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Centre Number

Student Number

SCEGGS Darlinghurst

2009

Preliminary Course
Semester 2 Examination

Mathematics Extension 1

Outcomes Assessed: P1-P8, PE1, PE2, PE3 and PE6
Task Weighting: 40%

General Instructions

- Reading time – 5 minutes
- Time allowed – 1½ hours
- Write using black or blue pen
- Write your Student Number at the top of this page
- This paper has **four** questions
- Attempt all questions on the pad paper provided
- Answer **all** questions and show all necessary working
- **Start each question on a new page**
- Marks will be deducted for careless or badly arranged work
- Mathematical templates, geometrical equipment and scientific calculators may be used
- **Do not** attach all questions together in one bundle

Total marks – 60

- Attempt Questions 1 – 4

Question	Comm	Reason	Calc	TOTAL
1		/4	/2	**/15**
2	/3	/4		**/16**
3		/5		**/13**
4		/3	/2	**/16**
TOTAL	/3	/16	/4	**/60**

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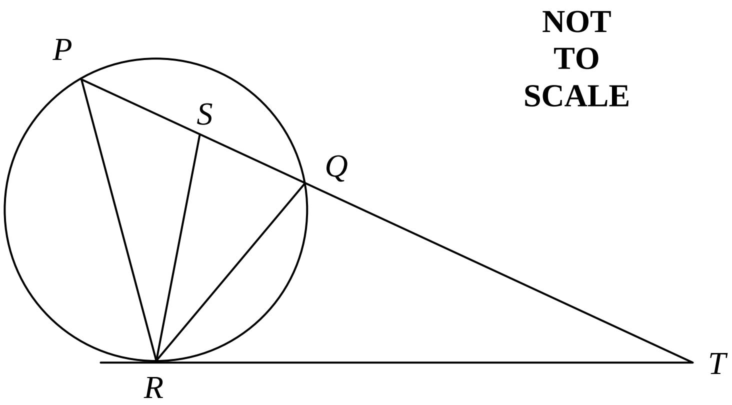
Marks

Question 1 (15 marks)

(a) The polynomial $x^3 + 2x^2 + 1$ is divided by $x + 3$. Calculate the remainder. **2**

(b) The interval *AB*, where *A* is $(4, 5)$ and *B* is $(19, -5)$ is divided internally in the ratio $2:3$ by the point $P(x, y)$. Find the values of *x* and *y*. **2**

(c)



In the diagram ΔPQR is drawn in a circle. The tangent to the circle at *R* meets *PQ* produced to *T*. *S* is a point on *PQ* such that *RS* bisects $\angle QRP$.

Copy the diagram.

(i) Explain why $\angle TRQ = \angle RPT$. **1**

(ii) Show that $TR = TS$, giving clear reasons. **3**

(d) A monic polynomial $P(x)$ of degree 4 has one root equal to 2 and a double root equal to -1. **2**

If the polynomial passes through the point $(0, 6)$ find the equation of $P(x)$.

Question 1 continues on the next page

Marks

Question 1 (continued)

(e) Let $f(x) = 3x^2 - 2x$. **2**

Use the definition $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

to find the derivative by first principles of $f(x)$ at the point $x = a$.

(f) By making an appropriate substitution, or otherwise, solve for *x*. **3**

$$2\left(x + \frac{1}{x}\right)^2 + \left(x + \frac{1}{x}\right) - 15 = 0$$

• Start a new page

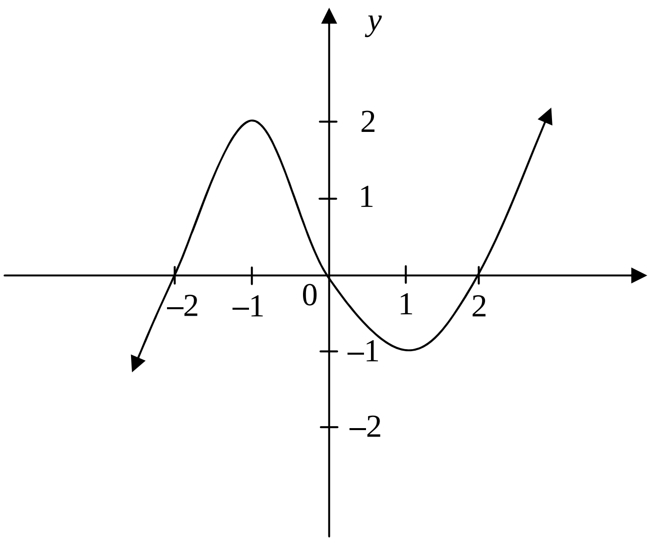
Marks

Question 2 (16 marks)

(a) Solve for θ, $0° \le \theta \le 360°$ **3**

$$\sin 2\theta = \sin\theta$$

(b) The curve $y = f(x)$ is shown below.



On the separate answer page attached, draw the graph of:

(i) $y = |\, f(x)\,|$ **1**

(ii) $y = f(x+2)$ **1**

(iii) $y = f(x) - 1$ **1**

Question 2 continues on the next page

Marks

Question 2 (continued)

(c) The polynomial $P(x) = 2x^3 + 3x^2 + kx - 2$ has roots α, β and γ.

(i) Find the value of $\alpha + \beta + \gamma$. **1**

(ii) Find the value of $\alpha\beta\gamma$. **1**

(iii) If one root is the reciprocal of the other, find the third root and hence find the value of *k*. **2**

(d) (i) Express $\sin x + \cos x$ in the form $R\cos(x - \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. **2**

(ii) Hence solve for $0° \le x \le 360°$ **2**

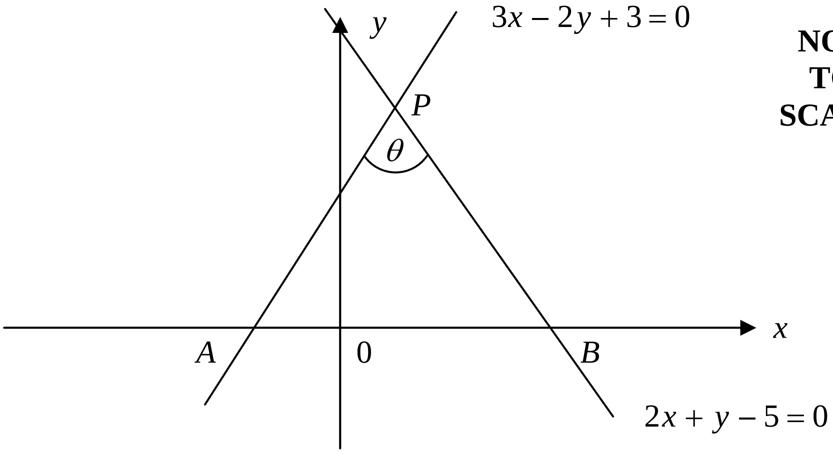
$$\sin x + \cos x = 1$$

(iii) Find all possible solutions of $\sin x + \cos x = 1$. **2**

Marks

Question 3 (13 marks)

(a)



The diagram shows the lines $3x - 2y + 3 = 0$ and $2x + y - 5 = 0$ intersecting at point P. The lines cut the x and y axes at A and B respectively.

(i) Find the co-ordinates of P. **2**

(ii) Find the size of the acute angle, θ, between the lines $3x - 2y + 3 = 0$ and $2x + y - 5 = 0$. Answer correct to the nearest minute. **3**

(b) (i) By using the substitution $t = \tan\frac{x}{2}$, show that **2**

$\sin x - 3\cos x - 2$

can be written as $\dfrac{t^2 + 2t - 5}{1 + t^2}$.

(ii) Hence, find correct to the nearest minute **3**

the values of x, $0° \le x \le 360°$ for which

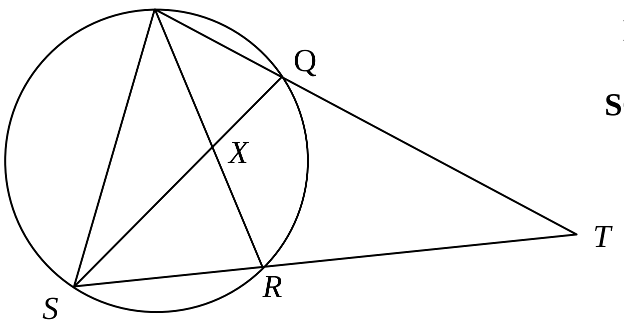
$\sin x - 3\cos x - 2 = 0$.

Question 3 continues on the next page

Marks

Question 3 (continued)

(c)



The points P, Q, R, S are placed on a circle of radius r such that PR and QS meet at X.
The lines PQ and SR are produced to meet at T, and $QXRT$ is a cyclic quadrilateral.

Copy or trace this diagram onto your answer page.

(i) Find the size of $\angle SQT$, giving reasons for your answer. **2**

(ii) Find an expression for the length of PS in terms of r. **1**

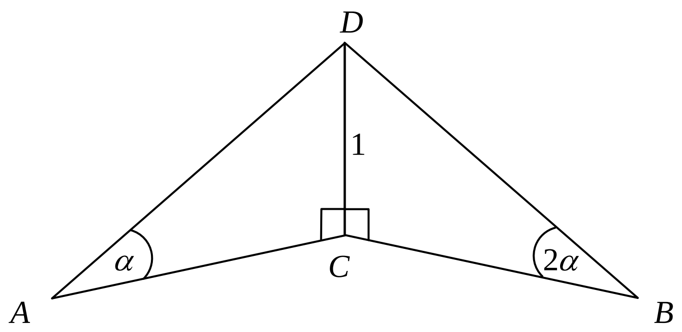
Marks

Question 4 (16 marks)

(a) Solve for x. **3**

$$\frac{3}{2x-1} \geq x$$

(b)



In the diagram CD is a vertical flagpole of height 1 metre which stands on horizontal ground.
The angles of elevation of the top D of the flagpole from points A and B on the ground are α and 2α respectively.

(i) Show that $BC = \dfrac{1-\tan^2\alpha}{2\tan\alpha}$. **1**

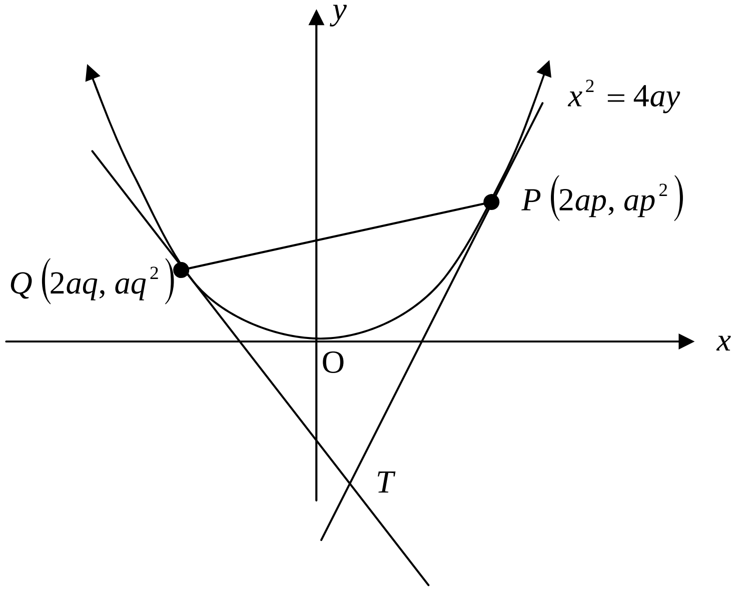
(ii) Show that $AC - BC = BD$. **3**

Question 4 continues on the next page

Marks

Question 4 (continued)

(c) $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ are two points on the parabola $x^2 = 4ay$.
The tangents at P and Q intersect at T.



(i) Show that the tangent at P is given by $y = px - ap^2$. **2**

(ii) Show that the tangents intersect at the point $T\left(a\left(p+q\right), apq\right)$. **2**

(iii) Show that the chord PQ has equation $y = \frac{1}{2}\left(p+q\right)x - apq$. **2**

(iv) The line PQ is a tangent to the parabola $x^2 = 2ay$. **2**

Show that $\left(p+q\right)^2 = 8pq$.

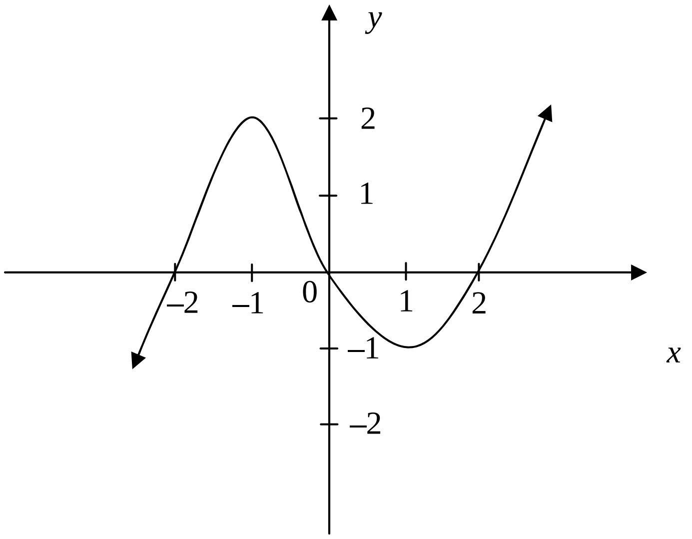
(v) Show that the Cartesian equation of the locus of T is a parabola. **1**

End of paper

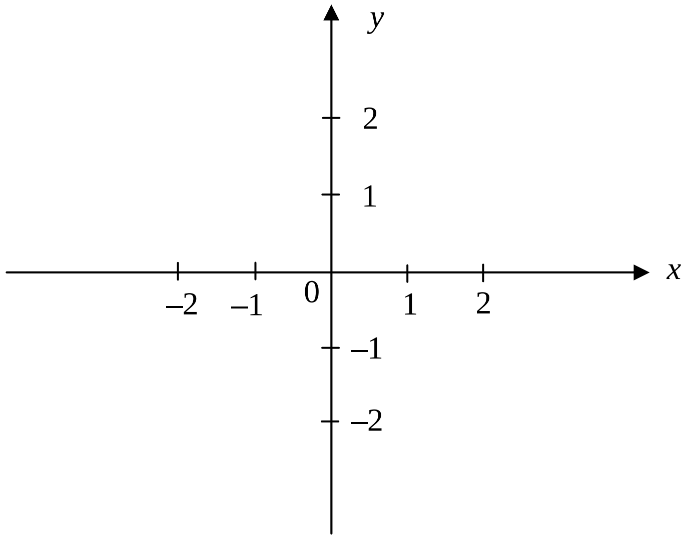
Student Number

Use this page to answer Question 2 (b)

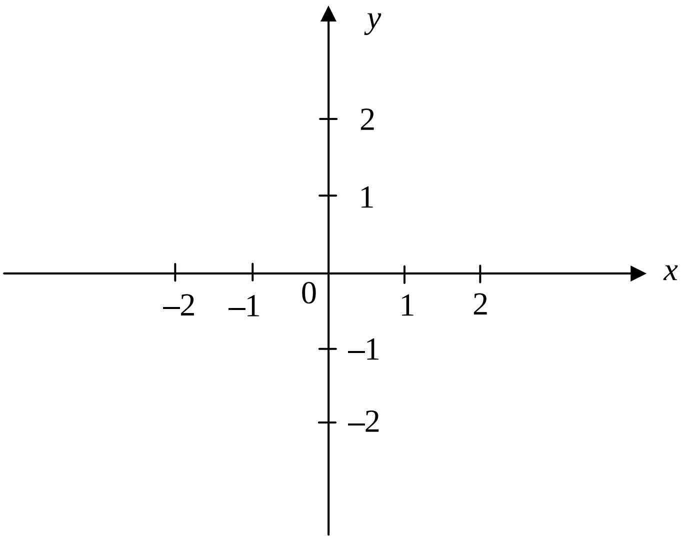
$y = f(x)$



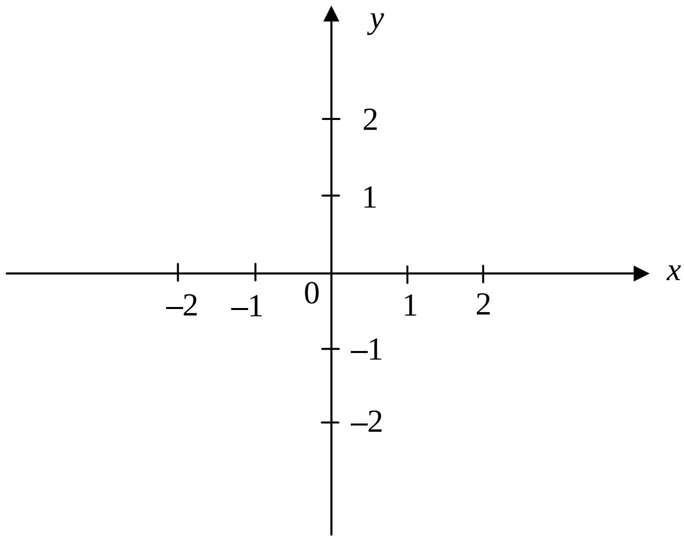
(i) $y = |f(x)|$



(ii) $y = f(x + 2)$



(iii) $y = f(x) - 1$



Year 11 Extension 1 Mathematics

Preliminary Semester 2 Exam 2009

	Comments

Question 1

a) $P(x) = x^3 + 2x^2 + 1$

Remainder

$P(-3) = (-3)^3 + 2(-3)^2 + 1$ ✓

$= -27 + 18 + 1$ ✓

$= -8$

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b) $A(4,5)$ $B(19,-5)$

$2:3$ Internal division

$$P = \left(\frac{3\times4 + 2\times19}{2+3}, \frac{3\times5 + 2\times-5}{2+3} \right)$$

$$= \left(\frac{50}{5}, \frac{5}{5} \right)$$

$$= (10, 1)$$ ✓✓

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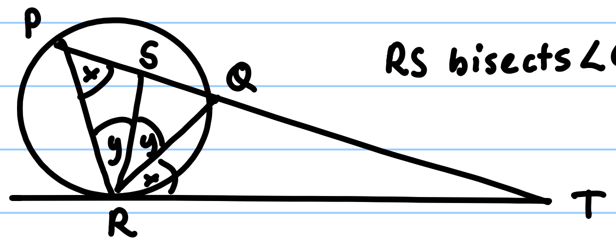
c) 

RS bisects $\angle QRP$

i) $\angle TRQ = \angle TPR$

(Angle between a tangent and a chord is equal to the angle in the alternate segment.) ✓

ii) Let $\angle TRQ = \angle TPR = x$

Let $\angle QRS = \angle SRP = y$ (equal angles since RS bisects $\angle QRP$)

$\angle TSR = \angle SPR + \angle SRP$
$= x + y$ (exterior angle of a triangle is equal to sum of opposite interior angles) ✓

$\angle SRT = x + y$ ✓

$\therefore$ $TR = TS$ (sides opposite equal angles are equal.) ✓ (Reas 4)

Question1 continued.

d)

$$P(x) = (x-2)(x+1)^2(x+k)$$

monic
coefficient of x^4 is one

must put extra factor because of degree 4

passes through $(0,6)$

$6 = 2\times(-1)^2\times k$

$-2k = 6$

$k = -3$

e)

$f(x) = 3x^2 - 2x$

$f(a) = 3a^2 - 2a$

$f(a+h) = 3(a+h)^2 - 2(a+h)$

$= 3(a^2+2ah+h^2) - 2a - 2h$

$= 3a^2 + 6ah + 3h^2 - 2a - 2h$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{3a^2+6ah+3h^2-2a-2h-(3a^2-2a)}{h}$$ ✓

$$= \lim_{h\to 0} \frac{6ah+3h^2-2h}{h}$$

$$= \lim_{h\to 0} \frac{h(6a+3h-2)}{h}$$

$$= \lim_{h\to 0} (6a+3h-2)$$

$= 6a - 2$ ✓

Question 1 (continued)

f) $2\left(x+\frac{1}{x}\right)^2 + \left(x+\frac{1}{x}\right) - 15 = 0$

Let $m = x + \frac{1}{x}$

$2m^2 + m - 15 = 0$

$(2m - 5)(m + 3) = 0$

$m = \frac{5}{2}$

$x + \frac{1}{x} = \frac{5}{2}$

$x^2 + 1 = \frac{5}{2}x$

$2x^2 + 2 = 5x$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2}, 2$

$m = -3$

$x + \frac{1}{x} = -3$

$x^2 + 1 = -3x$

$x^2 + 3x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{9 - 4.1.1}}{2}$

$= \frac{-3 \pm \sqrt{5}}{2}$

Question 2

a) $\sin 2x = \sin x$

$2\sin x\cos x - \sin x = 0$

$\sin x\,(2\cos x - 1) = 0$

$\sin x = 0$

$x = 0^\circ, 180^\circ, 360^\circ$ ✓

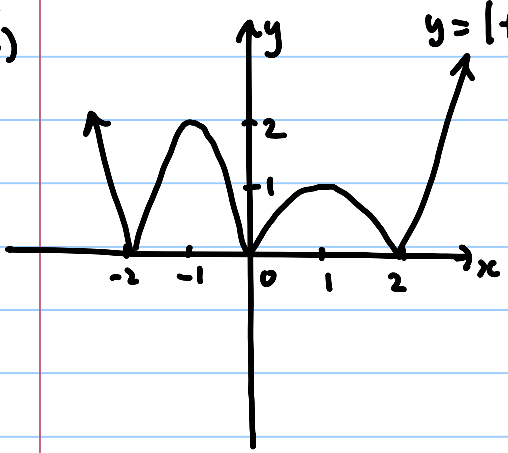
$\cos x = \frac{1}{2}$ ✓

quad 1 & 4

$x = 60^\circ, 300^\circ$ ✓

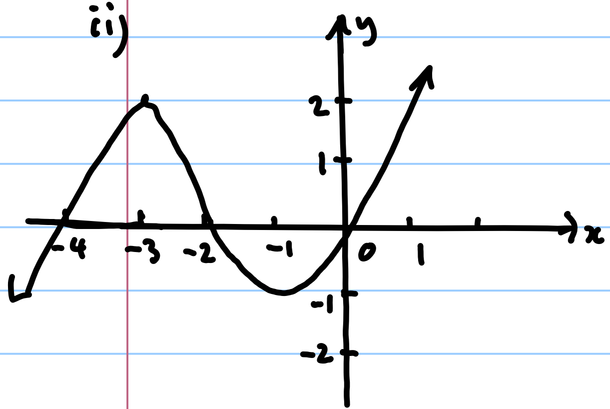
Many divided this line by $\sin x$, doing this eliminates one of the resulting equations. You must factorise then solve each equation. Also remember that 360° is in the domain

b) i)



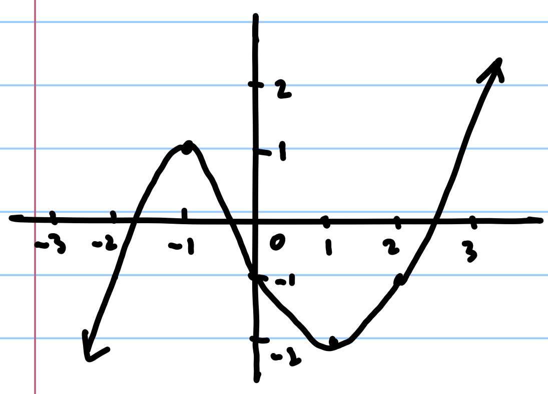
These were well done

ii)



$y = f(x+2)$

Shift left 2 units.



$y = f(x) - 1$

Shift down 1 unit

If your curve is shifted down then the intercepts will also change

Question 2 (continued)

c) $P(x) = 2x^3 + 3x^2 + kx - 2$

i) $\alpha + \beta + \gamma = -\frac{b}{a}$
$= -\frac{3}{2}$

Parts (i) and (ii) were well done.

ii) $\alpha\beta\gamma = -\frac{d}{a}$
$= \frac{2}{2}$
$= 1$

iii) One root is reciprocal $\Rightarrow$ Roots are $\alpha, \frac{1}{\alpha}, \gamma$

Read the question carefully it says one root is the reciprocal of the other, not the negative reciprocal

from part ii) R 2

$\alpha\beta\gamma = 1$
$\alpha . \frac{1}{\alpha} \gamma = 1$
$\gamma = 1$ ✓

Since this is a root of the polynomial $P(1) = 0$

$2 \times (1)^3 + 3 \times (1)^2 + k \times 1 - 2 = 0$
$2 + 3 + k - 2 = 0$
$k + 3 = 0$
$k = -3$ ✓

Question 2 (continued)

d) i)

$\sin x + \cos x = R\cos(x - \alpha)$

$= R(\cos x\cos\alpha + \sin x\sin\alpha)$

$= R\cos x\cos\alpha + R\sin x\sin\alpha$

Part (i) well done

Match the coefficients

$R\cos\alpha = 1$ ①

$R\sin\alpha = 1$ ②

Find R

$R^2 = 1^2 + 1^2$

$R = \sqrt{1+1}$

$= \sqrt{2}$ ✓

Find α

②/①

$\tan\alpha = 1$

$\alpha = 45°$ ✓

$\therefore \sin x + \cos x = \sqrt{2}\cos(x - 45°)$

ii) $\sin x + \cos x = 1$

$\sqrt{2}\cos(x - 45°) = 1$

$\cos(x - 45°) = \frac{1}{\sqrt{2}}$

quad 1 & 4

Check domain

$0° \leq x \leq 360°$

$-45° \leq x - 45° \leq 315°$

Remember to check the domain, often a solution is left out if this is not done

$\therefore x - 45° = -45°, 45°, 360° - 45°$

quad 4, quad 1, quad 4

$x = 0°, 90°, 360°$ ✓✓

iii) all solutions → general solution

$\cos(x - 45°) = \frac{1}{\sqrt{2}}$

R ②

$x - 45° = 360°n \pm \cos^{-1}\frac{1}{\sqrt{2}}$

$x = 360°n \pm 45° + 45°$

$\therefore x = 360°n$ or $x = 360°n + 90°$

where $n \in \mathbb{Z}$ ✓

Not well done, all possible solutions means general solution

Two methods

- learn the rule
- look for a pattern

Another Method

$\cos(x - 45°) = \frac{1}{\sqrt{2}}$ quadrants 1 or 4

in Q1: $x - 45° = 45°,\ 360° + 45°,\ 2\times 360° + 45° \ldots$

$x = 90°,\ 360° + 90°,\ 2\times 360° + 90°$

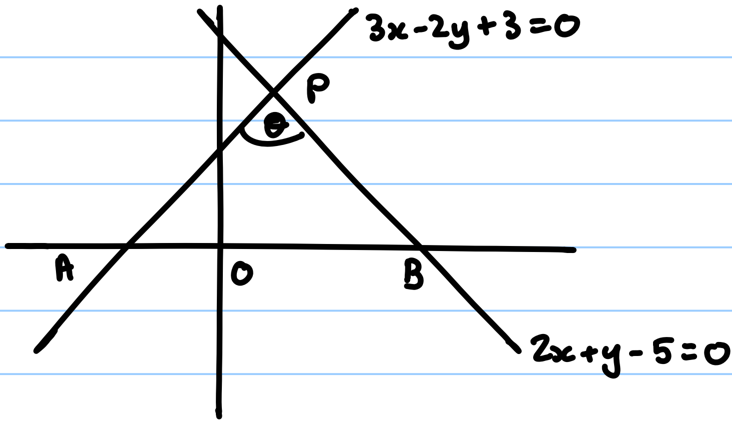
Q4: $x - 45° = 360 - 45°,\ 2\times 360 - 45°,\ \ldots$

$x = 360°,\ 2\times 360°,\ \ldots$

$\therefore\ x = 360°n + 90°$ or $x = 360°n$

Question 3

a)



i) $3x-2y+3=0$ ①

$2x+y-5=0$ ②

②×2 $4x+2y-10=0$ ③

$3x-2y+3=0$ ①

Add ① & ③ $7x-7=0$

$7x=7$

$x=1$

subst. into ① $3-2y+3=0$

$2y=6$

$y=3$ $\therefore P(1,3)$

ii) $3x-2y+3=0$

$m_1=-\frac{3}{-2}$

$=\frac{3}{2}$

$2x+y-5=0$

$m_2=-\frac{2}{1}$

$=-2$

$ax+by+c=0$
To find gradient without making y the subject
$m=-\frac{a}{b}$

Acute angle $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$=\left|\frac{\frac{3}{2}--2}{1+\frac{3}{2}\times-2}\right|$

$=\left|\frac{3\frac{1}{2}}{-2}\right|$

$=\left|-\frac{7}{4}\right|$

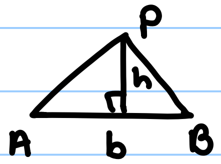
$\tan\theta=\frac{7}{4}$

$\theta=60°15'$

Reas 2

Hence Use Area $= \frac{1}{2}ab\sin C$

or otherwise



use $A = \frac{1}{2}bh$

You need to find points A, B and some lengths.

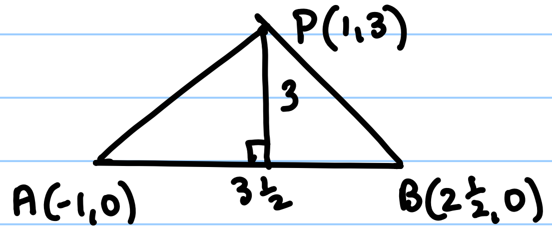
Point A put $y=0$ into $3x-2y+3=0$

$$3x+3=0$$
$$3x=-3$$
$$x=-1 \qquad A(-1,0)$$

Point B put $y=0$ into $2x+y-5=0$

$$2x-5=0$$
$$2x=5$$
$$x=2\tfrac{1}{2} \qquad B(2\tfrac{1}{2},0)$$

or otherwise is less work!



By counting horizontally and vertically

base $= 3\frac{1}{2}$

perpendicular height $= 3$ units

$$\therefore \text{ Area } \triangle APB = \tfrac{1}{2} \times 3\tfrac{1}{2} \times 3$$
$$= 5.25 \quad \text{units}^2$$

Question 3 (continued)

b) i) $t = \tan\frac{x}{2}$

$\sin x = \frac{2t}{1+t^2}$ $\quad \cos x = \frac{1-t^2}{1+t^2}$

$\sin x - 3\cos x - 2$

$= \frac{2t}{1+t^2} - 3\left(\frac{1-t^2}{1+t^2}\right) - 2$ ✓

$= \frac{2t - 3 + 3t^2 - 2(1+t^2)}{1+t^2}$

$= \frac{2t - 3 + 3t^2 - 2 - 2t^2}{1+t^2}$

$= \frac{t^2 + 2t - 5}{1+t^2}$ ✓

ii) $\sin x - 3\cos x - 2 = 0$

$\frac{t^2 + 2t - 5}{1+t^2} = 0$

$t^2 + 2t - 5 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4 - 4\times1\times-5}}{2}$

$= \frac{-2 \pm \sqrt{24}}{2}$ ✓

$t = \frac{-2+\sqrt{24}}{2}$

$\tan\frac{x}{2} = 1.449...$

Quad 1 & 3

(but Quad 3 angle will be too big when doubled)

$\frac{x}{2} = 55°24'$

$x = 110°48'$ ✓

$t = \frac{-2-\sqrt{24}}{2}$

$\tan\frac{x}{2} = -3.449...$

(Quad 2 and 4 but Quad 4 angle will be too big.)

$\frac{x}{2} = (180° - 73°50')$

$\frac{x}{2} = 106°10'$

$x = 212°20'$ ✓

Check domain

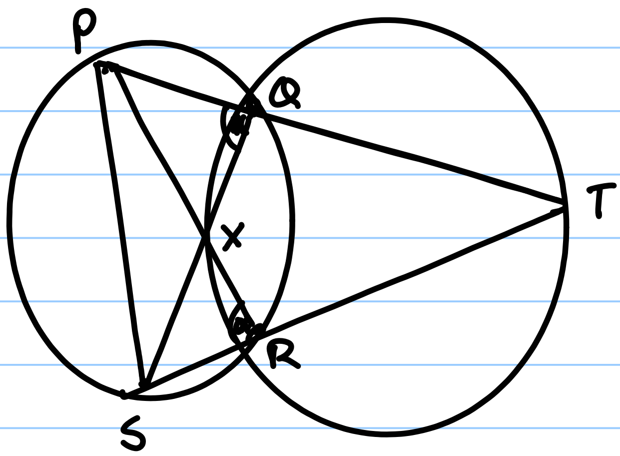
$0° \leq x \leq 360°$

$0° \leq \frac{x}{2} \leq 180°$

So look for answers in quadrants 1 & 2 only.

Question 3 (continued)

Reas 3



i) Let $\angle PQS = \angle PRS = x$
(angles in the same segment are equal)

$\angle XQT = 180° - \angle PQS = 180° - x$
$\angle TRP = 180° - \angle PRS = 180° - x$
(Angle sum of a straight line is 180°)
$\angle XQT = \angle TRP$
Since QXRT is a cyclic quadrilateral
$\angle XQT + \angle TRX = 180°$
$\therefore 2\angle XQT = 180°$
$\angle XQT = 90°$
$\therefore \angle SQT = 90°$

ii) Since $\angle SQT = 90°$
$\angle SQP = 90°$
(Angle sum straight line is 180°)
$\therefore$ PS is a diameter of the circle
(Angle subtended by the diameter is 90° at the circumference)
Since r is the radius of the circle
$PS = 2r$

Question 4

a) $\frac{3}{2x-1} \geqslant x$

Not well done

undefined when $x = \frac{1}{2}$ ✓

multiply both sides by $(2x-1)^2$

$$\frac{3}{2x-1} \times (2x-1)^2 \geqslant x(2x-1)^2$$

$$3(2x-1) \geqslant x(2x-1)^2$$

$$3(2x-1) - x(2x-1)^2 \geqslant 0$$

don't expand.
factorise it instead!

most students chose to expand at this point although this can be done it takes a lot more time and effort

Please factorise, it is much more efficient and you will be less likely to make a mistake

$$(2x-1)\left[3 - x(2x-1)\right] \geqslant 0$$

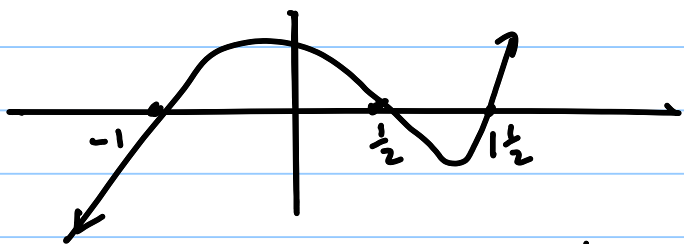
$$(2x-1)(3 - 2x^2 + x) \geqslant 0$$

factorise out -1 and then reverse signs

$$(2x-1) \cdot -1(2x^2 - x - 3) \geqslant 0$$

$$(2x-1)(2x^2 - x - 3) \leq 0$$

$$(2x-1)(2x-3)(x+1) \leq 0$$ ✓



$x \leq -1, \quad -\frac{1}{2} \leq x \leq 1\frac{1}{2}$

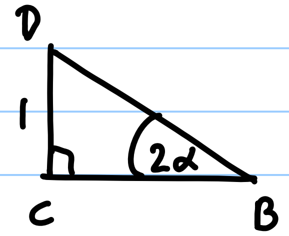
but $x \neq \frac{1}{2}$ $\therefore$ The solution is

$x \leq -1, \quad -\frac{1}{2} < x \leq 1\frac{1}{2}$ ✓

make sure you answer the question and incorporate any undefined values

Question 4 (continued)

b)



$$\tan 2\alpha = \frac{1}{BC}$$

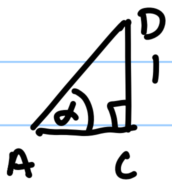
$$BC = \frac{1}{\tan 2\alpha}$$

$$= \frac{1}{\frac{2\tan\alpha}{1-\tan^2\alpha}}$$

$$= \frac{1-\tan^2\alpha}{2\tan\alpha}$$

Part (i) was well done

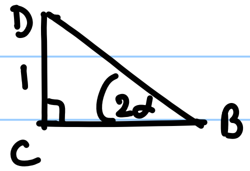
ii)



$$\tan\alpha = \frac{1}{AC}$$

$$AC = \frac{1}{\tan\alpha}$$

R
2



$$\sin 2\alpha = \frac{1}{BD}$$

$$BD = \frac{1}{\sin 2\alpha}$$

This was a tricky question.
You needed to establish an expression for AC and BD in terms of α then set the question up as a trig identity proof.
Problems arose when students wrote an expression for BD using pythagoras - although this can be done it makes the question very difficult.

Question 4 (continued)

Prove that $AC - BC = BD$

$$AC - BC = \frac{1}{\tan\alpha} - \left(\frac{1-\tan^2\alpha}{2\tan\alpha}\right)$$

$$= \frac{2 - 1 + \tan^2\alpha}{2\tan\alpha}$$

$$= \frac{1 + \tan^2\alpha}{2\tan\alpha}$$

you needed to use $1 + \tan^2 x = \sec^2 x$

$$= \frac{\sec^2\alpha}{2\tan\alpha}$$

$$= \frac{\frac{1}{\cos^2\alpha}}{2\frac{\sin\alpha}{\cos\alpha}}$$

$$= \frac{1}{\cos^2\alpha} \times \frac{\cos\alpha}{2\sin\alpha}$$

$$= \frac{1}{2\sin\alpha\cos\alpha}$$

$$= \frac{1}{\sin 2\alpha}$$

$$= BD$$

Question 4 (continued)

i) $x^2 = 4ay$

$y = \frac{x^2}{4a}$

$y' = \frac{2x}{4a}$

$= \frac{x}{2a}$

Calc ②

At $P(2ap, ap^2)$

$m_T = \frac{2ap}{2a}$

$= p$

Eqn tangent

$y - y_1 = m(x - x_1)$

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$ ✓

Parts (i) to (iii) were quite good

* Important note: if a question says show then you must show working eg. don't just write $m_T = p$ you must differentiate and substitute.

ii) Similarly tangent at Q

$y = qx - aq^2$ ②

$y = px - ap^2$ ①

Solve simultaneously

$px - ap^2 = qx - aq^2$

$px - qx = ap^2 - aq^2$

$x(p-q) = a(p^2 - q^2)$

$x = \frac{a(p-q)(p+q)}{p-q}$ ✓

$x = a(p+q)$

Subst. into ① $y = ap(p+q) - ap^2$

$= ap^2 + apq - ap^2$

$= apq$

$T(a(p+q), apq)$ ✓

Question 4 (continued)

iii) $P(2ap, ap^2)$ $Q(2aq, aq^2)$

Gradient PQ

$$m_{PQ} = \frac{aq^2 - ap^2}{2aq - 2ap}$$
$$= \frac{a(q^2-p^2)}{2a(q-p)}$$
$$= \frac{a(q-p)(q+p)}{2a(q-p)}$$
$$= \frac{p+q}{2}$$

Equation of chord PQ

$$y - y_1 = m(x - x_1)$$
$$y - ap^2 = \frac{(p+q)}{2}(x - 2ap)$$
$$y - ap^2 = \tfrac{1}{2}(p+q)x - \tfrac{1}{2}(p+q) \times 2ap$$
$$y - ap^2 = \tfrac{1}{2}(p+q)x - ap^2 - apq$$
$$y = \tfrac{1}{2}(p+q)x - apq$$

Alternate Method

Chord of contact: PQ is a chord of contact from the external point $T(a(p+q), apq)$

$$xx_0 = 2a(y + y_0)$$
$$xa(p+q) = 2a(y + apq)$$
$$x(p+q) = 2y + 2apq$$
$$2y = x(p+q) + 2apq$$
$$y = \tfrac{1}{2}(p+q)x + apq$$

Question 4 (continued)

(iv) $x^2 = 2ay$ ①
$y = \frac{1}{2}(p+q)x - apq$ ②

R
②

This was a tricky question: we have the line PQ and the parabola $x^2=2ay$. Start by solving these simultaneously

Solve simultaneously

$$x^2 = 2a\left(\tfrac{1}{2}(p+q)x - apq\right)$$
$$x^2 = a(p+q)x - 2a^2pq$$

$$x^2 - a(p+q)x + 2a^2pq = 0$$

This is a quadratic equation. Since the line PQ is a <u>tangent</u> to the parabola there is only one solution

* This is essential to solve this question

$\therefore$ Using the discriminant $\Delta = 0$

$$\Delta = b^2 - 4ac$$
$$= (-a(p+q))^2 - 4.1.2a^2pq$$
$$= a^2(p+q)^2 - 8a^2pq$$
$$= a^2\left[(p+q)^2 - 8pq\right]$$

Since $\Delta = 0$

$$a^2\left[(p+q)^2 - 8pq\right] = 0$$

$$(p+q)^2 - 8pq = 0$$
$$(p+q)^2 = 8pq$$

Question 4 (continued)

v) $T(a(p+q), apq)$

$x = a(p+q) \Rightarrow p+q = \frac{x}{a}$

$y = apq \Rightarrow pq = \frac{y}{a}$

You need to eliminate p&q. Use your x and y coordinates to get an expression for p+q and for pq

Using part iv)

$$(p+q)^2 = 8pq$$

$$\left(\frac{x}{a}\right)^2 = 8\frac{y}{a}$$

$$\frac{x^2}{a^2} = \frac{8y}{a}$$

$$x^2 = \frac{8y}{a} \times a^2$$

$$x^2 = 8ay \quad \checkmark$$

Substitute these expressions into the equation from part (v)

The locus of T is a parabola.