NAME MASTER

SYDNEY GRAMMAR SCHOOL

2015 Annual Examination

FORM V
MATHEMATICS EXTENSION 1

Monday 31st August 2015

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.

Total — 100 Marks

- All questions may be attempted.

Section I – 9 Marks

- Questions 1–9 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 91 Marks

- Questions 10–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Ten.

5A: DS	5B: RCF	5C: SO	5D: DNW
5E: DWH	5F: REJ	5G: SJE	5H: KWM

Checklist

- SGS booklets — 7 per boy
- Multiple choice answer sheet
- Candidature — 124 boys

Examiner
DS

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

The trapezoidal rule is used to approximate a definite integral. If $n + 1$ function values are used, then we are summing the areas of how many trapezia?

(A) $n - 1$

(B) n

(C) $n + 1$

(D) $n + 2$

QUESTION TWO

What is the gradient of the curve $y = -e^{-x}$ at its y-intercept?

(A) e

(B) $-e$

(C) 1

(D) -1

QUESTION THREE

What is the equation of the horizontal asymptote of the curve $y = \dfrac{x-2}{x-3}$?

(A) $y = 1$

(B) $y = \frac{2}{3}$

(C) $x = 3$

(D) $x = 2$

QUESTION FOUR

The definite integral $I = \displaystyle\int_{-2}^{2} \sqrt{4 - x^2}\, dx$ can be evaluated without finding a primitive of $\sqrt{4 - x^2}$. What is the exact value of I ?

(A) 4π

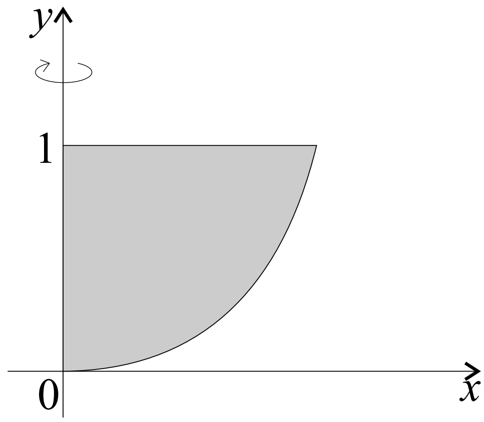
(B) 2π

(C) π

(D) $\frac{\pi}{2}$

Exam continues next page ...

QUESTION FIVE



The diagram above shows the region bounded by the parabola $y = x^2$, the line $y = 1$ and the y-axis. What is the volume of the paraboloid formed by rotating this region about the y-axis?

(A) $\frac{\pi}{5}$

(B) $\frac{\pi}{3}$

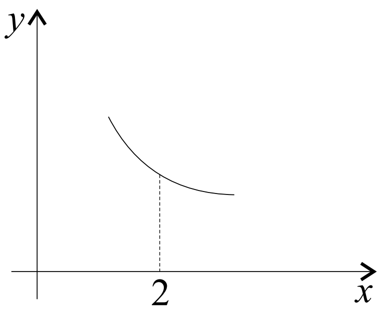
(C) $\frac{\pi}{2}$

(D) $\frac{2\pi}{3}$

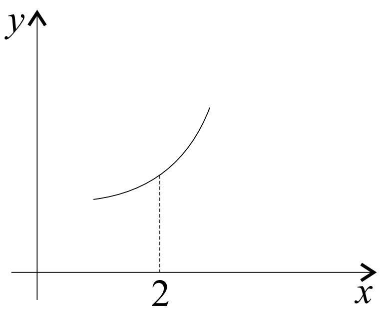
QUESTION SIX

A curve has equation $y = f(x)$. If $f'(2) < 0$ and $f''(2) > 0$, which diagram below shows the curve as it passes through the point where $x = 2$?

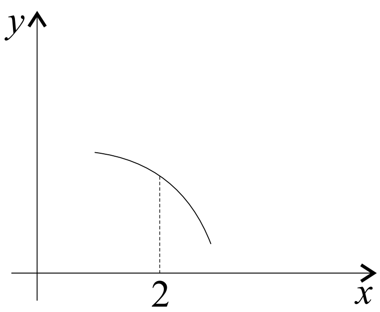
(A)



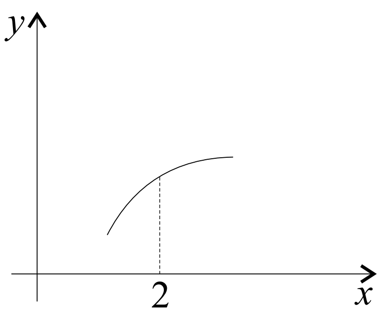
(B)



(C)



(D)



Exam continues next page ...

QUESTION SEVEN

By the chain rule, the derivative of $(x^2+1)^3$ is $6x(x^2+1)^2$. Which function is a primitive of $12x(x^2+1)^2$?

(A) $2x(x^2+1)^3$

(B) $2(x^2+1)^3$

(C) $\frac{1}{2}x(x^2+1)^3$

(D) $\frac{1}{2}(x^2+1)^3$

QUESTION EIGHT

Which expression is equivalent to $\dfrac{\sin\theta}{1+\cos\theta}$?

(A) $\operatorname{cosec}\theta + \cot\theta$

(B) $\dfrac{1-\sin\theta}{\cos\theta}$

(C) $\sin\theta + \tan\theta$

(D) $\dfrac{1-\cos\theta}{\sin\theta}$

QUESTION NINE

For $x > 0$, which expression is NOT equivalent to $e^{\ln x}$?

(A) $\ln(e^x)$

(B) $x^{\ln e}$

(C) $(\ln e)^x$

(D) $\dfrac{1}{e^{\ln \frac{1}{x}}}$

End of Section I

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION TEN (13 marks) Use a separate writing booklet. **Marks**

(a) Differentiate:

(i) $(5-2x)^4$ **1**

(ii) xe^{2x} **2**

(iii) $\log_e \sqrt{x}$ **2**

(b) Find:

(i) $\displaystyle\int \frac{1}{\sqrt{x}}\,dx$ **1**

(ii) $\displaystyle\int \frac{2}{3x+4}\,dx$ **1**

(iii) $\displaystyle\int \frac{2}{(3x+4)^2}\,dx$ **2**

(c) Evaluate $\displaystyle\int_e^{e^3} \frac{1}{x}\,dx$. **2**

(d) The function $f(x)$ is defined by: **2**

$$f(x) = \begin{cases} kx & \text{for } x < 2 \\ x^2 + 6 & \text{for } x \geq 2 \end{cases}$$

For what value of k is $f(x)$ continuous at $x = 2$?

QUESTION ELEVEN (13 marks) Use a separate writing booklet. **Marks**

(a) Find, in terms of k, the coordinates of the point that divides the interval joining $A(-2,-1)$ to $B(3,6)$ in the ratio $k : 1-k$. **2**

(b) Solve for x: **3**

$$\frac{5}{x-1} \geq 1$$

(c) If p is a positive integer, find an expression for the number of terms in the sequence **2**

$$p,\ p+2,\ p+4,\ \ldots,\ 3p.$$

(d) (i) Expand $(e^{2x}+2)^2$. **1**

(ii) Hence evaluate $\displaystyle\int_0^1 (e^{2x}+2)^2\,dx$. **2**

(e) (i) Show that $\dfrac{1}{x+2} - \dfrac{1}{x+3} = \dfrac{1}{(x+2)(x+3)}$. **1**

(ii) Hence find the exact value of $\displaystyle\int_{-1}^{1} \frac{1}{(x+2)(x+3)}\,dx$. **2**

QUESTION TWELVE (13 marks) Use a separate writing booklet. **Marks**

(a) A function has second derivative $y'' = 3x^3(x+3)^2(x-2)$. Determine the x-coordinates of any points of inflexion on its graph. **2**

(b) (i) Sketch the curve $y = 8x - 4x^3$, clearly indicating the x-intercepts. (There is no need to find the stationary or inflexion points.) **2**

(ii) Find the total area enclosed by the curve and the x-axis. **3**

(c) A function $f(x)$ is defined by the equation $f(x) = x + \dfrac{4}{x}$.

(i) Show that the function is odd. **1**

(ii) Find $f'(x)$. **1**

(iii) Show that the function has stationary points at $x = 2$ and $x = -2$. **1**

(iv) Classify the two stationary points. **2**

(v) Notice that $f(-2) = -4$ and $f(2) = 4$. So $f(-2) < f(2)$. Explain why this fact does not contradict the results in part (iii). **1**

Exam continues overleaf ...

QUESTION THIRTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Use Simpson's rule with five function values, as well as appropriate log laws, to show that $\displaystyle\int_1^5 \ln x\,dx \doteqdot \ln 57$. **4**

(b) A curve has gradient function $f'(x) = 6x^2 + px + q$. The curve has a stationary point at $(2, -4)$ and its y-intercept is 14. Find the values of p and q. **5**

(c) Suppose that the limiting sum of the series $v + v^2 + v^3 + \cdots$ is w.

(i) Write down a formula for w in terms of v. **1**

(ii) Hence find v in terms of w. **2**

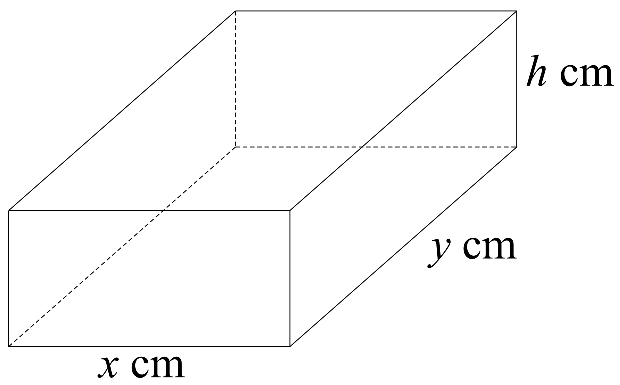
(iii) Explain why the limiting sum of the series $w - w^2 + w^3 - \cdots$ is v. **1**
(You may assume that $|v|$ and $|w|$ are both less than one.)

QUESTION FOURTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) Solve for x: **2**

$$\log_3 x + 2 = \log_3(x + 2)$$

(b)



A closed rectangular box has dimensions x cm, y cm and h cm, as shown in the diagram above. It is to be made from $300\,\text{cm}^2$ of thin sheet metal, and the perimeter of its base is to be 40 cm.

(i) Show that the volume V of the box is given by **3**

$$V = 150h - 20h^2.$$

(ii) Hence find the dimensions of the box that meets all the requirements and has the maximum possible volume. **4**

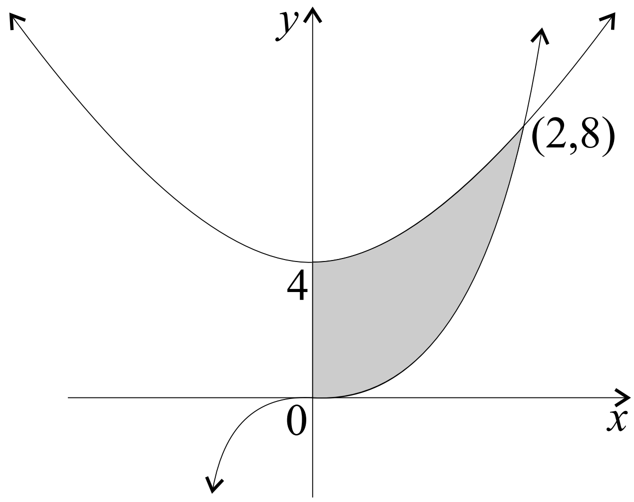
(c) One root of the quadratic equation $ax^2 + 2bx + c = 0$ is the reciprocal of the square of the other root. **4**

Prove that $a^3 + c^3 + 2abc = 0$.

Exam continues next page ...

QUESTION FIFTEEN (13 marks) Use a separate writing booklet. **Marks**

(a)



The shaded region $\mathcal{R}$ is bounded by the curves $y = x^2 + 4$ and $y = x^3$, and the y-axis, as shown in the diagram above.

(i) Calculate the area of $\mathcal{R}$. **3**

(ii) Determine the volume of the solid of revolution formed when $\mathcal{R}$ is rotated about the x-axis. **4**

(b) The function $y = P(x)$ is defined by $P(x) = (x - p)(x - q)(x - r)$, where p, q and r are distinct real numbers.

(i) Sketch a possible graph of $y = P(x)$. **2**
(Do NOT attempt to find the stationary or inflexion points.)

(ii) Expand $P(x)$ and write it in the form $ax^3 + bx^2 + cx + d$. **2**

(iii) By considering the equation $P'(x) = 0$, or otherwise, prove that **2**

$$(p + q + r)^2 > 3(pq + qr + rp).$$

Exam continues overleaf ...

QUESTION SIXTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) For any real number x, let $[x]$ denote the largest integer less than or equal to x.

For example $[2{\cdot}9] = 2$ and $[3] = 3$.

(i) Sketch the graph of $y = [x]$ for $0 \le x \le 5$. **2**

(ii) Find the value of $\displaystyle\int_0^5 [x]\,dx$. **1**

(b) Consider the function $y = \dfrac{\ln x}{x^n}$, where $n > 1$.

(i) State the domain of the function. **1**

(ii) Show that there is a stationary point at $x = e^{\frac{1}{n}}$. **2**

(iii) Determine the nature of the stationary point. **2**

(iv) Sketch the graph of the function. **2**
(There is no need to find the coordinates of the point of inflexion.)

(v) Explain why $\dfrac{\ln x}{x^n} < \dfrac{1}{ne}$ for $x > e^{\frac{1}{n}}$. **1**

(vi) Deduce that $e^{\frac{1}{n-1}} > \dfrac{n}{n-1}$. **2**

End of Section II

END OF EXAMINATION

BLANK PAGE

The following list of standard integrals may be used:

$$\int x^n \, dx = \frac{1}{n+1} \, x^{n+1}, \;\; n \neq -1; \;\; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \;\; x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} \, e^{ax}, \;\; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \;\; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \;\; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \;\; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \;\; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \;\; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \;\; a > 0, \;\; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \;\; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

$$\text{NOTE}: \;\; \ln x = \log_e x, \;\; x > 0$$

NAME: ....

CLASS: ....MASTER: ....

SYDNEY GRAMMAR SCHOOL

2015
Annual Examination
FORM V
MATHEMATICS EXTENSION 1
Monday 31st August 2015

- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

Question Nine

A ◯ B ◯ C ◯ D ◯

SOLUTIONS TO FORM 5 Ext 1 ANNUAL 2015 (Total: 100)

(1) n trapezia (B)

(2) $y' = e^{-x}$ (C)

(3) $y = 1$ (A)

(4) $\frac{1}{2}\pi(2)^2 = 2\pi$ (B)

(5) $\pi\int_0^1 y\,dy = \frac{\pi}{2}$ (C)

(6) Decreasing and concave up. (A)

(7) $2(x^2+1)^3$ (B)

(8) $\frac{\sin\theta}{1+\cos\theta} \cdot \frac{1-\cos\theta}{1-\cos\theta}$

$= \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$ (D)

(9) $(\ln e)^x = 1^x$ (C)

ONE EACH

(10)(a)(i) $-8(5-2x)^3$ ✓

(ii) $vu' + uv'$ ✓

$= e^{2x}.1 + x.2e^{2x}$ ✓

$= e^{2x}(1+2x)$ ✓

(iii) $\frac{d}{dx}\left(\frac{1}{2}\ln x\right) = \frac{1}{2x}$ ✓

(b)(i) $\int x^{-\frac{1}{2}}\,dx$

$= 2x^{\frac{1}{2}} + c$ ✓

$= 2\sqrt{x} + c$

(ii) $\frac{2}{3}\int \frac{3}{3x+4}\,dx$

$= \frac{2}{3}\ln(3x+4) + c$ ✓

(iii) $2\int(3x+4)^{-2}\,dx$ ✓

$= \frac{2(3x+4)^{-1}}{-1(3)} + c$ ✓

$= \frac{-2}{3(3x+4)} + c$

(c) $[\ln x]_e^{e^3}$ ✓

$= \ln e^3 - \ln e$

$= 3\ln e - \ln e$

$= 2$ ✓

(d) For continuity at $x = 2$

$k(2) = 2^2 + 6$ ✓

$2k = 10$

$k = 5$ ✓

(11)(a) $\left(\dfrac{3k-2(1-k)}{k+(1-k)}, \dfrac{6k-(1-k)}{k+(1-k)}\right)$

$= (5k-2,\ 7k-1)$

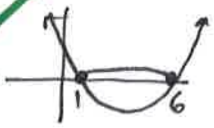
(b) Multiply both sides by $(x-1)^2$.

$5(x-1) \geq (x-1)^2,\ x \neq 1$

$(x-1)(x-1-5) \leq 0$

$(x-1)(x-6) \leq 0$

$1 < x \leq 6$



(c) Let $T_n = 3p$

then $a+(n-1)d = 3p$

$p + 2(n-1) = 3p$

$2(n-1) = 2p$

$n = p+1$

So there are $(p+1)$ terms.

(d)(i) $(e^{2x}+2)^2 = e^{4x} + 4e^{2x} + 4$

(ii) $\int_0^1 (e^{4x} + 4e^{2x} + 4)\,dx$

$= \left[\frac{1}{4}e^{4x} + 2e^{2x} + 4x\right]_0^1$

$= \frac{1}{4}e^4 + 2e^2 + 4 - \left(\frac{1}{4} + 2 + 0\right)$

$= \frac{1}{4}e^4 + 2e^2 + \frac{7}{4}$

$= \frac{1}{4}(e^4 + 8e^2 + 7)$

(e)(i) LHS $= \dfrac{x+3-(x+2)}{(x+2)(x+3)}$

$= \dfrac{1}{(x+2)(x+3)}$

$=$ RHS

(ii) $\int_{-1}^{1}\left(\dfrac{1}{x+2} - \dfrac{1}{x+3}\right)dx$

$= \left[\ln(x+2) - \ln(x+3)\right]_{-1}^{1}$

$= \left[\ln\left(\dfrac{x+2}{x+3}\right)\right]_{-1}^{1}$

$= \ln\frac{3}{4} - \ln\frac{1}{2}$

$= \ln\frac{3}{2}$

(12)(a)

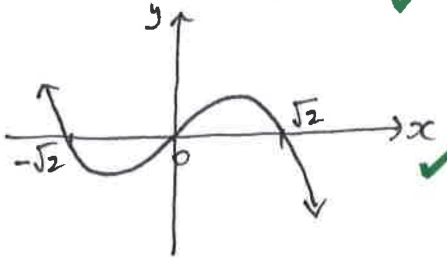
x	-4	-3	-1	0	1	2	3
y''	+	0	+	0	−	0	+

The concavity changes at $x=0$ and $x=2$. (But NOT at $x=-3$.)

(b)(i) $y = 4x(2-x^2)$

$= 4x(\sqrt{2}+x)(\sqrt{2}-x)$

x-intercepts at $0, \pm\sqrt{2}$.



(ii) $A = 2\int_0^{\sqrt{2}} (8x - 4x^3)\,dx$

$= 2\left[4x^2 - x^4\right]_0^{\sqrt{2}}$

$= 2(8-4)$

$= 8\ u^2$

(c)(i) $f(-x) = -x + \frac{4}{-x} = -\left(x + \frac{4}{x}\right) = -f(x)$

(ii) $f(x) = x + 4x^{-1}$

so $f'(x) = 1 - 4x^{-2}$

$= 1 - \frac{4}{x^2}$

(iii) Let $f'(x) = 0$ for stationary points.

Then $1 - \frac{4}{x^2} = 0$

$x^2 = 4$

$x = \pm 2$.

(iv) $f''(x) = 8x^{-3}$

so $f''(-2) < 0$ and $f''(2) > 0$.

So there is a maximum turning point at $x=-2$ and a minimum turning point at $x=2$.

(v) $(-2,-4)$ and $(2,4)$ lie on different branches of the curve, because there is a vertical asymptote at $x=0$.

(13)(a)

x	1	2	3	4	5
$y=\ln x$	0	$\ln 2$	$\ln 3$	$\ln 4$	$\ln 5$
	(y_0)	(y_1)	(y_2)	(y_3)	(y_4)

$$\int_1^5 \ln x\,dx \doteqdot \frac{h}{3}\left(y_0+y_4+4(y_1+y_3)+2y_2\right)$$
$$= \frac{1}{3}\left(\ln 5 + 4(\ln 2+\ln 4)+2\ln 3\right)$$
$$= \frac{1}{3}\ln\left(5\times 2^4\times 4^4\times 3^2\right)$$
$$= \ln 184320^{\frac{1}{3}}$$
$$= \ln 56.91\ldots$$
$$\doteqdot \ln 57$$

(b) $f'(2)=0$ so $24+2p+q=0$ (1)

$f(x)=2x^3+\frac{1}{2}px^2+qx+c$

$f(0)=14$ so $c=14$

$f(2)=-4$ so $16+2p+2q+14=-4$

so $17+p+q=0$ (2)

(1)−(2): $7+p=0$

so $p=-7$ and $q=-10$.

(c)(i) We are given that $\frac{v}{1-v}=w$.

(ii) So $v = w - wv$
$$v+wv=w$$
$$v(1+w)=w$$
$$v=\frac{w}{1+w}$$

(iii) $v = \frac{w}{1-(-w)}$
$$= w - w^2 + w^3 - \ldots$$

(assuming that both series converge)

(14)(a) $2=\log_3(x+2)-\log_3 x$

$\log_3 \frac{x+2}{x} = 2$

$\frac{x+2}{x}=9$

$x+2=9x$

$x=\frac{1}{4}$

(b)(i) Given $2x+2y=40$

so $x+y=20$ (1)

and given $2xy+2xh+2yh=300$

so $xy+h(x+y)=150$ (using (1))

so $xy+20h=150$

so $xy=150-20h$. (2)

$V=xyh$

$=(150-20h)h$ (using (2))

$=150h-20h^2$

(ii) $\frac{dV}{dh}=150-40h$

Let $\frac{dV}{dh}=0$ for stationary points.

Then $h=\frac{15}{4}$.

$\frac{d^2V}{dh^2}=-40<0$,

so V is maximised when $h=\frac{15}{4}$.

When $h=\frac{15}{4}$,

$xy=75$ and $x+y=20$.

By inspection, $x=15$ and $y=5$ (or vice versa),

so the box has base 15 cm by 5 cm and its height is 3.75 cm.

(c) Let the roots be α and $\frac{1}{\alpha^2}$.

Then $\alpha+\frac{1}{\alpha^2}=\frac{-2b}{a}$ (1)

and $\alpha\cdot\frac{1}{\alpha^2}=\frac{c}{a}$

so $\alpha=\frac{a}{c}$ (2)

Substituting (2) into (1):

$\frac{a}{c}+\frac{c^2}{a^2}=\frac{-2b}{a}$

Multiply both sides by a^2c:

$a^3+c^3=-2bac$

so $a^3+c^3+2abc=0$.

(15)(a)(i)

$\text{Area} = \int_0^2 (x^2+4-x^3)\,dx$

$= \left[\frac{1}{3}x^3 + 4x - \frac{1}{4}x^4\right]_0^2$

$= \frac{8}{3} + 8 - 4$

$= \frac{20}{3}\ u^2$

(ii)

$\text{Volume} = \pi\int_0^2 \left((x^2+4)^2 - (x^3)^2\right)dx$

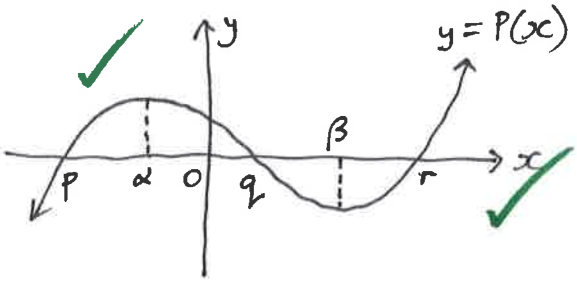
$= \pi\int_0^2 (x^4+8x^2+16-x^6)\,dx$

$= \pi\left[\frac{1}{5}x^5 + \frac{8}{3}x^3 + 16x - \frac{1}{7}x^7\right]_0^2$

$= \pi\left(\frac{32}{5} + \frac{64}{3} + 32 - \frac{128}{7}\right)$

$= \frac{4352\pi}{105}\ u^3$

(b)(i)



(ii) Expanding, we have

$P(x) = x^3 - (p+q+r)x^2 + (pq+qr+rp)x - pqr$

so $P'(x) = 3x^2 - 2(p+q+r)x + (pq+qr+rp)$

The equation $P'(x)=0$ has two distinct real roots (α and β in the diagram above),

so $\Delta > 0$

so $4(p+q+r)^2 - 12(pq+qr+rp) > 0$

so $(p+q+r)^2 > 3(pq+qr+rp)$

(ii) (OTHERWISE)

$(p-q)^2 > 0$

so $p^2+q^2 > 2pq$ ①

Similarly $q^2+r^2 > 2qr$ ②

and $r^2+p^2 > 2rp$ ③

① + ② + ③ :

$2p^2+2q^2+2r^2 > 2pq+2qr+2rp$

so $p^2+q^2+r^2 > pq+qr+rp$ ④

Now,

$(p+q+r)^2 = p^2+q^2+r^2+2(pq+qr+rp)$

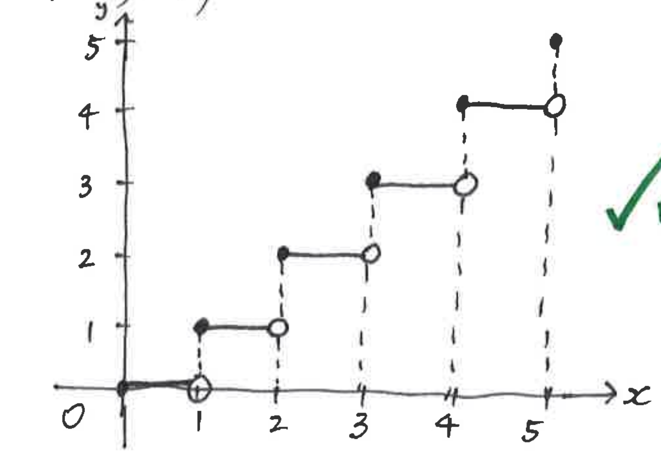
From ④, it follows that

$(p+q+r)^2 > pq+qr+rp+2(pq+qr+rp)$

so

$(p+q+r)^2 > 3(pq+qr+rp)$.

(16)(a)(i)



(ii) $\int_0^5 [x]\,dx = 0+1+2+3+4$
$= 10$

(b)(i) Domain is $x>0$.

(ii) $y' = \dfrac{x^n \cdot \frac{1}{x} - nx^{n-1} \cdot \ln x}{(x^n)^2}$

$= \dfrac{x^{n-1}(1-n\ln x)}{x^{2n}}$

Let $y'=0$ for stationary points.
$x>0$, so the only solution occurs when $\ln x = \frac{1}{n}$
i.e. when $x = e^{\frac{1}{n}}$.

(iii)

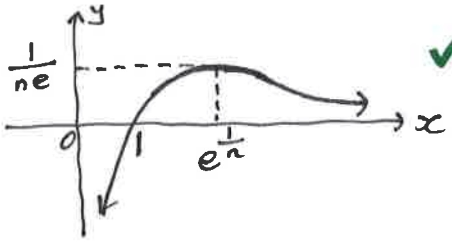
x	$e^{\frac{1}{2n}}$	$e^{\frac{1}{n}}$	$e^{\frac{2}{n}}$
y'	$\dfrac{e^{\frac{n-1}{2n}}(1-\frac{1}{2})}{e}$	0	$\dfrac{e^{\frac{2n-2}{n}}(1-2)}{e^4}$
	/	—	\

There is a maximum turning point at $x = e^{\frac{1}{n}}$.

(some working or explanation required)

(iv) x-intercept at $(1,0)$.
When $x = e^{\frac{1}{n}}$,
$y = \frac{1}{n} \div (e^{\frac{1}{n}})^n$
$= \frac{1}{ne}$.

As $x \to \infty$, $y \to 0^+$.
As $x \to 0^+$, $y \to -\infty$.



(v) From the graph, when $x > e^{\frac{1}{n}}$, $y < \frac{1}{ne}$, (since the max y-value is $\frac{1}{ne}$)
so when $x > e^{\frac{1}{n}}$, $\dfrac{\ln x}{x^n} < \dfrac{1}{ne}$.

(vi) Let $x = e^{\frac{1}{n-1}} \left(> e^{\frac{1}{n}}\right)$,
then $\frac{1}{n-1} \div e^{\frac{n}{n-1}} < \frac{1}{ne}$
so $\dfrac{n}{n-1} < e^{\frac{n}{n-1}-1}$
so $e^{\frac{1}{n-1}} > \dfrac{n}{n-1}$.