SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2012
YEAR 11 Mathematics Yearly

Mathematics Extension Continuers

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Section is to be returned in a separate bundle.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.
- Answer must be given in simplest exact form.

Total Marks – 70

- Attempt questions 1-15

Examiner: *P. Bigelow*

Section I (10 marks)

Answer this section on the Multiple Choice Answer Sheet

(1) Evaluate $\lim_{a \to 4} \frac{a^2-16}{a+4}$

(A) 0

(B) 8

(C) –4

(D) 4

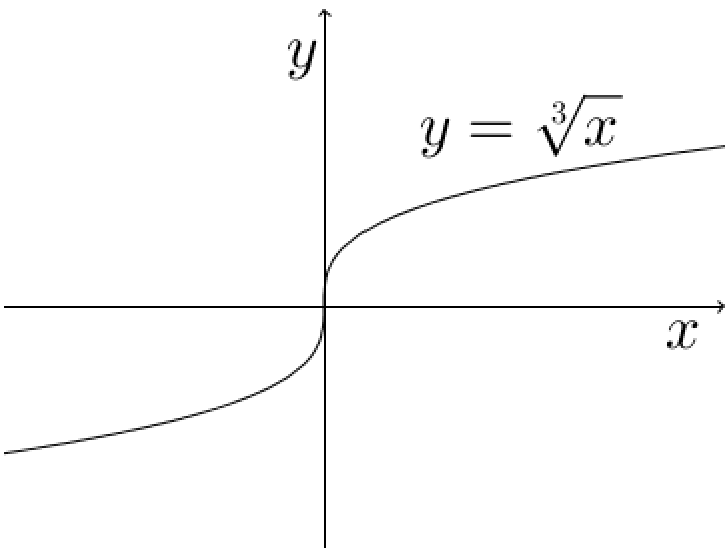
(2) At the origin:

(A) $y'' > 0$

(B) $y'' < 0$

(C) $y'' = 0$

(D) y'' is undefined



(3) A pair of dice are rolled, the probability the sum of the uppermost faces being greater than 7 is:

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{5}{12}$

(D) $\frac{1}{3}$

(4) The full solution to $\frac{4}{x} > 1$ is:

(A) $x < 4$

(B) $x > 4$

(C) $0 < x < 4$

(D) $x < 0, x > 4$

(5) Which of the following is <u>not</u> equal to $\cos 2\theta$

(A) $\cos^2\theta - \sin^2\theta$

(B) $1 - 2\sin^2\theta$

(C) $2\cos^2\theta - 1$

(D) $1 - 2\cos^2\theta$

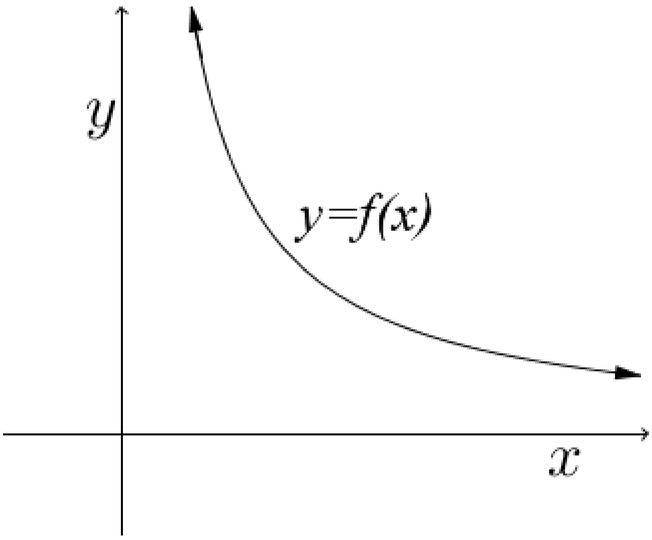
(6) Which is true for $y = f(x)$?

(A) $f'(x) < 0,\ f''(x) > 0$

(B) $f'(x) < 0,\ f''(x) < 0$

(C) $f'(x) > 0,\ f''(x) > 0$

(D) $f'(x) > 0,\ f''(x) < 0$



(7) $3^x \times 2^x =$

(A) 5^x

(B) 5^{2x}

(C) 6^{2x}

(D) 6^x

(8) If $\log_5 x = 4$ then x is:

(A) 20

(B) 25

(C) 625

(D) 125

(9) A coin is tossed three times. The probability of getting 2 Heads and a Tail is

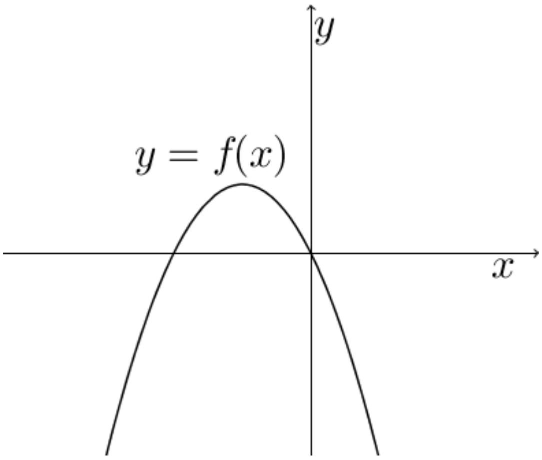
(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

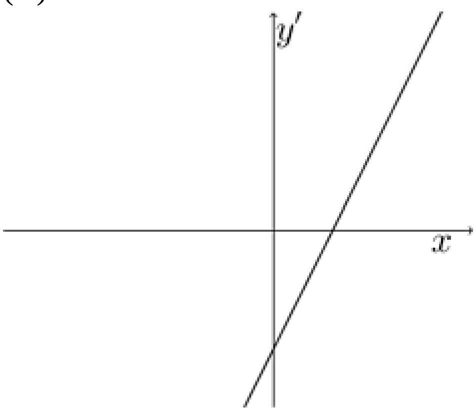
(C) $\frac{3}{8}$

(D) $\frac{5}{8}$

(10) Given the graph of $y = f(x)$ which is a possible sketch of $y = f'(x)$



(A)



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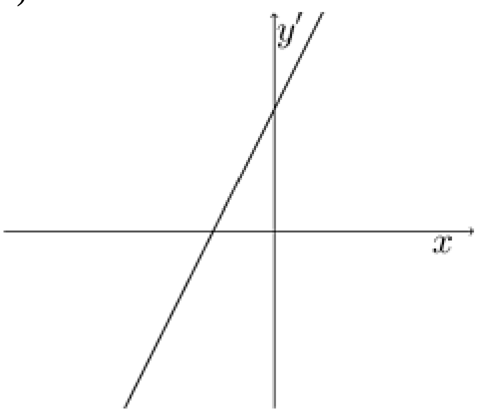
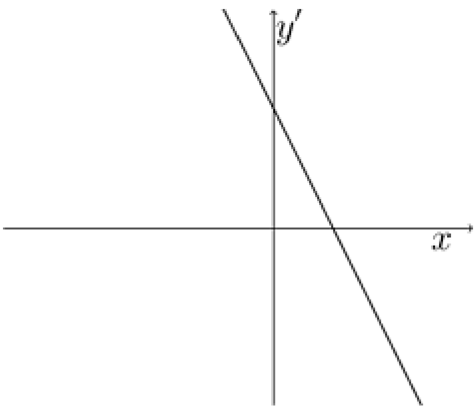
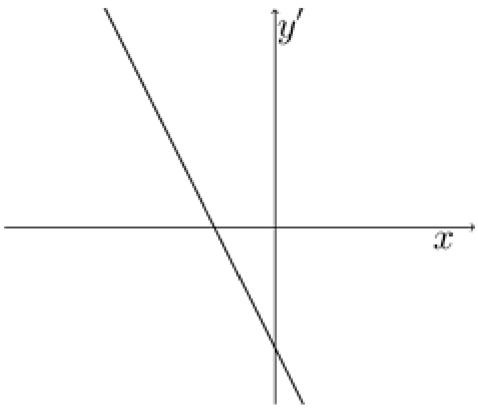


(C)



(D)



Section II (60 marks)

Answer each Question in a new Writing Booklet

Question 11 **[12 marks]**

(a) Simplify: $\sin 70° \cos 40° + \sin 40° \cos 70°$ **[2]**

(b) Expand $\cos(3A - 2B)$ **[2]**

(c) From an urn containing 4 white and 5 brown balls, two balls are selected without replacement. Find the probability of selecting **[6]**

(i) two white balls.

(ii) two different colours.

(iii) at least one brown.

(d) Simplify **[2]**

$$\frac{\sin 2A}{1 + \cos 2A}$$

End of Question 11

Answer each Question in a new Writing Booklet

Question 12 [12 marks]

(a) **[3]**

(i) Show that $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

(ii) Hence find in simplest exact form the value of $\tan\frac{\pi}{12}$.

(b) Simplify **[2]**

(i) $\log_6 3 + \log_6 2$

(ii) $\log_5 100 - \log_5 4$

(c) Solve for x **[2]**

$$5^{3x-4} = 25^{x-2}$$

(d) If $f'(x) = 3x^2 + 2x + 4$ and $f(1) = 7$, find $f(-1)$. **[3]**

(e) If $f(x) = x^2 + 3x$ find $f'(x)$ from first principles. **[2]**

End of Question 12

Answer each Question in a new Writing Booklet

Question 13 [12 marks]

(a) Solve the following inequations and plot the solutions on separate number lines. **[4]**

(i) $\frac{3}{|2x-1|} > 1$

(ii) $\frac{4}{2x-1} \leq \frac{1}{x+2}$

(b) Find $\frac{dy}{dx}$ for the following **[8]**

(i) $y = \sqrt{7 + x^2}$

(ii) $y = x\sqrt{2x - 1}$

(iii) $y = \sqrt[3]{x^2} + \sqrt{x^3}$

(iv) $y = \frac{5x}{4+x^2}$

End of Question 13

Answer each Question in a new Writing Booklet

Question 14 [12 marks]

(a) Find **[2]**

$$\lim_{x\to\infty} \frac{4x+3x^2}{5x-2x^2}$$

(b) If **[2]**

$$f(x) = x^2 + \frac{1}{x^2}$$

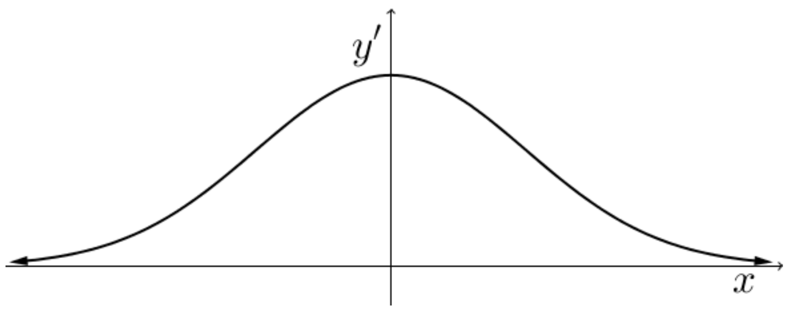
find

(i) $f'(x)$

(ii) $f''(x)$

(c) For what values of x is $f(x) = x^3 - 3x^2 - 7x + 10$ concave up? **[2]**

(d) Given that the graph below is the gradient function of $y = f(x)$.
Sketch $y = f(x)$. **[2]**



(e) If $\tan A = \frac{3}{4}$ and $\cos B = \frac{5}{13}$, where A and B are acute. Find: **[4]**

(i) $\sin 2A$.

(ii) $\cos(A - B)$.

End of Question 14

Answer each Question in a new Writing Booklet

Question 15 [12 marks]

(a) **[4]**

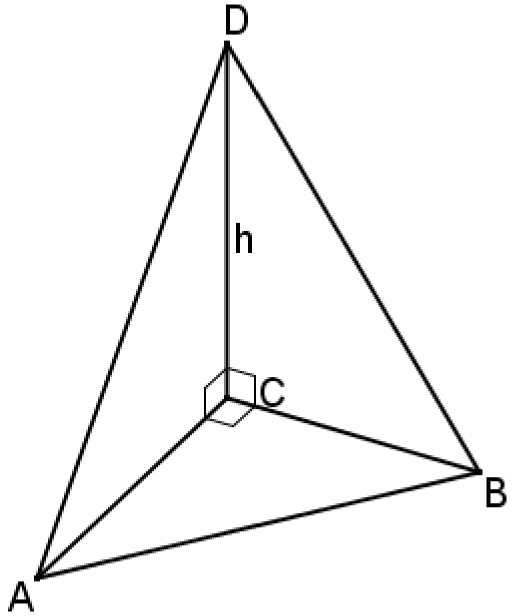
(i) If $\theta + \varphi = 45°$ show that

$$\tan\theta + \tan\varphi = 1 - \tan\theta\tan\varphi$$

(ii) By letting $\theta = \varphi$ show that $t^2 + 2t - 1 = 0$ where $t = \tan 22\frac{1}{2}°$.

(iii) Hence find the exact value of $\tan 22\frac{1}{2}°$.

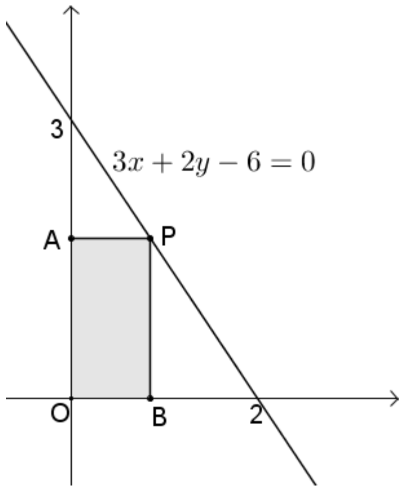
(b) A tower CD is of height *h* metres. From a point A due South of the base the angle of elevation of D is 32°. From B, due East of the base the angle of elevation of D is 28°. Given that A and B are 500 metres apart: find the height *h* to the nearest metre. **[4]**



(c) Two sides of a rectangle OAPB lie on the x and y axes. The vertex opposite the origin lies in the first quadrant and is on **[4]**

$$3x + 2y - 6 = 0$$

Find the maximum area of the rectangle.



End of Exam

Student Number:____________________

Mathematics Extension 2
Trial HSC 2012

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ✗ B ✗ C ○ D ○

Section I: **Multiple choice answer sheet.**

Completely colour the cell representing your answer. Use black pen.

1. ● (B) (C) (D)
2. (A) (B) (C) ●
3. (A) (B) ● (D)
4. (A) (B) ● (D)
5. (A) (B) (C) ●
6. ● (B) (C) (D)
7. (A) (B) (C) ●
8. (A) (B) ● (D)
9. (A) (B) ● (D)
10. (A) (B) (C) ●

Q11 - 2012 YR 11 - CONTINUERS - YEARLY.

a) $\sin 70 \cos 40 + \sin 40 \cos 70$

$= \sin(70+40)$ 2

$= \underline{\underline{\sin(110)}}$.

b) $\cos(3A-2B)$.

$= \cos 3A \cos 2B$ 2

$+ \sin 3A \sin 2B$.

$— = 1\frac{1}{2}$.

c) 4W 5B

i) $P(2 \text{ white}) = \frac{1}{6}$. 2

ii) $P(2 \text{ difft}) = P(WB) + P(BW)$.

$= \left(\frac{4}{9} \times \frac{5}{8}\right) + \left(\frac{5}{9} \times \frac{4}{8}\right)$

$= \frac{5}{9}$ 2

iii) $P(\text{at least 1 brown})$

$= 1 - P(\text{no brown})$

$= 1 - \frac{1}{6}$.

$= \frac{5}{6}$. 2

d) $\frac{\sin 2A}{1 + \cos 2A}$.

$= \frac{2 \sin A \cos A}{1 + [2\cos^2 A - 1]}$.

$= \frac{2 \sin A \cancel{\cos A}}{2\cos^{\cancel{2}} A}$.

$= \frac{\cancel{2} \sin A}{\cancel{2} \cos A}$.

$= \underline{\underline{\tan A}}$ 20

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12 (a) (i) show $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$

RHS $\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi - 2\pi}{12} = \frac{\pi}{12}$ (1)

(ii) $\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$$= \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$$ rationalized

$$\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{3 - 2\sqrt{3} + 1}{3 - 1} = \frac{4 - 2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}(2 - \sqrt{3})}{\cancel{2}}$$

$$= 2 - \sqrt{3}$$ (2)

(12) (b) (i) $\log_6 3 + \log_6 2 = \log_6 (3\times 2)$

$= \log_6 6 = 1$ (1)

(ii) $\log_5 100 - \log_5 4 = \log_5\left(\frac{100}{4}\right)$

$= \log_5 25$

$= \log_5 5^2 = 2\log_5 5 = 2$ (1)

(12) (c) $5^{3x-4} = 25^{x-2}$

$5^{3x-4} = 5^{2(x-2)} \Rightarrow 3x - 4 = 2x - 4$

$x = 0$ (2)

(d) $f'(x) = 3x^2 + 2x + 4$

$f(x) = \frac{3x^3}{3} + \frac{2x^2}{2} + 4x + c$

$f(x) = x^3 + x^2 + 4x + c$

$f(1) = 7 \qquad 7 = 1 + 1 + 4 + c$

$c = 1$

$f(x) = x^3 + x^2 + 4x + 1$ (3)

So $f(-1) = -1 + 1 - 4 + 1 = 2 - 5 = -3$

(e) $f'(x) = \lim\limits_{h\to 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h}$

$\lim\limits_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h - \cancel{x^2} - \cancel{3x}}{h}$

$\lim\limits_{h\to 0} \frac{\cancel{h}(2x + \cancel{h} + 3)}{\cancel{h}}$

$f'(x) = 2x + 3$ (2)

(13) (a) (i) $\frac{3}{|2x-1|} > 1 \quad x \neq \frac{1}{2}$

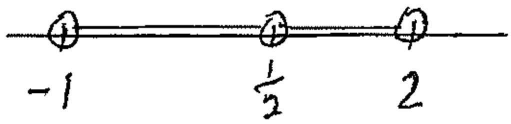
$3 > 1|2x-1|$

$|2x-1| < 3$

$2x-1 < 3 \qquad 2x-1 > -3$

$2x < 4 \qquad 2x > -2$

$x < 2 \qquad x > -1$ (2)



(ii) $\frac{4}{(2x-1)} \leq \frac{1}{(x+2)} \qquad x \neq \frac{1}{2},\ x \neq -2$

$\times (2x-1)^2(x+2)^2 \qquad (2x-1)^{\cancel{2}}(x+2)^2 \times \frac{4}{\cancel{(2x-1)}} \leq \frac{1}{\cancel{(x+2)}} \times (2x-1)^2(x+2)^2$

$4(2x-1)(x+2)^2 \leq (2x-1)^2(x+2)$

$4(2x-1)(x+2)^2 - (2x-1)^2(x+2) \leq 0$

$(2x-1)(x+2)\left[4(x+2) - (2x-1)\right] \leq 0$

$(2x-1)(x+2)\left[4x+8-2x+1\right] \leq 0$

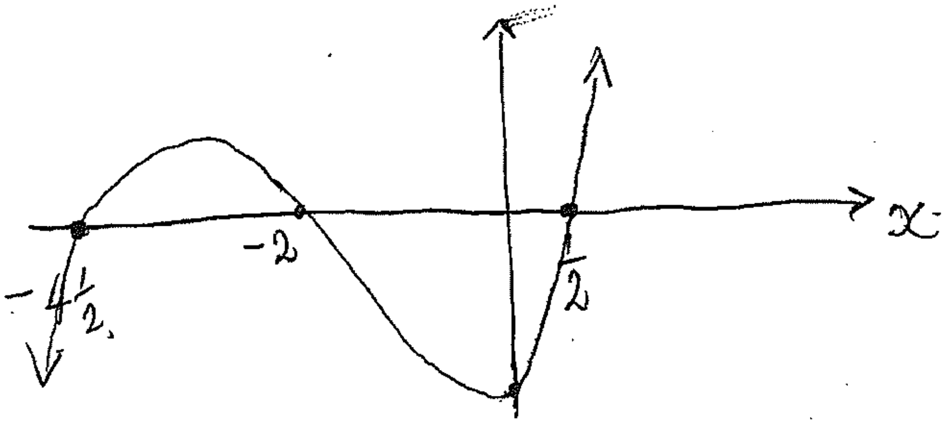
$(2x-1)(x+2)\left[2x+9\right] \leq 0$

below or touching

$x \leq -4\frac{1}{2}$, $-2 < x < \frac{1}{2}$

with restriction

(2)



(13) (b) (i) $y = \sqrt{7+x^2}$

$$= (7+x^2)^{\frac{1}{2}}$$

$$y' = \cancel{\tfrac{1}{2}}(7+x^2)^{-\frac{1}{2}} \times \cancel{2}x$$

$$= \frac{x}{\sqrt{7+x^2}}$$ (2)

(ii) $y = x.\sqrt{2x-1}$

$$= x(2x-1)^{\frac{1}{2}}$$

$$y' = x \times \cancel{\tfrac{1}{2}}(2x-1)^{-\frac{1}{2}} \times \cancel{2} + (2x-1)^{\frac{1}{2}} \times 1$$

$$= \frac{x}{\sqrt{2x-1}} + \frac{\sqrt{2x-1}}{1} = \frac{x+2x-1}{\sqrt{2x-1}} = \frac{3x-1}{\sqrt{2x-1}}$$ (2)

(iii) $y = \sqrt[3]{x^2} + \sqrt{x^3}$

$$= x^{2/3} + x^{3/2}$$

$$y' = \frac{2}{3}x^{-\frac{1}{3}} + \frac{3}{2}x^{\frac{1}{2}}$$

$$= \frac{2}{3\sqrt[3]{x}} + \frac{3\sqrt{x}}{2}$$ (2)

(iv) $y = \dfrac{5x}{4+x^2}$

$$y' = \frac{(4+x^2)\times 5 - 5x \times 2x}{(4+x^2)^2}$$ (2)

$$= \frac{20+5x^2-10x^2}{(4+x^2)^2} = \frac{20-5x^2}{(4+x^2)^2} \text{ OR } \frac{5(4-x^2)}{(4+x^2)^2}$$

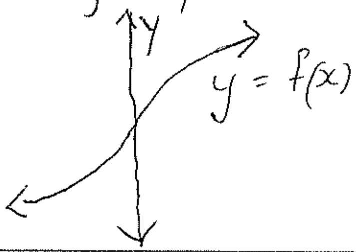
14 (a) $$\lim_{x\to\infty} \frac{4x+3x^2}{5x-2x^2} = \lim_{x\to\infty}\left(\frac{\frac{4}{x}+3}{\frac{5}{x}-2}\right)$$

$$= -\frac{3}{2}$$

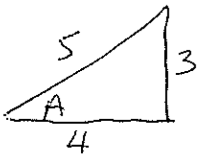
(b) $$f(x) = x^2 + x^{-2}$$
$$f'(x) = 2x - 2x^{-3} = 2x - \frac{2}{x^3}$$
$$f''(x) = 2 + 6x^{-4} = 2 + \frac{6}{x^4}$$

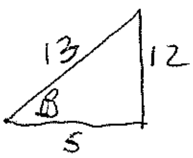
(c) $f(x)$ concave up when $f''(x) > 0$
$$f(x) = x^3 - 3x^2 - 7x + 10$$
$$f'(x) = 3x^2 - 6x - 7$$
$$f''(x) = 6x - 6$$
Concave up $\quad 6x - 6 > 0$
$$x > 1$$

(d) $y' > 0$ y increasing
y' increasing y concave up
y' decreasing y concave down



(e)





$$\sin 2A = 2\sin A\cos A = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

$$\cos(A-B) = \cos A\cos B + \sin A \sin B$$
$$= \frac{4}{5}\times\frac{5}{13} + \frac{3}{5}\times\frac{12}{13}$$
$$= \frac{56}{65}$$

15(a)(i) $\tan(\theta+\alpha) = \dfrac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$

$\tan 45^\circ = 1 = \dfrac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$

$\therefore \tan\theta + \tan\alpha = 1 - \tan\theta\tan\alpha$

(ii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$2\theta = 45 \quad \theta = 22\tfrac{1}{2} \quad t = \tan 22\tfrac{1}{2}$

$1 = \dfrac{2t}{1-t^2}$

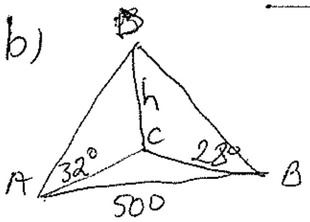
$2t = 1 - t^2$

$t^2 + 2t - 1 = 0$

(iii) from (ii) $t = \dfrac{-2 \pm \sqrt{4 - 4\times 1\times -1}}{2}$

$t = -1 \pm \sqrt{2} \qquad t = \sqrt{2} - 1 \ (t > 0)$

(b)



$\tan 32 = \dfrac{h}{AC} \qquad \tan 28 = \dfrac{h}{BC}$

$AC^2 + BC^2 = 500^2$

$h^2\cot^2 32 + h^2\cot^2 28 = 500^2$

$h^2 = \dfrac{500^2}{\cot^2 32 + \cot^2 28}$

$h = 202\text{m}$ nearest m

(c) $A = xy, \quad 2y = 6 - 3x$

$y = 3 - \tfrac{3}{2}x$

$A = 3x - \tfrac{3}{2}x^2$

$\dfrac{dA}{dx} = 3 - 3x$

max $3 - 3x = 0$

$x = 1, \quad y = \tfrac{3}{2}$

Max $A = \tfrac{3}{2}\,u^2$