SYDNEY TECHNICAL HIGH SCHOOL

(Celebrating 100 Years of Public Education)

YEAR 11 YEARLY EXAMINATION

PRELIMINARY HSC ASSESSMENT TASK 3

2011

Mathematics Extension 1

General Instuctions

- Working time - 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Diagrams are not drawn to scale

Total marks - 66

- Attempt Questions 1 – 6
- All questions are of equal value
- Start each question on a new page

Name :________________________

Teacher :________________________

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	**TOTAL**

Question 1 (11 marks)

(a) Express the following parametric equations in Cartesian form. 1

$$x = t + 1 \ , \ y = t(t + 4)$$

(b) Solve $\frac{x}{x+2} < 0$ 2

(c) Find, correct to the nearest degree, the acute angle between the lines 2

$2x - y + 1 = 0$ and $3x - 2y + 4 = 0$

(d) Write down the equation of a monic polynomial of degree 3 that is 2

an odd function and has a root at $x = 2$.

(e) If $\alpha\,, \beta \textit{ and } \gamma$ are the roots of the equation $2x^3 - 4x^2 + 6x + 8 = 0$

find the value of

(i) $\alpha + \beta + \gamma$ 1

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ 1

(iii) $\alpha\beta\gamma$ 1

(iv) $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$ 1

Question 2 (11 marks) - Start a new page

(a) Evaluate $\lim\limits_{x \to \infty} \frac{x^2-x-2}{3x^2-3x+1}$ 1

(b) Solve the equation $2x = \sqrt{7x+11}$ 3

(c) If θ is acute and $cos\ 2\theta = \frac{3}{4}$, find the exact value of $cos\theta$. 2

(d) Factorise fully the polynomial $2x^3 + 11x^2 + 17x + 6$ 3

(e) Find the values of a and b if the solution of the inequality 2

$|x-a| \leq b$ is given by $2 \leq x \leq 5$.

Question 3 (11 marks) - Start a new page

(a) Find the values of A, B and C if 3

$2x^2 + 3x - 4 \equiv A(x-1)^2 + Bx(x-2) + C(x-1)$

(b) Find the value of k if the line $2x - y + 2 + k(x + 3y - 1) = 0$ 2

is parallel to the line $x + 2y + 4 = 0$.

(c) Use the substitution $t = tan\frac{x}{2}$ to solve the equation 4

$2sinx - 5cosx = 2$ for $0° \leq x \leq 360°$ correct to the nearest degree.

(d) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$.

(i) Show that the gradient of the chord PQ is given by $\frac{p+q}{2}$. 1

(ii) Find the equation of the chord PQ. 1

Question 4 (11 marks) - Start a new page

(a) Find the vertex and focus of the parabola $y = \frac{1}{8}x^2 + x$ 2

(b) Find the remainder when $x^3 + 4x - 4$ is divided by $x^2 - 1$ 2

(c) Find the value, or values of k if the line $y = kx - 16$ 3

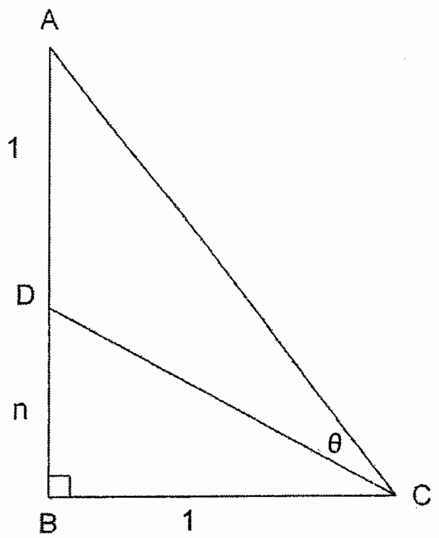
is a tangent to the parabola $y = x^2 - 2x$.

(d) Solve the equation $2sin2\theta = 3tan\theta$ for $0 \leq \theta \leq 2\pi$. 4

(Give your answers in radians)

Question 5 (11 marks) - Start a new page

(a) 3



In triangle ABC, $angle\ ABC = 90°$, $AD = BC = 1\ unit$ and $BD = n\ units$.

If angle $ACD = \theta$ find an expression for $\tan\theta$ in terms of n.

(b) Consider the variable point $P(4p, 2p^2)$ on the parabola $x^2 = 8y$.

(i) Prove that the equation of the tangent at P is $y = px - 2p^2$. 2

(ii) If the tangent at P meets the y axis at the point T, 1

find the coordinates of T.

(iii) If S is the focus of the parabola $x^2 = 8y$, show that $TS = PS$. 2

(iv) If the point M divides the interval PT internally in the ratio $1:3$, 3

find the locus of M as the point P varies.

Question 6 (11 marks) - Start a new page

(a) Let A and B be the fixed points $(5, 6)$ and $(-1, 0)$ respectively and P be the variable point (x, y).

(i) Show that the locus of all points P, such that the distance from P to A is twice the distance from P to B, is a circle. **2**

(ii) Find the centre and radius of this circle. **2**

(b) The equation $x^3 + 6x^2 - x + m = 0$ has one root equal to the sum of the other two roots.

(i) Find the value of m. **2**

(ii) Solve the equation. **2**

(c) Simplify $\dfrac{\tan 2\theta - \tan\theta}{\cot\theta + \tan 2\theta}$ **3**

Ext-1 Solutions 2011

Teacher's Name: **Student's Name/N°:**

1\. a. $f = x - 1$

$\therefore \; y = (x-1)(x+3)$

b. $\dfrac{x}{x+2} < 0$

$-2 < x < 0$

c. $m_1 = 2 \qquad m_2 = \dfrac{3}{2}$

$\tan\theta = \dfrac{2 - \frac{3}{2}}{1 + 2 \times \frac{3}{2}}$

$= \dfrac{\frac{1}{2}}{4}$

$= \dfrac{1}{8}$

$\therefore \; \theta = 7^\circ$

d. $y = x(x-2)(x+2)$

e. i) $\alpha + \beta + \gamma = 2$

ii) $\alpha\beta + \alpha\gamma + \beta\gamma = 3$

iii) $\alpha\beta\gamma = -4$

iv) $\dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \dfrac{2}{-4}$

$= -\dfrac{1}{2}$

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2. a) $\frac{1}{3}$

b)
$$4x^2 = 7x + 11$$
$$4x^2 - 7x - 11 = 0$$
$$(4x - 11)(x + 1) = 0$$
$$x = \frac{11}{4}, -1$$

but -1 does not satisfy original equation

$$\therefore \ x = \frac{11}{4}$$

c)
$$\cos 2\theta = 2\cos^2\theta - 1$$
$$\frac{3}{4} = 2\cos^2\theta - 1$$
$$\cos^2\theta = \frac{7}{8}$$
$$\cos\theta = \sqrt{\frac{7}{8}} \quad (+\text{ve as } \theta \text{ is acute})$$

d)
$$P(-2) = -16 + 44 - 34 + 6$$
$$= 0$$

$\therefore \ (x+2)$ is a factor

$$\therefore \ (x+2)(2x^2 + 7x + 3)$$
$$= (x+2)(2x+1)(x+3)$$

e)
$$-b \le x - a \le b$$
$$a - b \le x \le a + b$$

$\therefore \ a - b = 2$ $\qquad$ $\therefore \ a = 3\frac{1}{2}, \ b = 1\frac{1}{2}$

$a + b = 5$

Teacher's Name: **Student's Name/N°:**

3. a) $2x^2+3x-4 \equiv A(x-1)^2 + Bx(x-2) + C(x-1)$

let $x=1$: $1 = -B \Rightarrow B=-1$

look at x^2: $2 = A+B \Rightarrow A=3$

let $x=0$: $-4 = A-C \Rightarrow C=7$

b) parallel $\Rightarrow$ equal gradients

$$-\frac{1}{2} = \frac{-(2+k)}{3k-1}$$

$$3k-1 = 4+2k$$

$$k=5$$

c) $2\sin x - 5\cos x = 2$

$$2\left(\frac{2t}{1+t^2}\right) - 5\left(\frac{1-t^2}{1+t^2}\right) = 2$$

$$4t - 5 + 5t^2 = 2 + 2t^2$$

$$3t^2 + 4t - 7 = 0$$

$$(3t+7)(t-1) = 0$$

$$t = -\frac{7}{3}, 1$$

$$\tan\frac{x}{2} = -\frac{7}{3}, 1$$

$$\frac{x}{2} = 113.2°, 45°$$

$$x = 226°, 90°$$

d) i) $m = \dfrac{ap^2 - aq^2}{2ap - 2aq}$

$$= \frac{a(p-q)(p+q)}{2a(p-q)}$$

$$= \frac{p+q}{2}$$

Teacher's Name: **Student's Name/N°:**

ii) $y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$y - ap^2 = \left(\frac{p+q}{2}\right)x - ap^2 - apq$

$y = \left(\frac{p+q}{2}\right)x - apq$

4. a) $y = \frac{1}{8}x^2 + x$

$8y = x^2 + 8x$

$8y + 16 = (x+4)^2$

$4 \times 2(y+2) = (x+4)^2$

$\therefore$ Vertex is $(-4, -2)$

focus is $(-4, 0)$

b)
$$\begin{array}{r|l} & x \\ \hline x^2 - 1 & x^3 + 4x - 4 \\ & x^3 - x \\ \hline & \quad\quad 5x - 4 \end{array}$$

$\therefore$ remainder $= 5x - 4$

Teacher's Name: **Student's Name/N°:**

c) Solve simultaneously (1 solution)

$$x^2 - 2x = kx - 16$$
$$x^2 - 2x - kx + 16 = 0$$
$$x^2 - (2+k)x + 16 = 0$$

1 solution $\Rightarrow \Delta = 0$

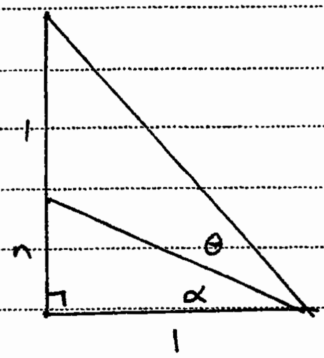
$$(2+k)^2 - 4 \times 16 = 0$$
$$k^2 + 4k - 60 = 0$$
$$(k+10)(k-6) = 0$$
$$k = -10,\ 6$$

d) $$2\sin 2\theta = 3\tan\theta$$
$$4\sin\theta\cos\theta = \frac{3\sin\theta}{\cos\theta}$$
$$4\sin\theta\cos^2\theta - 3\sin\theta = 0$$
$$\sin\theta(4\cos^2\theta - 3) = 0$$
$$\sin\theta = 0 \qquad \cos\theta = \pm\frac{\sqrt{3}}{2}$$

$$\theta = 0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Teacher's Name: ______________ **Student's Name/Nº:** ______________

5. a) $\tan\alpha = n$ $\quad\quad \tan(\theta+\alpha) = n+1$



$$\tan(\theta+\alpha) = n+1$$

$$\frac{\tan\theta + \tan\alpha}{1-\tan\theta\tan\alpha} = n+1$$

$$\frac{\tan\theta + n}{1-n\tan\theta} = n+1$$

$$\tan\theta + n = n+1 - n^2\tan\theta - n\tan\theta$$

$$\tan\theta + n^2\tan\theta + n\tan\theta = 1$$

$$\tan\theta = \frac{1}{n^2+n+1}$$

b) i) $y = \frac{x^2}{8}$

$y' = \frac{x}{4}$ $\quad$ when $x = 4p$

$$m_T = \frac{4p}{4}$$

$$= p$$

$$\therefore\ y - 2p^2 = p(x-4p)$$

$$y - 2p^2 = px - 4p^2$$

$$y = px - 2p^2$$

ii) when $x = 0$

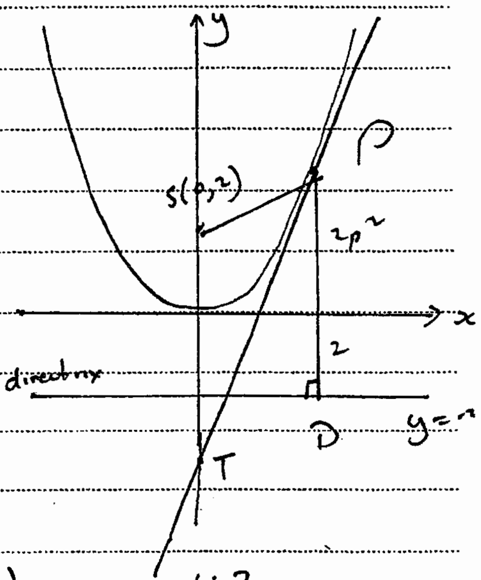
$$y = -2p^2$$

$$\therefore\ T(0, -2p^2)$$

Teacher's Name: **Student's Name/Nº:**

iii)

$TS = 2 + 2p^2$



$PS = PD$ (by definition)

$= 2 + 2p^2$

$= TS$

iv) $P(4p, 2p^2)$ $T(0, -2p^2)$ $1:3$

$\therefore M\left(\dfrac{3\times 4p + 0}{4}, \dfrac{3\times 2p^2 + 1\times -2p^2}{4}\right)$

$M(3p, p^2)$

$\therefore x = 3p, \quad y = p^2$

$\therefore p = \dfrac{x}{3}$

$\therefore y = \dfrac{x^2}{9}$ is the locus.

Teacher's Name: ______________ **Student's Name/N°:** ______________

6. a) i) $PA = 2 \times PB$

$$\sqrt{(x-5)^2 + (y-6)^2} = 2 \times \sqrt{(x+1)^2 + y^2}$$

$$x^2 - 10x + 25 + y^2 - 12y + 36 = 4\left[x^2 + 2x + 1 + y^2\right]$$

$$3x^2 + 18x + 3y^2 + 12y = 57$$

$$x^2 + 6x + y^2 + 4y = 19$$

which is a circle

ii) $x^2 + 6x + 9 + y^2 + 4y + 4 = 19 + 9 + 4$

$$(x+3)^2 + (y+2)^2 = 32$$

$\therefore$ Centre $(-3, -2)$

radius $\sqrt{32}$ units

b) let roots be $\alpha, \beta, \alpha + \beta$

i) $\alpha + \beta + \alpha + \beta = -6$ (sum of roots)

$$\alpha + \beta = -3$$

$\therefore -3$ is a root

sub into equation

$$(-3)^3 + 6(-3)^2 - (-3) + m = 0$$

$$m = -30$$

ii) $x^3 + 6x^2 - x - 30 = 0$ has -3 as a root

$\therefore (x+3)$ is a factor

$\therefore (x+3)(x^2 + 3x - 10) = 0$ by inspection

$$(x+3)(x+5)(x-2) = 0$$

$\therefore x = -3, -5, 2$

Teacher's Name: Student's Name/N°:

c) $\dfrac{\tan 2\theta - \tan\theta}{\cot\theta + \tan 2\theta}$

$$= \frac{\dfrac{2\tan\theta}{1-\tan^2\theta} - \tan\theta}{\dfrac{1}{\tan\theta} + \dfrac{2\tan\theta}{1-\tan^2\theta}}$$

$$= \frac{2\tan^2\theta - \tan^2\theta\,(1-\tan^2\theta)}{1-\tan^2\theta + 2\tan^2\theta}$$

$$= \frac{\tan^2\theta + \tan^4\theta}{1+\tan^2\theta}$$

$$= \tan^2\theta$$