SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2010
Year 11 Yearly Examination

Mathematics Extension Continuers
(3 Unit)

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question.
- All answers to be given in simplified exact form unless otherwise stated.
- Hand in your answers in 3 separate bundles:

  Section A (Question 1 and Question 2),

  Section B (Question 3 and Question 4) and

  Section C (Question 5 and Question 6)

Total Marks – 82

- Attempt questions 1-6
- All questions are **NOT** of equal value.

Examiner: *P. Bigelow*

Section A – Start a new booklet.

Question 1 (14 marks). **Marks**

a) Solve $x^2 + 2x - 8 = 0$. 1

b) Find T_{10} of $5 + 9 + 13 + 17 + ...$ 1

c) If $f(y) = 9 - y^2$, find: 2

(i) $f(-2)$

(ii) $f(y+1)$

d) State the domain of $f(x) = \sqrt{3-x}$. 1

e) Write down the equation of the parabola with focus (0,5) and directrix $y + 5 = 0$. 1

f) Find x in the following: 2

(i) $\log_x 36 = 2$

(ii) $\log_8 128 = x$

g) Find the exact value of $\tan 15^\circ$. 2

h) Solve: 4

(i) $|3x - 1| > 7$

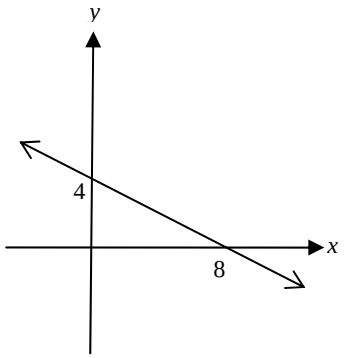
(ii) $\frac{4}{x} \leq \frac{3}{x+1}$

End of Question 1

Question 2 (12 Marks). **Marks**

a) Find the sum of the first 9 terms of $2-1+\frac{1}{2}-\frac{1}{4}+...$ 1

b) Write down the equation of the line represented in the diagram: 2



c) State whether the following functions are ODD, EVEN or NEITHER: 3

(i) $f(x)=\frac{4}{9+x^2}$

(ii) $f(x)=\frac{x}{9+x}$

(iii) $f(x)=\frac{-x}{9+x^2}$

d) Solve $2^{3x+2}=64$. 2

e) By considering $0.\dot{5}\dot{4}$ as an infinite geometric series, express $0.\dot{5}\dot{4}$ as a fraction in simplest form. 2

f) Simplify: 2

(i) $\sin 5A\cos 2A-\cos 5A\sin 2A$

(ii) $2\sin 3\theta\cos 3\theta$

End of Question 2

End of Section A

Section B – Start a new booklet.

Question 3 (11 marks). **Marks**

a) Sketch the following on separate diagrams (showing essential features): 4

(i) $xy = -4$

(ii) $y = 2^{-x}$

(iii) $y = \sqrt{4 - x^2}$

(iv) $x^2 + y^2 - 6y = 0$

b) State whether the following quadratics are INDEFINITE, POSITIVE DEFINITE or NEGATIVE DEFINITE: 3

(i) $2x^2 + 3x + 7$

(ii) $6 - x - x^2$

(iii) $x^2 - 9x - 8$

c) Solve: $4^x - 9(2)^x + 8 = 0$. 2

d) An interval *PQ* is divided externally in the ratio 4:3 by the point *S*. Find *S*, if *P* is (4,3) and *Q* is (-1,9). 2

End of Question 3

Question 4 (16 marks). **Marks**

a) Graph the region defined by the intersection of $y \geq x^2$ and $x + y \leq 2$. 2

b) Find the acute angle between the lines: $2x - y - 5 = 0$ and $x - 3y + 3 = 0$. 2

c) Find $f'(x)$ in the following: 6

(i) $f(x) = 3x^2 - x + 1$

(ii) $f(x) = (1 - 5x)^6$

(iii) $f(x) = x\sqrt{1+x}$

(iv) $f(x) = \dfrac{3x-1}{3x+2}$

d) If $\sin x = \dfrac{3}{4}$ and $\dfrac{\pi}{2} \leq x \leq \pi$, find the exact value of $\sin 2x$. 2

e) If α and β are the roots of $2x^2 - 6x - 1 = 0$, find: 4

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

(iv) $\alpha^2 + \beta^2$

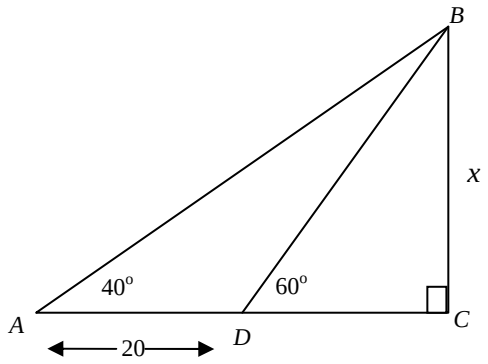
End of Question 4

End of Section B

Section C – Start a new booklet.

Question 5 (15 marks). **Marks**

a) Find the value of x correct to 3 significant figures. 3



b) Sketch $(x-2)^2 = 8(y+1)$, showing vertex, focus and directrix. 2

c) (i) Express $\sin\theta + \cos\theta$ in the form $R\sin(\theta+\alpha)$ where $R>0$ and $0^\circ < \alpha < 90^\circ$. 4

(ii) Hence, solve $\sin\theta + \cos\theta = 1$ for $0 < \theta < 2\pi$.

d) Find the general solution of $\tan x = 1$ 2

e) (i) Find the axis of symmetry of $y = 4 + x - x^2$ 2

(ii) Hence, or otherwise, find the maximum value of $4 + x - x^2$ 2

End of Question 5

Question 6 (14 marks). **Marks**

a) Prove the following: 4

(i) $\cos^4 x - \sin^4 x = \cos 2x$

(ii) $\dfrac{\sin 2A}{1-\cos 2A} = \cot A$

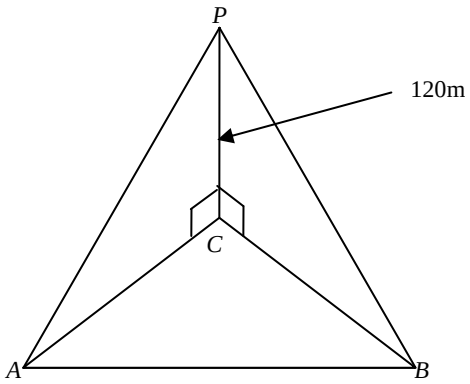
b) Given $5^x = 13$, find x correct to two decimal places. 2

c) A couple wish to save for a deposit on a home. They need to save \$20,000 over a 5 year period. They deposit \$*P*, every month, into an account which is paying 9%p.a., compounding monthly. 4

(i) Show that $20000 = P\left(1.0075 + 1.0075^2 + ... + 1.0075^{60}\right)$.

(ii) Find P to the nearest dollar.

d) Two boats at A and B are observed from the top P of a vertical cliff CP of height 120 metres. A is on a bearing of 195°T from the cliff and its angle of depression from P is 22°. B is on a bearing of 161°T from the cliff and its angle of depression from P is 27°. 4



(i) Find $\angle ACB$.

(ii) Use the cosine rule to find the distance between the boats (to the nearest metre).

End of Question 6.

End of Section C.

End of Examination.

Mathematics Extension Continuers 2010 – Section A:

Question 1:

a) $x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, \quad 2$

b) $a = 5,\ d = 4$

$T_{10} = 5 + 9 \times 4$

$T_{10} = 41$

c) $f(y) = 9 - y^2$

(i) $f(-2) = 9 - (-2)^2$

$f(-2) = 5$

(ii) $f(y+1) = 9 - (y+1)^2$

$= 9 - (y^2 + 2y + 1)$

$= 9 - y^2 - 2y - 1$

$= 8 - 2y - y^2$

d) $f(x) = \sqrt{3-x}$

Domain: $x \leq 3$

e) $x^2 = 20y$

f)

(i) $log_x 36 = 2$

$36 = x^2$

$x = 6$

(ii) $\log_8 128 = x$

$\log_8 8^{7/3} = x$

$\frac{7}{3}\log_8 8 = x$

$x = \frac{7}{3}$

g) $\tan 15° = \tan(45° - 30°)$

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$

$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= 2 - \sqrt{3}$

Question 2:

a) $2-1+\frac{1}{2}-\frac{1}{4}+\cdots$

$a=2$

$r=-\frac{1}{2}$

$$S_9=\frac{2\left(1-\left(-\frac{1}{2}\right)^9\right)}{1+\frac{1}{2}}$$

$$S_9=\frac{171}{128}$$

b) $m=-\frac{1}{2}$

$y-0=-\frac{1}{2}(x-8)$

$2y=-x+8$

$x+2y-8=0$

c)

(i) $f(x)=\frac{4}{9+x^2}$

$$f(-x)=\frac{4}{9+(-x)^2}$$

$$=\frac{4}{9+x^2}$$

Since $f(x)=f(-x)$

$\therefore f(x)$ is even

(ii) $f(x)=\frac{x}{9+x}$

$$f(-x)=\frac{-x}{9-x}$$

$$-f(x)=-\frac{x}{9+x}$$

Since $f(x)\neq f(-x)\neq -f(x)$

$\therefore f(x)$ is neither.

(iii) $f(x)=\frac{-x}{9+x^2}$

$$f(-x)=\frac{-(-x)}{9+(-x)^2}$$

$$=\frac{x}{9+x^2}$$

$$-f(x)=-\frac{-x}{9+x^2}$$

$$=\frac{x}{9+x^2}$$

Since $f(-x)=-f(x)$

$\therefore f(x)$ is odd.

d) $2^{3x+2}=64$

$2^{3x+2}=2^6$

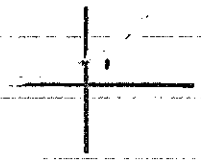
$3x+2=6$

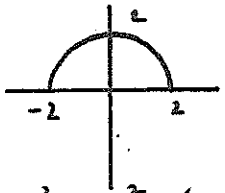
$$x=\frac{4}{3}$$

QUESTION THREE

a)

i) 

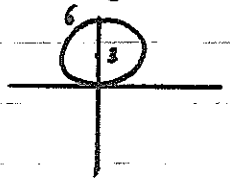
ii) 

iii) 

iv) $x^2 + y^2 - 6y = 0$

$x^2 + y^2 - 6y + 9 = 9$

$x^2 + (y-3)^2 = 9$



b) $y = 2x^2 + 3x + 7$, $\Delta = -47$

i) Positive Definite

ii) $y = -x^2 - x + 6$, $\Delta = 25$

Indefinite

iii) $y = x^2 - 9x - 8$, $\Delta = 113$

Indefinite

c) $4^x - 9(2^x) + 8 = 0$

$(2^x)^2 - 9(2^x) + 8 = 0$

let $u = 2^x$

$u^2 - 9u + 8 = 0$

$(u-8)(u-1) = 0$

$u = 8, 1$

$x = 3, 0$

d) $P(4,3)$ $Q(-1,9)$

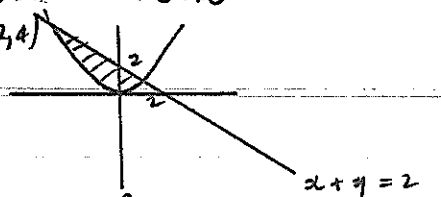
$m:n = 4:-3$

$X = \frac{-3\times4 + 4\times-1}{1}$, $Y = \frac{-3\times3 + 4\times9}{1}$

$= -16$ $= 27$

$(X, Y) = (-16, 27)$

QUESTION FOUR

a) 

$y \geq x^2$, $x + y \leq 2$

b) $y = 2x - 5$ $\tan A = 2$

$y = \frac{1}{3}x + 1$ $\tan B = \frac{1}{3}$

$\tan(A-B) = \frac{2 - \frac{1}{3}}{1 + \frac{2}{3}} = 1$

$A - B = 45^\circ = \frac{\pi}{4}$

c) i) $y = 3x^2 - x + 1$ $y' = 6x - 1$

ii) $y = (1-5x)^6$ $y' = -30(1-5x)^5$

iii) $y = x(1+x)^{1/2}$, $y' = x \cdot \frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}$

$y' = \frac{x}{2\sqrt{1+x}} + \sqrt{1+x}$

$= \frac{x + 2(1+x)}{2\sqrt{1+x}} = \frac{3x+2}{2\sqrt{1+x}}$

iv) $y = \frac{3x-1}{3x+2}$, $y' = \frac{(3x+2)3 - 3(3x-1)}{(3x+2)^2}$

$= \frac{9}{(3x+2)^2}$

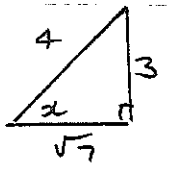
d) $\sin x = \frac{3}{4}$ $\frac{\pi}{2} \leq x \leq \pi$

$x \approx 131^\circ$, $2x \approx 263^\circ$ Hence $\sin 2x < 0$

$\sin 2x = 2\sin x \cos x$

$= 2 \times \frac{3}{4} \times \frac{-\sqrt{7}}{4}$

$= -\frac{3\sqrt{7}}{8}$



e) $2x^2 - 6x - 1 = 0$, $\alpha + \beta = 3$, $\alpha\beta = -\frac{1}{2}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{-\frac{1}{2}} = -6$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 9 - 2(-\frac{1}{2})$

$= 10$.

SECTION C

Q5

(a) $\tan 60° = \frac{x}{DC}$.

$DC = \frac{x}{\tan 60°}$

$\tan 40° = \frac{x}{20+DC}$.

So $(20+DC) = \frac{x}{\tan 40°}$.

$20 + \frac{x}{\tan 60°} = \frac{x}{\tan 40°}$

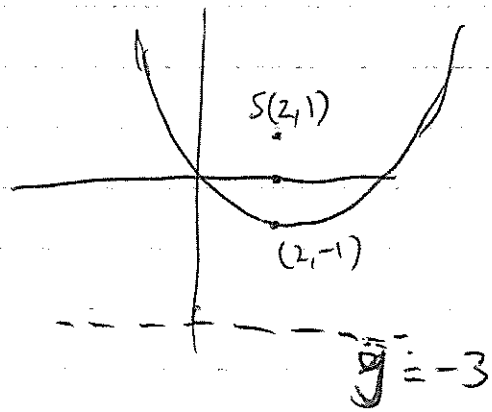
$20 = x\left(\frac{1}{\tan 40°} - \frac{1}{\tan 60°}\right)$.

$x = \frac{20}{\frac{1}{\tan 40°} - \frac{1}{\tan 60°}}$.

$= \frac{20(\tan 40 \tan 60)}{\tan 60 - \tan 40}$

$= 32.6$.

(b)



(c)(i) $R\sin(\theta+\alpha) = R\sin\theta\cos\alpha + R\sin\alpha\cos\theta$

$R\sin\alpha = 1$

$R\cos\alpha = 1$

$\tan\alpha = 1.$

$\alpha = 45°.$

$R^2 = 2$

$R = \sqrt{2}.$

$\sqrt{2}\sin(\theta+45°).$

(ii) $\sqrt{2}\sin(\theta+45°) = 1$

$\sin(\theta+45°) = \frac{1}{\sqrt{2}}.$

~~$\theta+45° = 135°$~~)

$\theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$)

$\theta = \frac{\pi}{2}.$

(d) $\tan x = 1$

$x = \frac{\pi}{4} + \pi n \qquad n \in \mathbb{Z}$

(e) (i) $y = 4 + x - x^2$.

$$x = \frac{-b}{2a}.$$

$$x = \frac{-1}{-2}.$$

$$x = \frac{1}{2}.$$

(ii) $4 + \frac{1}{2} - \frac{1}{4}$.

$= 4\frac{3}{4}$.

Q6 (a) (i) $\cos^4 x - \sin^4 x = (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$.

$= \cos^2 x - \sin^2 x$

$= \cos 2x$.

(ii) $\frac{\sin 2A}{1 - \cos 2A} = \frac{2\sin A \cos A}{\cos^2 A + \sin^2 A - \cos^2 A + \sin^2 A}$.

$= \frac{2\sin A \cos A}{2\sin^2 A}$

$= \frac{\cos A}{\sin A}$

$= \cot A$.

(b) $5^x = 13$.

$x = \frac{\log 13}{\log 5}$.

$= 1.60$.

(c) (i) $A_1 = P \times 1.0075$.

$A_2 = (A_1 + P)\,1.0075$.

$= P \times 1.0075^2 + P \times 1.0075$

$A_3 = P \times 1.0075^3 + P \times 1.0075^2 + P \times 1.0075$.

$A_{60} = P(1.0075^{60} + 1.0075^{59} + \ldots + 1.0075)$.

Since $A_{60} = 20000$.

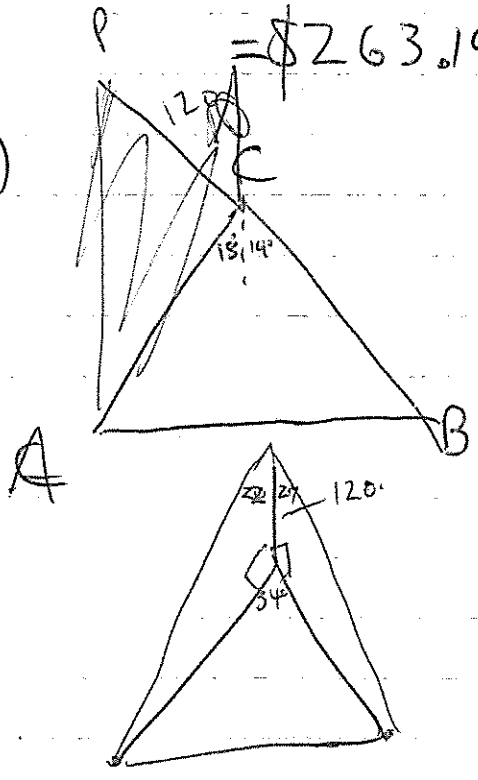
Then $20000 = P(1.0075 + \ldots + 1.0075^{60})$.

(ii) $20000 = P\left(\dfrac{1.0075(1-(1.0075)^{60})}{1-1.0075}\right)$

$$P = \frac{20000\,(-0.0075)}{1.0075\,(1-1.0075^{60})}.$$

$P = \$263.19$.

(d)



(i) 34°.

(ii) $\tan 22 = \dfrac{AC}{120} \Rightarrow AC = 120\tan 22$

$\tan 27 = \dfrac{BC}{120} \Rightarrow BC = 120\tan 27$.

$AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \cos 34$.

$AB = 34.26$.