SYDNEY TECHNICAL HIGH SCHOOL

Mathematics Extension 1

YEAR 11 YEARLY EXAMINATION
SEPTEMBER 2008

General Instructions

- Working time allowed – 90 minutes
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions
- All questions are of equal value

NAME :________________________

TEACHER :____________________

QUESTION 1	QUESTION 2	QUESTION 3	QUESTION 4	QUESTION 5	TOTAL

Question 1 (solutions must be in black or blue pen) Marks

a) Sketch $y = (x+1)^2(x-3)$ clearly showing all intercepts. 2

b) Simplify $\dfrac{2+\frac{1}{x}}{2x^2+x}$ 2

c) Solve $\dfrac{3x}{x+1} \le 1$ 2

d) Expand and simplify $(\cos A - \sin A)(\cos A + \sin A)$ 2

expressing your answer in terms of $2A$.

e) If α, β and δ are the roots of $2y^3 - 8y^2 + 3y + 1 = 0$ 4

find the value of i) $\alpha + \beta + \delta$

ii) $\alpha\beta + \alpha\delta + \beta\delta$

iii) $\alpha\beta\delta$

iv) $\alpha^2 + \beta^2 + \delta^2$

Question 2 (Start a new page) Marks

a) Find the coordinates of the point that divides the interval from $(-4,2)$ to $(6,9)$ internally in the ratio $4:1$. **2**

b) Find the remainder when $x^4 - 2x^2 + 5$ is divided by $x^2 + x - 2$. **2**

c) Find the equation of the parabola with focus $(0,6)$ and directrix $y = 2$. **2**

d) Find the locus of the set of points $P(x, y)$ which are equidistant from the points $A(1,8)$ and $B(5,-2)$. **2**

e) i) Express $3\sin\theta - 2\cos\theta$ in terms of t where $t = \tan\frac{\theta}{2}$. **2**

ii) Hence, or otherwise solve

$3\sin\theta - 2\cos\theta = 2$ for $0° < \theta < 180°$ (nearest degree) **2**

Question 3 (Start a new page)

Marks

a) A curve has parametric equations $x = \frac{t}{3}$, $y = 4t^2$. 1

Find the Cartesian equation for this curve.

b) Find the acute angle between the lines $2x - 3y + 4 = 0$ 2

and $x + y - 6 = 0$ giving your answer correct to the nearest degree.

c) Factorise fully $x^3 + 7x^2 + 2x - 40$ 3

d) i) Sketch the region in the number plane defined by 2

$$y \le | 2x - 3 |$$

ii) For what values of m does the equation 1

$| 2x - 3 | = mx$ have 2 solutions.

e) Find the value of a given that 1

$x^3 - 2x^2 + a \equiv (x+3)Q(x) + 3$ where $Q(x)$ is a polynomial.

f) By rationalising the numerator, find the exact value of the following limit 2

$$\lim_{x \to 0} \frac{\sqrt{x+5} - \sqrt{5}}{x}$$

Question 4 (Start a new page) Marks

a) i) Derive the equation of the tangent to the parabola $x^2 = 8y$ at the point $P(4p, 2p^2)$. 2

ii) Find the point of intersection of the two tangents to the parabola $x^2 = 8y$ drawn from the points $P(4p, 2p^2)$ and $Q(4q, 2q^2)$. 2

b) Write down the equation of a monic polynomial of degree 3, which is an odd function and has a root at $x = 3$. 2

c) Find the value of m given that the equation $x^3 + mx + 2 = 0$ has a double root. 2

d) If $f(x-1) = x^2 - 4x$ find $f(a)$ in simplest terms. 1

e) i) Sketch $y = f(x)$ given 2

$$f(x) = \begin{cases} x^2 + 1, & \text{for } x < 0 \\ 1 - x^2, & \text{for } 0 \le x \le 1 \\ x - 1, & \text{for } x > 1 \end{cases}$$

ii) At which point or points is the function in part i) not differentiable. 1

Question 5 (Start a new page)

Marks

a) Solve $3^{2x} - (1+\sqrt{3})3^{x} + \sqrt{3} = 0$ 2

b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$.

i) Find an expression in terms of a and p for the distance PS. 1

where S is the focus of the parabola.

ii) Find the equation of the chord PQ. 2

iii) If this chord passes through the point $M(0, 4a)$ show that $pq = -4$ 1

iv) The chord PQ which passes through the point M meets 3

the directrix of $x^2 = 4ay$ at the point A.

Find the coordinates of A in terms of a and p.

c) From a point A which lies due North of a tower and at ground level, 3

the top of the tower has an angle of elevation of $24°$.

From a point B which lies due East of the same tower and is also at

ground level, the top of the tower has an angle of elevation of $34°$.

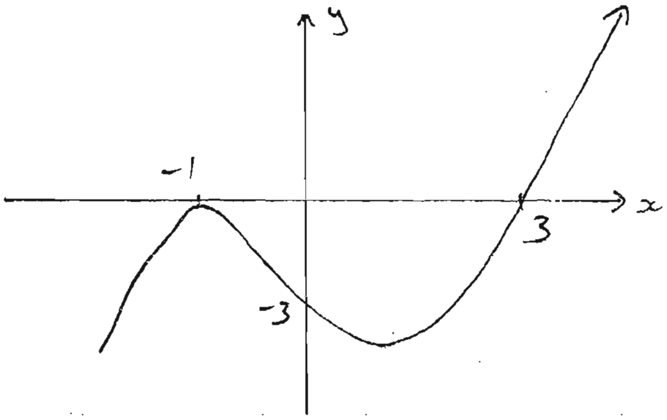
Calculate the bearing of point B from point A.

End of Paper

SOLUTIONS

QUESTION 1

a)



b) $\dfrac{\frac{2x+1}{x}}{x(2x+1)} = \dfrac{1}{x^2}$

c) $\dfrac{3x}{x+1} \leq 1$

$-1 < x \leq \frac{1}{2}$

d) $\cos^2 A - \sin^2 A$

$= \cos 2A$

) i) 4

ii) $\frac{3}{2}$

iii) $-\frac{1}{2}$

iv) $(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= 4^2 - 2\left(\frac{3}{2}\right)$

$= 13$

QUESTION 2

a) $\left(4, 7\frac{3}{5}\right)$

b)

$$
\begin{array}{r}
x^2 - x + 1 \\
x^2+x-2 \,\big)\, \overline{x^4 \quad - 2x^2 \quad + 5} \\
x^4 + x^3 - 2x^2 \\
\hline
-x^3 \quad\quad + 5 \\
-x^3 - x^2 + 2x \\
\hline
x^2 - 2x + 5 \\
x^2 + x - 2 \\
\hline
-3x + 7
\end{array}
$$

$\therefore$ Remainder $= -3x + 7$

c) $x^2 = 8(y-4)$

d) $PA = PB$

$\sqrt{(x-1)^2 + (y-8)^2} = \sqrt{(x-5)^2 + (y+2)^2}$

$x^2 - 2x + 1 + y^2 - 16y + 64$
$= x^2 - 10x + 25 + y^2 + 4y + 4$

$8x - 20y + 36 = 0$
$2x - 5y + 9 = 0$

e) i) $3\left(\dfrac{2t}{1+t^2}\right) - 2\left(\dfrac{1-t^2}{1+t^2}\right)$

$= \dfrac{6t - 2 + 2t^2}{1+t^2}$

ii) $\dfrac{2t^2 + 6t - 2}{1+t^2} = 2$

$6t = 4$
$t = \frac{2}{3}$
$\therefore \theta = 67°$

QUESTION 3

a) $x = \frac{t}{3} \Rightarrow t = 3x$

$y = 4t^2$

$\therefore y = 4(3x)^2$

$y = 36x^2$

b) $m_1 = \frac{2}{3}$ $m_2 = -1$

$\tan\theta = \frac{\left|\frac{2}{3} - -1\right|}{1 + (\frac{2}{3} \times -1)}$

$= 5$

$\therefore \theta = 79^\circ$

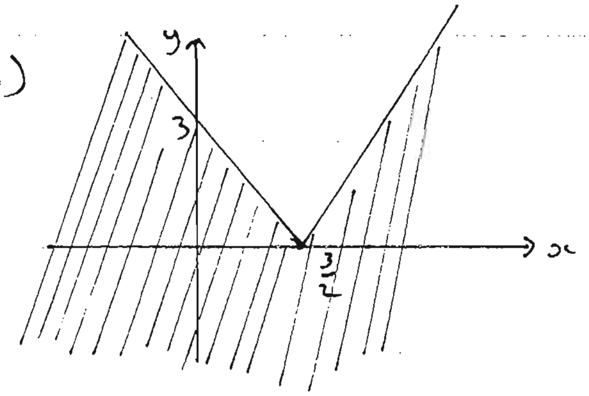
c) $P(2) = 8 + 28 + 4 - 40$
$= 0$

$\therefore (x-2)$ is a factor

$\therefore (x-2)(x^2 + 9x + 20)$

$= (x-2)(x+4)(x+5)$

d) i)



ii) $0 < m < 2$

e) $P(-3) = 3$

$-27 - 18 + a = 3$

$a = 48$

f) $\frac{\sqrt{x+5} - \sqrt{5}}{x} \times \frac{\sqrt{x+5} + \sqrt{5}}{\sqrt{x+5} + \sqrt{5}}$

$= \frac{x + 5 - 5}{x(\sqrt{x+5} + \sqrt{5})}$

$= \frac{1}{\sqrt{x+5} + \sqrt{5}}$

$\therefore \lim_{x \to 0} \frac{1}{\sqrt{x+5} + \sqrt{5}}$

$= \frac{1}{2\sqrt{5}}$

QUESTION 4

a) i) $y = \frac{x^2}{8}$

$\frac{dy}{dx} = \frac{2x}{8}$

$= \frac{x}{4}$

when $x = 4p$
$\frac{dy}{dx} = \frac{4p}{4}$

$m_T = p$

$\therefore$ equation of tangent
$y - 2p^2 = p(x - 4p)$

$y = px - 2p^2$

ii) $y = px - 2p^2$
$y = qx - 2q^2$

$\therefore px - qx - 2p^2 + 2q^2 = 0$

$x(p-q) = 2(p-q)(p+q)$

$x = 2(p+q)$

$\therefore y = 2p(p+q) - 2p^2$

$= 2pq$

$\therefore$ pt is $(2(p+q), 2pq)$

b) $P(x) = x(x-3)(x+3)$

c) $x^3 + mx + 2 = 0$

let roots be α, α, β

$\therefore$
$2\alpha + \beta = 0$ ①
$\alpha^2 + 2\alpha\beta = m$ ②
$\alpha^2\beta = -2$ ③

Solving ① and ③ simultaneously

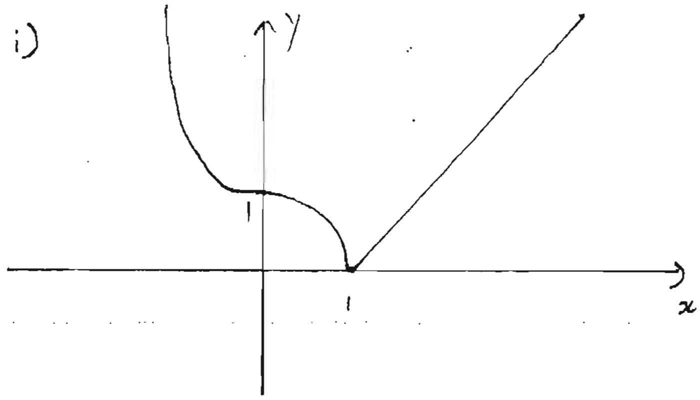
$\beta = -2\alpha \Rightarrow \alpha^2(-2\alpha) = -2$
$\alpha^3 = 1$
$\alpha = 1$
$\therefore \beta = -2$

$\therefore m = -3$

d) $f(a) = (a+1)^2 - 4(a+1)$

$= a^2 - 2a - 3$

e) i)



ii) not differentiable at $x = 1$.

QUESTION 5

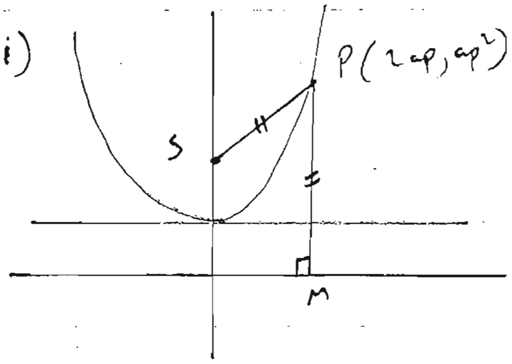
a) let $u = 3^x$

$u^2 - (1+\sqrt{3})u + \sqrt{3} = 0$

$(u-1)(u-\sqrt{3}) = 0$
$u = 1, \sqrt{3}$

$\therefore 3^x = 1 \qquad 3^x = \sqrt{3}$
$x = 0 \qquad x = \frac{1}{2}$

b) i)



$PS = PM$ (by definition)
$= ap^2 + a$

i) $m_{PQ} = \dfrac{ap^2 - aq^2}{2ap - 2aq}$

$= \dfrac{p+q}{2}$

equation

$y - ap^2 = \left(\dfrac{p+q}{2}\right)(x - 2ap)$

$y = \frac{1}{2}(p+q)x - apq$

iii) sub $(0, 4a)$

$4a = \frac{1}{2}(p+q)0 - apq$

$4a = -apq$

$pq = -4$

iv) sub $y = -a$ into chord

$-a = \frac{1}{2}(p+q)x - apq$

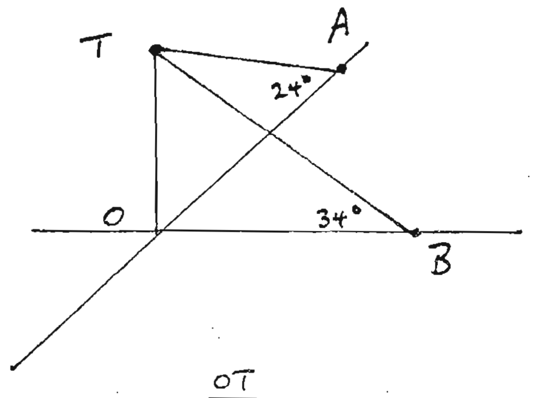
$x = \dfrac{2a(pq-1)}{p+q}$

sub $q = \dfrac{-4}{p}$

$= \dfrac{2a(-5)}{p - \frac{4}{p}}$

$= \dfrac{-10ap}{p^2 - 4}$

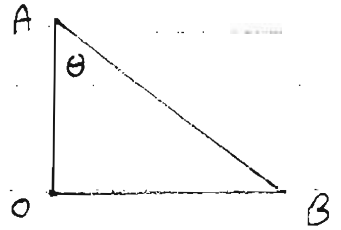
c)



$\tan 24° = \dfrac{OT}{OA}$

$\therefore \quad OA = \dfrac{OT}{\tan 24°}$

also $OB = \dfrac{OT}{\tan 34°}$



$\tan\theta = \dfrac{OB}{OA}$

$= \dfrac{\frac{OT}{\tan 34°}}{\frac{OT}{\tan 24°}}$

$= \dfrac{\tan 24°}{\tan 34°}$

$\theta = 33°26'$

$\therefore$ bearing is $146° 34'$