2}$

$= -2$ ✓

iii) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$ ✓

$= 0^2 - 2\times -2$

$= 4$ ✓

There is no excuse for not knowing these formula. Be careful with coefficients $P(x) = 2x^3 - 4x - 7 \Rightarrow b = 0$

c) $AC \times BC = EC \times DC$ (product of the intercepts of secants through a point are constant) ✓

$20 \times 12 = x(x+1)$

$x^2 + x - 240 = 0$ ✓

$(x-15)(x+16) = 0$

$x = 15$ as $x > 0$ ✓

Comm - 3

Only a few students could recall this property

d) $P(-3) = 0$ $P(-1) = 12$

$\therefore (-3)^3 + a\times(-3)^2 + b\times -3 + 6 = 0$ ✓

$-27 + 9a - 3b + 6 = 0$

$9a - 3b = 21$

$(-1)^3 + a(-1)^2 + b\times -1 + 6 = 12$ ✓

$-1 + a - b + 6 = 12$

$a - b = 7$

$\therefore 9a - 3b = 21 \ldots ①$

$a - b = 7 \ldots ② \times 3$

$3a - 3b = 21 \ldots ③$

$① - ③ \quad 6a = 0$

$a = 0$

using ② $b = -7$ ✓

Reas - 3

Some students confused the concepts of factor and remainder

Q2 a) i) $\sqrt{3}\cos\theta + \sin\theta \equiv A\sin(\theta + \alpha)$

$\equiv A\sin\theta\cos\alpha + A\cos\theta\sin\alpha$

$\therefore A\cos\alpha = 1$

$A\sin\alpha = \sqrt{3}$

$A^2\sin^2\alpha + A^2\cos^2\alpha = (\sqrt{3})^2 + 1^2$

$A^2 = 3 + 1$

$= 4$ ✓

$A = 2$ ✓ $A > 0$

$\frac{A\sin\alpha}{A\cos\alpha} = \frac{\sqrt{3}}{1}$

$\tan\alpha = \frac{\sqrt{3}}{1}$

$\alpha = 60^\circ$ ✓

$\therefore \sqrt{3}\cos\theta + \sin\theta = 2\sin(\theta + 60^\circ)$

Done very well. Just be careful with the auxiliary angle. I saw 30° a few times!

ii) $2\sin(\theta + 60^\circ) = -\sqrt{3}$

$\sin(\theta + 60^\circ) = -\frac{\sqrt{3}}{2}$

$\theta + 60^\circ$ lies in the 3rd & 4th quadrant

$\theta + 60^\circ = 240^\circ$ $\theta + 60^\circ = 300$

$\theta = 180^\circ$ or $\theta = 240^\circ$

✓ ✓

Comm - 2

Need to practise solving trig. equations. The quadrant work was poor.

b) i) $m_{l_1} = 1$ $m_{l_2} = -\sqrt{3}$

$\tan \angle ACB = \left| \dfrac{1 - -\sqrt{3}}{1 + 1 \times -\sqrt{3}} \right|$ ✓

$= \left| \dfrac{1+\sqrt{3}}{1-\sqrt{3}} \right|$

$= \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ ✓

First line was done very well but a majority of students didn't realise the impact of the | | sign.

ii) $\tan\theta = 1$ $\tan\phi = -\sqrt{3}$

$\theta = 45^\circ$ $\phi = 120$ ✓

$\phi = \angle ACB + \theta$ (exterior angle equals sum of two opposite interior angles) ✓

$120^\circ = \angle ACB + 45^\circ$

$\angle ACB = 75^\circ$

Again done well but many students left out the reasons (NO PENALTY)

iii) $\tan 75^\circ = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ ✓ Reas - 5

c) i) $\dfrac{13!}{2!\,4!\,2!\,2!\,2!} = 16\,216\,200$ ✓ Comm - 1

(O A N B R)

Done well

ii) No of words with A's together $= \dfrac{10!}{2!\,2!\,2!\,2!} = 226800$ ✓

$\therefore P(\text{A's together}) = \dfrac{226800}{16\,216\,200}$

$= \dfrac{2}{143}$ ✓ Reas - 2

Many students made an addition error i.e. $\dfrac{9!}{2!2!2!2!}$ or forgot to divide by the 4 '2!'s'

Q3 a) i) $P(0) = (0-2)(0-1)^2(0+2)^3$

$= -16$ ✓

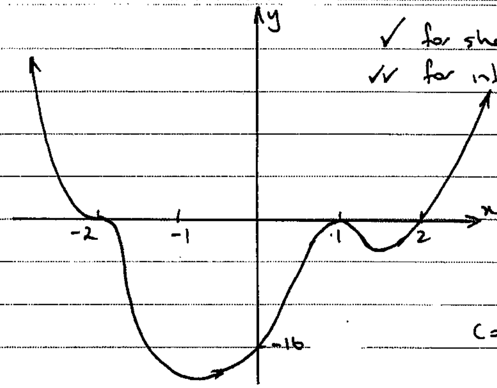
ii) y-int : $x = 0$

$y = -16$

x-int : $y = 0$

$x = 2$ $x = 1$ $x = -2$

multiplicity 1 2 3



✓ for shape
✓✓ for intercepts

Comm - 3

Students who used calculus were generally less successful

b) i) No of committees $= {}^{14}C_5$

$= 2002$ ✓

ii) No of committees with majority of men ✓

$= {}^8C_3 \times {}^6C_2 + {}^8C_4 \times {}^6C_1 + {}^8C_5 \times {}^6C_0$ ✓

$= 840 + 420 + 56$

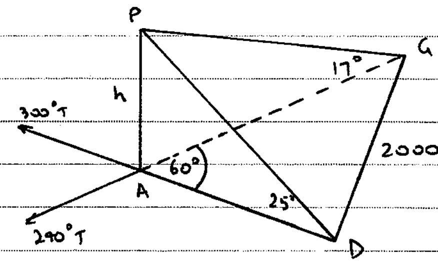
$= 1316$

$\therefore P(\text{majority of men}) = \dfrac{1316}{2002}$

$= \dfrac{94}{143}$ ✓ Reas - 3

Some students used permutations instead of combinations

c) i)



✓ Comm - 1

Students needed to be convincing. Some students clearly "fudged" from the answer.

ii) $\tan 65^\circ = \dfrac{AD}{h}$ $\tan 73^\circ = \dfrac{AG}{h}$

$AD = h\tan 65^\circ$ $AG = h\tan 73^\circ$ ✓

cosine rule : $DG^2 = AD^2 + AG^2 - 2 \times AD \times AG \times \cos 60^\circ$ ✓

$2000^2 = h^2\tan^2 65^\circ + h^2\tan^2 73 - 2h^2\tan 65 \tan 73 \cos 60$

$= h^2(\tan^2 65^\circ + \tan^2 73^\circ - 2\tan 65 \tan 73 \cos 60^\circ)$

$h^2 = \dfrac{2000^2}{\tan^2 65^\circ + \tan^2 73^\circ - 2\tan 65 \tan 73^\circ \cos 60^\circ}$ ✓

- Angles must be clearly identified - 3 letters
- Only a few students were able to show how 60° was calculated
- More supporting working are required.

$$h = \frac{2000}{\sqrt{\tan^2 65° + \tan^2 73° - 2\tan 65° \tan 73° \cos 60°}}$$ Recs-3

iii) $h = 695$ (to 3 sig. figs) ✓

Q4 a) i) No of arrangements $= 8!$ ✓

$= 40320$

ii) No of arrangements $= 4! \times 2! \times 2! \times 2! \times 2!$ ✓ ✓

$= 384$

iii) _ _ _ _ _ _ _ _

T M M M

T M M } 6 ways

T M

∴ No of arrangements $= 6 \times 2 \times 3! \times 2! \times 2! \times 2!$ ✓ ✓

$= 576$ Recs-5

This was done well.

parts ii) & iii) were done poorly but they were tricky. Have a look at the solution and ask questions

b) let $\angle YXZ = \alpha$ and $\angle YZX = \beta$

∴ $\angle XYZ = 180 - (\alpha + \beta)$ (angle sum of a triangle is 180°) ✓

$\angle ZWV = \beta$ (angle at tangent equals angle in the alternate segment) ✓

$\angle XWV = \alpha$ (" " ")

$\angle XWZ = \angle XWV + \angle ZWV$

$= \alpha + \beta$

∴ $\angle XWZ + \angle XYZ = 180$

∴ XYZW is a cyclic quadrilateral as opposite angles are supplementary. ✓

Comm-3

TANGENTS FROM DIFFERENT CIRCLES ARE NOT EQUAL IN LENGTH !!! Everyone except 2 or 3 people thought they did.

c) i) $P(x) = x^3 - x^2 - 12x$

$= x(x^2 - x - 12)$

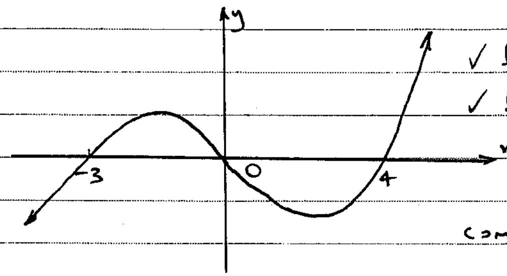
$= x(x-4)(x+3)$

∴ y-int: $x=0$ $P(0)=0$

x-int $y=0$ $P(x)=0$

$x(x-4)(x+3)=0$

$x=0$ $x=-3$ $x=4$



✓ for shape

✓ for intercepts

Comm-2

Done very well. A few people graphed it the wrong way.

ii) $x^2 \times (x-1) \geq \frac{12}{x} \times x^2$

$x^3 - x^2 \geq 12x$

$x^3 - x^2 - 12x \geq 0$ ✓

∴ from the graph $-3 \leq x < 0$ and $x \geq 4$ ✓

Recs-2

Many students missed the link between i) & ii) because they ended up with $x^2 - x - 12 \geq 0$. You have to multiply through by x^2 not x to keep $\geq$

Q5 a) let $m = 2^a$ and $n = 3^b$

∴ $2^a + 3^b = 17$ $\quad 2^{a+2} - 3^{b+1} = 5$

$m + n = 17$ $\quad 2^2 \times 2^a - 3 \times 3^b = 5$

$4m - 3n = 5$

∴ $4m - 3n = 5$ ...①

$m + n = 17$ ...② ×3 ✓

$3m + 3n = 51$ ...③

① + ③

$7m = 56$

$m = 8$

from ② $n = 9$

∴ $2^a = 8$ $\quad 3^b = 9$

$a = 3$ $\quad b = 2$ ✓ Recs-2

Only a few students were successful in this question.

Alternative solutions are possible but only a couple of students were able to correctly find 'a' and 'b'.

A couple of "lucky" students "chanced" upon the correct answer - by trial and error.

b) $LHS = \frac{\sin 5x}{\sin x} - \frac{\cos 5x}{\cos x}$

$= \frac{\sin 5x \cos x - \cos 5x \sin x}{\sin x \cos x}$ ✓

$= \frac{\sin(5x - x)}{\sin x \cos x}$ ✓

$= \frac{\sin 4x}{\frac{1}{2}\sin 2x}$

$= \frac{2\sin 2x \cos 2x}{\frac{1}{2}\sin 2x}$ ✓

$= 4\cos 2x$ Reas - 3

Very few students could get to this line - Always look at the pattern.

c) i) $x = \pm 1$ ✓

ii) $\lim_{x \to \infty} \frac{x^2}{x^2 - 1}$

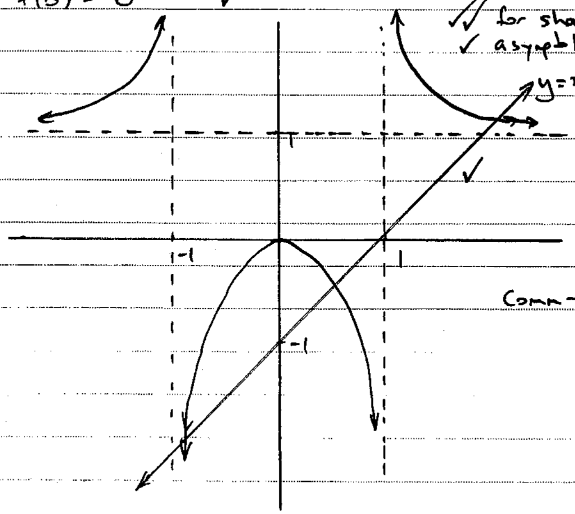
$= \lim_{x \to \infty} \frac{x^2/x^2}{x^2/x^2 - \frac{1}{x^2}}$

$= \lim_{x \to \infty} \frac{1}{1 - \frac{1}{x^2}}$

$= 1$ ✓

iii) $f(0) = 0$ ✓

iv)



✓✓ for shape
✓ asymptotes

Comm - 3

Well done by most students

v)

v) Solve simultaneously

$y = \frac{x^2}{x^2 - 1} \quad \ldots ①$ $\quad y = x - 1 \ldots ②$

① = ②

$\frac{x^2}{x^2 - 1} = x - 1$

$x^2 = (x-1)(x^2-1)$

$x^2 = x^3 - x - x^2 + 1$

$0 = x^3 - 2x^2 - x + 1$

$\therefore$ pts of intersection of $y = \frac{x^2}{x^2-1}$ and $y = x - 1$ are the solutions to $x^3 - 2x^2 - x + 1 = 0$

$\therefore$ 3 solutions. ✓

Reas - 1

A clear statement of the reason was required to obtain this mark.