SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

SEPTEMBER 2007

Yearly Examination

YEAR 11

Mathematics Extension (Continuers)

General Instructions

- Reading Time – 5 Minutes.
- Working time – 60 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Start each NEW section in a separate answer booklet.

Total Marks − 60

- Attempt questions 1 − 4

Examiner: R. Boros

Total marks 60

Attempt questions 1 to 4

Answer each **Section** in a **Separate** writing booklet

Section A (Use a SEPARATE writing booklet)

Question 1 (14 marks)

(a) Find the acute angle between the lines $y = 2x + 1$ and $y = -x + 1$, correct to the nearest minute. $\boxed{2}$

(b) Consider the polynomial $K(x) = 4x^3 + tx^2 + 2x - 1$. Given that $x + 1$ is a factor of $K(x)$, find the value of t. $\boxed{2}$

(c) The parametric equations of a curve are $x = \dfrac{2}{t}$ and $y = 2t^2$. What is the cartesian equation for the curve? $\boxed{2}$

(d) For the parabola $(x - 3)^2 = 6y + 12$, find the:

i. coordinates of the vertex $\boxed{1}$

ii. coordinates of the focus $\boxed{1}$

iii. equation of the directrix $\boxed{1}$

(e) Find the coordinates of the point Q which divides the interval joining $A(2, -3)$ and $B(-4, 1)$ externally in the ratio $1 : 3$. $\boxed{3}$

(f) Sketch the graph of $y = x^2(x - 2)^3$ without the use of calculus. $\boxed{2}$

Question 2 (14 marks)

(a) Differentiate $f(x) = 5 - x^2$ by using first principles. $\boxed{2}$

(b) Given that $\sin\alpha = \dfrac{7}{25}$ and $\cos\beta = -\dfrac{3}{5}$ where α and β are obtuse angles, find the exact value of:

i. $\sin 2\alpha$ $\boxed{1}$

ii. $\cos(\alpha + \beta)$ $\boxed{2}$

(c) In a class of 30 students, 22 study Chemistry, 18 study Physics and 13 study both Chemistry and Physics. If a student is chosen at random, what is the probability that the student studies Chemistry or Physics? $\boxed{2}$

(d) i. Express $\sin x - \sqrt{3}\cos x$ in the form $A\sin(x - \alpha)$, with $A > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. $\boxed{2}$

ii. Find the solutions to $\sin x - \sqrt{3}\cos x = \dfrac{2}{\sqrt{2}}$ for $0 \le x \le 2\pi$. $\boxed{2}$

(e) Solve the inequation $\dfrac{1}{x+2} \le \dfrac{1}{x+3}$. $\boxed{3}$

Section B (Use a SEPARATE writing booklet)

Question 3 (15 marks)

(a) Solve $\log_{27} 16 = x \log_3 2$. $\boxed{2}$

(b) Prove the trigonometric identity $\dfrac{\cos 2x}{(\cos x + \sin x)^3} = \dfrac{\cos x - \sin x}{1 + \sin 2x}$. $\boxed{3}$

(c) A committee of three is to be chosen from a group of four males and five females. The committee must include at least one male and at least one female. How many different commitees can be formed? $\boxed{2}$

(d) A maths teacher pays \$1000 into a superannuation fund at the beginning of each year. Compound interest is paid at 9%p.a. on the investment.

i. Show that the first \$1000 invested becomes \$20413.97 to the nearest cent after 35 years. $\boxed{1}$

ii. What will be the value of the investment at the end of 35 years? Answer correct to the nearest dollar. $\boxed{3}$

(e) Given that the cubic equation $2x^3 + 6x - 1 = 0$ has real roots α, β and γ. Evaluate:

i. $\alpha^3\beta^3\gamma^2 + \alpha^3\beta^2\gamma^3 + \alpha^2\beta^3\gamma^3$. $\boxed{2}$

ii. $\dfrac{\alpha}{\beta\gamma} + \dfrac{\beta}{\alpha\gamma} + \dfrac{\gamma}{\alpha\beta}$. $\boxed{2}$

Question 4 (17 marks)

(a) The derivative of $x\sqrt{x^2+3}$ is $\dfrac{ax^2+b}{\sqrt{x^2+3}}$, where a and b are constants. Find the value of a and b. $\boxed{3}$

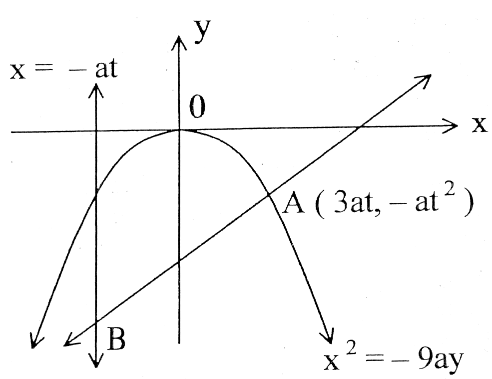
(b) The student council at a local school consists of 4 boys and 2 girls. In how many ways can they sit next to each other around a circular table for a meeting if:

i. there are no restrictions. $\boxed{1}$

ii. the girls are not to sit next to each other. $\boxed{2}$

(c) Find the general solution of the equation $\tan 2\theta = \tan\theta$ in radians. $\boxed{3}$

(d)



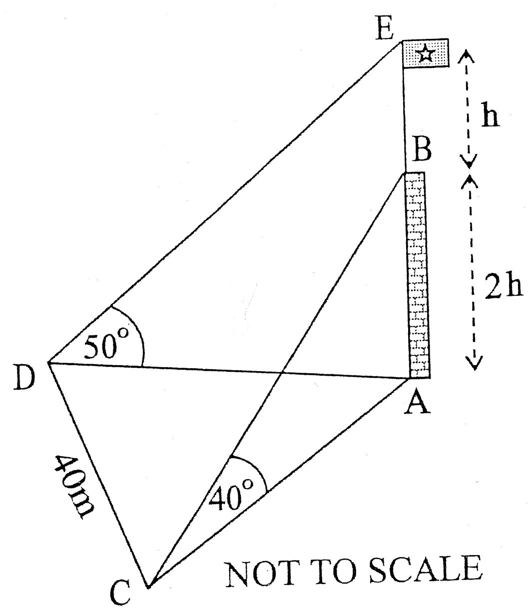
The point $A(3at, -at^2)$ is a variable point on the parabola $x^2 = -9ay$. The normal at A meets the line $x = -at$ at the point B.

i. Show that the equation of the normal to the parabola at A is $\boxed{2}$

$3x - 2ty = 2at^3 + 9at$.

ii. Find the coordinates of B. $\boxed{1}$

(e)



A building AB of height $2h$ metres has a flag pole of height h metres on top of it. From a point C, due south of the building, the angle of elevation of the top of the building is $40°$. From a point D, due west of the building, the angle of elevation of the top of the flagpole is $50°$. The points C and D are on the same level as A and they are 40 metres apart.

i. Find expressions for AC and AD in terms of h. $\boxed{2}$

ii. Show that $h = \dfrac{40}{\sqrt{4\cot^2 40° + 9\cot^2 50°}}$. $\boxed{2}$

iii. Find to the nearest degree, the true bearing of D from C. $\boxed{1}$

End of paper

Section A

Question 1.

(a) $\tan \alpha = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$.

$= \left| \frac{2 - (-1)}{1 + (2)(-1)} \right|$

$= 3$.

$\tan \alpha = 3$

$\alpha = 71^\circ 34'$

(b). $K(-1) = -4 + t - 2 - 1 ; = 0$.

$t = 7$.

(c) $x = \frac{2}{t} \Rightarrow t = \frac{2}{x}$.

So

$y = 2\left(\frac{2}{x}\right)^2$

$y = \frac{8}{x^2}$.

(d) $(x-3)^2 = 4\left(\frac{3}{2}\right)(y+2)$.

(i) Vertex $(3, -2)$.

(ii) Focus $(3, -\frac{1}{2})$.

(iii) Directrix $y = -\frac{7}{2}$ or $-3\frac{1}{2}$

(e) $\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n} \right)$

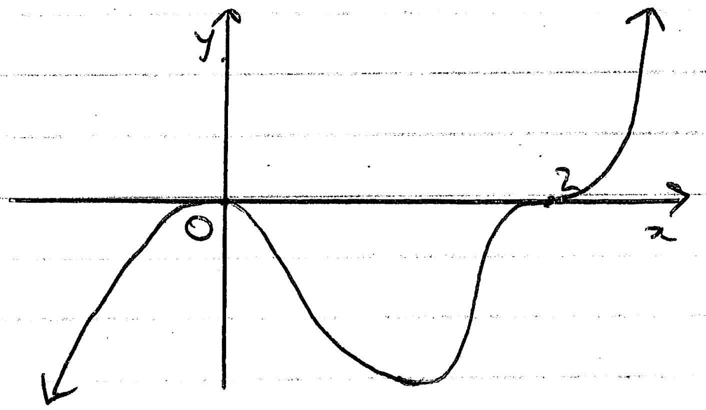
External division $m : -n$

$\left(\frac{-3\times 2 + 1\times -4}{1-3}, \frac{-3\times -3 + 1\times 1}{1-3} \right)$

$= \left(\frac{6-4}{-2}, \frac{9+1}{-2} \right)$

$= (-1, -5)$.

(f)



QUESTION 2.

(a)
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5-(x+h)^2 - (5-x^2)}{h}$$

$$= \lim_{h \to 0} \frac{5-x^2-2xh-h^2-5+x^2}{h}$$

$$= \lim_{h \to 0} -2x+h.$$

$$= -2x.$$

(b) $\sin\alpha = \frac{7}{25} \Rightarrow \cos\alpha = -\frac{24}{25}.$

$\cos\beta = -\frac{3}{5} \Rightarrow \sin\beta = \frac{4}{5}.$

(i) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$= 2 \times \frac{7}{25} \times -\frac{24}{25}$$

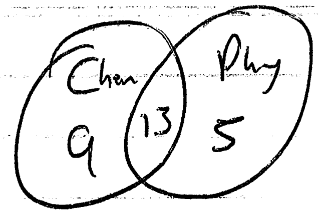
$$= -\frac{336}{625}.$$

(ii) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$

$$= -\frac{24}{25} \times -\frac{3}{5} - \frac{7}{25} \times \frac{4}{5}$$

$$= \frac{44}{125}.$$

(c)



$$\frac{27}{30} = \frac{9}{10}.$$

(d)(i) $A\sin(x-\alpha) = A\cos\alpha\sin x - A\sin\alpha\cos x.$

So $A\cos\alpha = 1$
$A\sin\alpha = \sqrt{3}.$

Thus $\tan\alpha = \sqrt{3}.$ and $A^2\cos^2\alpha + A^2\sin^2\alpha = 1+3.$
$\alpha = \frac{\pi}{3}$ $A^2 = 4$
$A = 2.$

$\sin x - \sqrt{3}\cos x \equiv 2\sin\left(x - \frac{\pi}{3}\right).$

(ii). $2\sin\left(x-\frac{\pi}{3}\right) = \frac{2}{\sqrt{2}}$

$$\sin\left(x-\frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

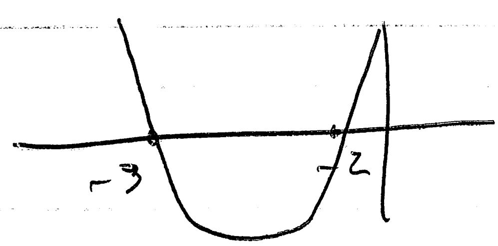
$$x - \frac{\pi}{3} = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$x = \frac{7\pi}{12}, \frac{13\pi}{12}.$$

(e) $\frac{1}{x+2} - \frac{1}{x+3} \leq 0.$

$$\frac{(x+3)-(x+2)}{(x+3)(x+2)} \leq 0$$

$$\frac{1}{(x+3)(x+2)} \leq 0.$$



$$(x+3)(x+2) \leq 0.$$

$$-3 \leq x \leq -2.$$

QUESTION. 3

a) $\log_{27} 16 = x \log_3 2$.

$LHS = \log_{27} 16$.

$= \dfrac{\log_3 16}{\log_3 27}$.

$= \dfrac{\log_3 2^4}{3}$.

$= \dfrac{4}{3} \log_3 2$.

$\therefore \quad x = \dfrac{4}{3}$.

b) $\dfrac{\cos 2x}{(\cos x + \sin x)^3} = \dfrac{\cos x - \sin x}{1 + \sin 2x}$.

$LHS = \dfrac{\cos 2x}{(\cos x + \sin x)^3}$

$= \dfrac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^3}$

$\boxed{= \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^3}}$

$= \dfrac{\cos x - \sin x}{(\cos x + \sin x)^2}$.

$= \dfrac{\cos x - \sin x}{\cos^2 x + 2\sin x \cos x + \sin^2 x}$

b) continued.

$= \dfrac{\cos x - \sin x}{1 + \sin 2x}$.

$= RHS$

c) 2 women 1 man $= {}^5C_2 \times {}^4C_1$.

+ 2 men 1 woman $= {}^4C_2 \times {}^5C_1$.

$= 10 \times 4 + 6 \times 5$

$= \underline{\underline{70 \text{ ways}}}$

d)

i) $\underline{\underline{1000(1.09)^{35} = 20413.97}}$

ii) 1^{st} yr $= 1000(1.09)$

2^{nd} Year $= 1000(1.09) + (1000)(1.09)^2$.

3rd Yr $= 1000(1.09) + 1000(1.09)^2 + 1000(1.09)^3$.

35^{th} yr $= 1000(1.09 + 1.09^2 + \dots + 1.09^{35})$

$= 1000\left(\dfrac{a(r^n - 1)}{r - 1}\right)$ $\quad a = 1.09$. $\quad r = 1.09$ $\quad n = 35$.

$= 1000\left(\dfrac{1.09(1.09^{35} - 1)}{1.09 - 1}\right)$.

$= \underline{\underline{\$235,124.72}}$

QUESTION 3 (CONTINUED).

e) $2x^3 + 0x^2 + 6x - 1 = 0$

i) $\alpha^3 \times \beta^3 \times \gamma^2 + \alpha^3\beta^2\gamma^3 + \alpha^2\beta^3\gamma^3$

$= \alpha^2\beta^2\gamma^2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= (\alpha\beta\gamma)^2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$\alpha\beta\gamma = \dfrac{-d}{a} = \dfrac{1}{2}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \dfrac{c}{a} = \dfrac{6}{2} = 3$

$\therefore (\alpha\beta\gamma)^2 \times (\alpha\beta + \alpha\gamma + \beta\gamma) = \left(\frac{1}{2}\right)^2 \times 3$

$= \dfrac{3}{4}$

ii) $\dfrac{\alpha}{\beta\gamma} + \dfrac{\beta}{\alpha\gamma} + \dfrac{\gamma}{\alpha\beta}$

$= \dfrac{\alpha^2 + \beta^2 + \gamma^2}{\alpha\beta\gamma}$

$= \dfrac{(\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)}{\alpha\beta\gamma}$

$\Rightarrow \alpha + \beta + \gamma = \dfrac{-b}{a} = 0$

$\alpha\beta + \beta\gamma + \alpha\gamma = \dfrac{c}{a} = 3$

$\alpha\beta\gamma = \dfrac{-d}{a} = \dfrac{1}{2}$

e) ii) CONTINUED

$= \dfrac{0 - 3}{\frac{1}{2}}$

$= -6$

B:3 2007 YR11 - Yearly - Continuers - SECTION B.

QUESTION 4.

a) Let $y = x\sqrt{x^2+3}$.

$\frac{dy}{dx} = vu' + uv'$

where $u = x$ $\quad u' = 1$.

$v = (x^2+3)^{\frac{1}{2}}$. $\quad v' = \frac{1}{2}(2x)(x^2+3)^{-\frac{1}{2}}$.

$\frac{dy}{dx} = (x^2+3)^{\frac{1}{2}}(1) + (x)(x)(x^2+3)^{-\frac{1}{2}}$

$= \sqrt{x^2+3} + \frac{x^2}{\sqrt{x^2+3}}$

$= \frac{x^2+3+x^2}{\sqrt{x^2+3}}$

$= \frac{2x^2+3}{\sqrt{x^2+3}}$

$= \frac{ax^2+b}{\sqrt{x^2+3}}$

$a = 2 \quad b = 3$.

b) i) No of ways around a table

$= (n-1)!$

$= 5!$

$= 120$ ways

b) (continued).

ii) $\underline{1}$ $\underline{{}^4C_1}$ $\underline{{}^3C_1}$ $\underline{{}^3C_1}$ $\underline{{}^2C_1}$ $\underline{1}$

↑ fix.

$= 1 \times 4 \times 3 \times 3 \times 2 \times 1$.

$= 72$ ways.

c) $\tan 2\theta = \tan\theta$.

$\frac{2\tan\theta}{1-\tan^2\theta} = \tan\theta$.

$2\tan\theta = \tan\theta - \tan^3\theta$

$\tan^3\theta + \tan\theta = 0$

$\tan\theta.(\tan^2\theta + 1) = 0$.

either:

$\tan^2\theta + 1 = 0$ which can't happen so

$\tan\theta = 0$.

thus this happens when

$\sin\theta = 0$.

$\therefore$ General solution is.

$\theta = n\pi \quad n \in \mathbb{Z}$.

QUESTION 4 (continued)

d) i) $x = -at$ $A(3at, -at^2)$

$x^2 = -9ay$

$y =$

gradient of tangent

$\frac{dy}{dx} = \frac{2x}{-9a} = m_1$

gradient of normal is.

$m_2 = \frac{-1}{m_1}$

$= \frac{9a}{2x}$ @ $(3at, -at^2)$

$= \frac{9a}{2(3at)}$

Point gradient formula.

$y - y_1 = m(x - x_1)$

$y + at^2 = \frac{9a}{2(3at)}(x - 3at)$

$6aty + 6a^2t^3 = 9ax - 27a^2t$

$6ty + 6at^3 = 9x - 27at$

$2ty + 2at^3 = 3x - 9at$

$3x - 2ty = 2at^3 + 9at$

d) ii)

$3x - 2ty = 2at^3 + 9at$

at $x = -at$.

$-3(at) - 2ty = 2at^3 + 9at$

$-2ty = 2at^3 + 9at + 3at$

$y = \frac{-2at^{\cancel{3}2} - 12a\cancel{t}}{2\cancel{t}}$

$y = -at^2 - 6a$.

Co-ordinates of B

$(-at, -at^2 - 6a)$

4 e) i) AC ⇒

$\tan 40 = \frac{2h}{AC}$

$AC = \frac{2h}{\tan 40}$

AD ⇒

$\tan 50 = \frac{3h}{AD}$

$AD = \frac{3h}{\tan 50}$

3:5) 2007 Yr11 Yearly - Continuers - SECTION B.

QUESTION 4 (CONTINUED)

e) continued.

ii). By Pythagoras.

$$40^2 = \left(\frac{2h}{\tan 40}\right)^2 + \left(\frac{3h}{\tan 50}\right)^2$$

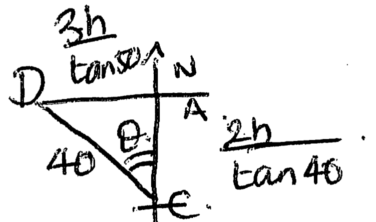
$$40^2 = \frac{4h^2}{\tan^2 40} + \frac{9h^2}{\tan^2 50}.$$

$$\boxed{40^2 = h^2(4\cot^2 40 + 9\cot^2 50)}$$

$$h^2 = \frac{40^2}{4\cot^2 40 + 9\cot^2 50}$$

$$h = \frac{40}{\sqrt{4\cot^2 40 + 9\cot^2 50}}$$

f)



$$\tan\theta = \frac{3h}{\tan 50} \quad \frac{2h}{\tan 40}$$

$$= \frac{3h}{\tan 50} \times \frac{\tan 40}{2h}.$$

$$= \frac{3}{2}\,\frac{\tan 40}{\tan 50}$$

$\tan\theta = 1.0561.$

$\theta = 46.56.$

$\therefore$ Bearing of D from C is $360^\circ - 46.56^\circ$

$= 313^\circ$ (nearest degree)