SYDNEY BOYS' HIGH

YEAR 11 - YEARLY EXAM

3 UNIT - MATHEMATICS

SEPTEMBER 1991

EXAMINER P.R. BIGELOW

TIME 2 HOURS

INSTRUCTIONS

* Show all necessary working
* Submit answers in 3 separate sections clearly labelled A,B AND C
* Approved calculators may be used

SECTION A (30 Marks)

(1) Solve a) $3x = x^2$

b) $\frac{4}{3x} + \frac{3}{2x} = -\frac{1}{6}$

(2) Find the domain of the following functions

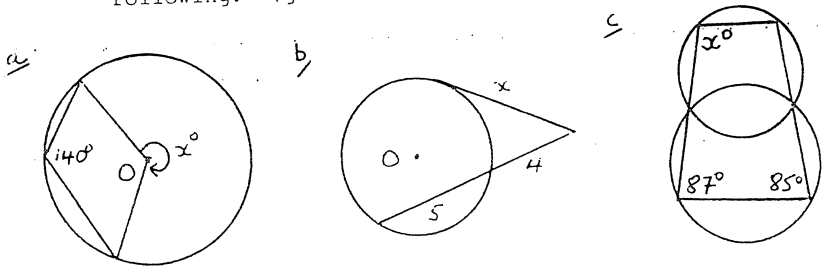
a) $y = \frac{1}{\sqrt{x+1}}$

b) $f(x) = \sqrt{2x+1} - \sqrt{6-x}$

(3) Factorise $3x^2 - 5x - 2$

(4) Find $\lim_{x \to 8} \frac{x^2 - 64}{x-8}$

(5) Find the value of the pronumeral in each of the following. (give a brief reason)



(6) Determine whether the following functions are odd, even or neither

a) $y = 3 - x^2$ b) $y = x^5 - x$ c) $y = 2^x - 1$

(7) Evaluate $\frac{f(x+h) - f(x)}{h}$ when $f(x) = 1-2x^2$

(8) Simplify $x^2 + \frac{1}{x^2}$ where $x = 1 - \sqrt{3}$

(9) Three people were surveyed to determine the day on which their birthday fell in 1988.

Find the probability that

a) all three had their birthday on the same day of the week.

b) at least one had their birthday on a Friday.

(10) Find all real numbers x such that $\frac{x-2}{6-x} \geqslant 1$

(11) Find the co-ordinates of the point $P(x,y)$ which divides the interval AB externally in the ratio 7:2 where A is $(1,-3)$ and B is $(6,7)$

(12) Sketch $\{ (x,y) : y < \sqrt{4-x^2} \}$

SECTION B (30 Marks)

(13) Differentiate with respect to x

a) $2x^3 - 3x - 5$

b) $(11-3x)^{10}$

c) $\frac{x}{1+x}$

d) $x\sqrt{1-x}$

e) $\sqrt[3]{(1+3x)^2}$

(14) P and Q are points on the curve $xy=1$.

The x-values of P and Q are $\frac{1}{1+h}$ and $1+h$ respectively.

Show that the equation of PQ is $x+y = 1+h+\frac{1}{1+h}$

(15) Find the acute angle between the lines
$y-2x$ and $3x + y - 5 = 0$ (answer to nearest minute)

(16) Find the distance between the lines
$x - y + 1 = 0$ and $x - y - 3 = 0$

(17) If $f(x) = 1 + \sqrt{3x-2}$ evaluate $f^1(1)$

(18) The first three terms of an arithmetic series are 50, 43 and 36

a) Write down an expression for the n'th term

b) If the last term of the series is -27
How many terms are there in the series?

c) Find the sum of the series

(19) Sketch the region in the Cartesian plane determined by the points (x,y) satisfying
$|y - 2x| \leq 2$

(20) a) Evaluate $\log_8 0.25$ b) If $\log_a b = 0.36$
find $\log_a \sqrt{a^2b}$

(21) Find the size of each interior angle of a regular 24 - sided polygon.

(22) Solve for x $|x^2 - 5| = 5x + 9$

(23) Simplify $\log 2 + \log 4 + \log 8 + ... + \log 512$

(24) Show that the curves $y=x^2$ and $2y = 2x^2 + 5x + 5$ cut at one point and hence by finding the slopes of the two tangents at the common point, show that the curves intersect at right angles.

SECTION C (40 Marks)

(25) The equation $(x - 3y + 5) + k(x + 2y) = 0$
represents a family of straight lines passing through a fixed point P.

a) For what value of k is one of the lines in the family parallel to the straight line $x + y = 2$?

b) For what value of k does one of the lines in the family pass through the centre of the circle $x^2 + y^2 - 10y + 21 = 0$

c) Find the co-ordinates of P

(26) Find the least positive integral n such that
$(\frac{5}{7})^n < 10^{-4}$

(27) Find the general solution of $\sin 2\theta = \sin\theta$

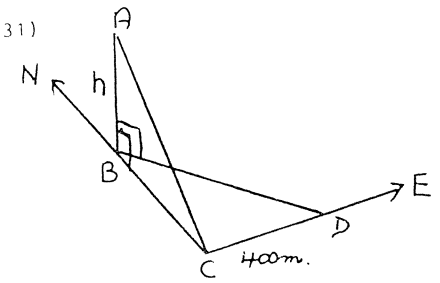
(28) Use the expansion of $R\cos(x+\alpha)$ to find all values of x in the domain $0^o \leq x \leq 360^o$ for which
$4\cos x - 3\sin x = -1$ (give answer to nearest degree)

(29) a) Write down the exact values of $\sin 45^o$, $\cos 45^o$, $\sin 30^o$ and $\cos 30^o$, then find expressions in surd form for $\sin 15^o$ and $\cos 15^o$.

b) Prove that $\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x}$
hence show that $\tan 7\frac{1}{2}^o = (\sqrt{3} - \sqrt{2})(\sqrt{2} - 1)$

(30) Find $\lim_{x \to -4} \frac{\sqrt{12 - x} - 4}{x + 4}$

(31)



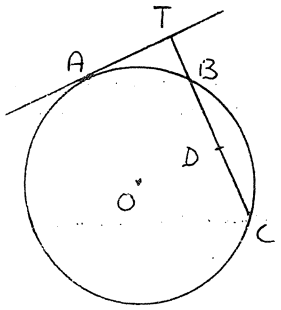
In the sketch B,C and D are points on ground level with D 400 metres due East of C and B is due North of C.

At C, the angle of elevation of A is 10°

If h metres is the height of the mast show that $h = 400\ (\cot^2 10^{\circ} - \cot^2 12^{\circ})^{-\frac{1}{2}}$. Hence find the height of the mast to the nearest metre.

(32) If $\sum_{r=0}^{\infty} 2^{rx} = 2$, find the value of x

(33)



A, B & C are points on the circumference of a circle centre 0.

The tangent at A meets CB produced at T.

D is the mid-point of BC

(i) Copy the diagram

(ii) Prove that ATDO is a cyclic quadrilateral

(iii) Prove that $\angle AOT = \angle ADT$