SYDNEY BOYS HIGH SCHOOL

3 UNIT MATHEMATICS

Year 11 Yearly Examination

September 2000

Time Allowed: 90 minutes

Total Marks: 72

Examiner: Mr R Dowdell

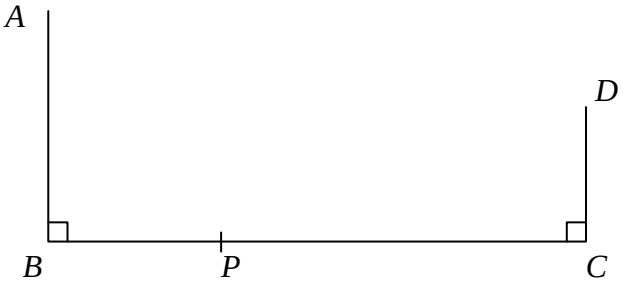
INSTRUCTIONS:

- Attempt *all* questions.
- *All* questions are of equal value.
- All necessary working should be shown in every question. Full marks may not be awarded if work is careless or badly arranged.
- Return your answers in 4 booklets. Each booklet must show your name.
- If required, additional Writing Booklets may be obtained from the Examination Supervisor upon request.

Question 1: (18 marks) Marks

(a) Find the point $P(x, y)$ which divides the interval joining $X(-2, 7)$ and $Y(3, 17)$ internally in the ratio 3:2. 2

(b) 3



AB = 9 cm, BC = 8cm, CD =7 cm

If $AP = PD$, calculate the length of BP.

(c) If $x = 2 + \sin\alpha$ and $y = 4 + 3\cos\alpha$, find a relationship between x and y which does not involve α. 2

(d) For $P(x) = 2x^3 - 7x^2 - 7x + 30$, 4

(i) evaluate $P(3)$;

(ii) evaluate $P(-2)$;

(iii) find all the zeroes of $P(x)$.

(e) If α, β and γ are the roots of $7x^3 + 5x^2 - 11x + 2$, evaluate 7

(i) $\alpha + \beta + \gamma$;

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$;

(iii) $\alpha\beta\gamma$;

(iv) $\alpha^2 + \beta^2 + \gamma^2$;

(v) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$;

(vi) $(\alpha + 1)(\beta + 1)(\gamma + 1)$.

Question 2: (18 marks) START A NEW BOOKLET Marks

(a) Find the acute angle (to the nearest degree) between the lines $y = 5x - 4$ and $y = -x + 3$. 2

(b) If $\tan A$ and $\tan B$ are the roots of the equation $3x^2 - 5x - 1 = 0$, find the value of $\tan(A + B)$. 2

(c) Find the general solution of the equation $\sin 2x = 2\cos^2 x$ 4

(d) A monic cubic polynomial leaves a remainder of $x + 8$ when divided by $x^2 + 4$ and when divided by x leaves a remainder of -4. 4

Find the polynomial in the form $ax^3 + bx^2 + cx + d$.

(e) Solve $\dfrac{x-3}{x^2 - x} \geq -2$. 6

Graph your solution on a number line.

Question 3: (18 marks) START A NEW BOOKLET Marks

(a) Show that $\sin 8\theta \sin 2\theta \equiv \sin^2 5\theta - \sin^2 3\theta$. 4

(b) Solve the equation $x^2 + 2x - 4 + \dfrac{3}{x^2 + 2x} = 0$ 4

(c) If $\cos A = \frac{7}{9}$ and $\sin B = \frac{1}{3}$, $0 \le A \le \frac{\pi}{2}$ and $0 \le B \le \frac{\pi}{2}$, 4

(i) show, without a calculator, that $A = 2B$;

(ii) find the value of $\cos(A + B)$ in simplest surd form.

(d) The elevation of a hill at a place *P* due east of it is 48°, and at a place *Q* due south of *P* the elevation is 30°. If the distance from *P* to *Q* is 500 metres, find the height of the hill (correct to 3 significant figures). 6

Question 4: (18 marks) START A NEW BOOKLET Marks

(a) If $f(x) = \dfrac{\sin(x-\frac{\pi}{4}) + \sin(x+\frac{\pi}{4})}{\cos(x-\frac{\pi}{4}) - \cos(x+\frac{\pi}{4})}$, 6

(i) comment on the value of $f(0)$;

(ii) simplify the expression for $f(x)$;

(iii) sketch $y = f(x)$ for $-2\pi \le x \le 2\pi$.

(b) (i) Simplify the square of $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ and hence state the positive square root of $\dfrac{2+\sqrt{3}}{4}$. 6

(ii) Given that θ is acute and that $\cos\theta = \dfrac{\sqrt{6}-\sqrt{2}}{4}$, find the exact value of $\sin\theta$.

(iii) Hence, or otherwise, evaluate $\sin 2\theta$ and deduce the exact value(s) of θ, expressing your answer in radians.

(c) (i) Show that the distance from (p, q) to the line $y = x$ is given by 6

$$d = \frac{|p-q|}{\sqrt{2}}.$$

(ii) A point $P(x, y)$ moves such that its distance from the line $y = x$ is equal to its distance from the point $A(-2, 2)$.

(α) Show that the equation of the locus of P is $x^2 + 8x + y^2 - 8y + 2xy + 16 = 0$.

(β) What type of curve does this locus represent?

END OF PAPER