SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 YEARLY EXAMINATION

PRELIMINARY HSC ASSESSMENT TASK 3

2012

Mathematics Extension 1

General Instructions

- Working time – 90 minutes
- Write using black or blue pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Diagrams are not drawn to scale

Total marks

- Attempt Questions 1- 6
- All questions are of equal value
- Start each question on a new page

Name: ______________________

Teacher: ______________________

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	TOTAL

QUESTION 1 **MARKS**

a) $A(-7,5)$ and $B(3,1)$ are two points. 2
Find the co-ordinates of the point P which divides the interval AB externally in the ratio 3:1

b) Solve $\frac{x}{x^2-4} \geq 0$ 2

c) If $x^2 - 7x + 3 = 0$ has two real roots $\propto$ and β, find the value of

i) $\propto + \beta$ 1

ii) $\propto \beta$ 1

iii) $\propto^3 + \beta^3$ 3

d) If α, β and δ are the roots of $x^3 + 2x^2 + 4x - 5 = 0$ find the value of $(\propto -1)(\beta - 1)(\delta - 1)$ 2

QUESTION 2 **MARKS**

a) For the function $f(x) = \frac{2x-1}{1-x^2}$

i) state the behaviour of the curve as x approaches ∞ and $-\infty$ 2

ii) write down the equations of any horizontal or vertical asymptotes 2

iii) sketch the curve showing all important features 2

b) i) Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $A\cos(\theta - \propto)$ where $A > 0$ and $0 \leq \propto \leq \frac{\pi}{2}$ 2

ii) Solve $\cos\theta + \sqrt{3}.\sin\theta = 1$ for $0 \leq \theta \leq 2\pi$ 3

QUESTION 3

MARKS

a) Find the acute angle to the nearest degree between the vertical line 3
$x = 2$ and $3x - 2y + 1 = 0$

b) Derive the equation of the normal at the point $(2ap, ap^2)$ on the parabola $x^2 = 4ay$ 3

c) Factorise fully the polynomial $P(x) = x^3 - x^2 - 10x - 8$ 3

d) Solve $|2x - 1| = 4x + 3$ 2

QUESTION 4

MARKS

a) i) Show that $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$ 1

ii) Hence or otherwise find the exact value of $2 \sin 285 \cos 45$ 2

b) Find values of k such that $x^2 + (k + 1)x + 4$ is positive definite 3

c) Find the area of a regular pentagon of side length 6m (give answer to the nearest whole number) 3

d) Find the vertex and focus of the parabola $y^2 = -3x + 6$ 2

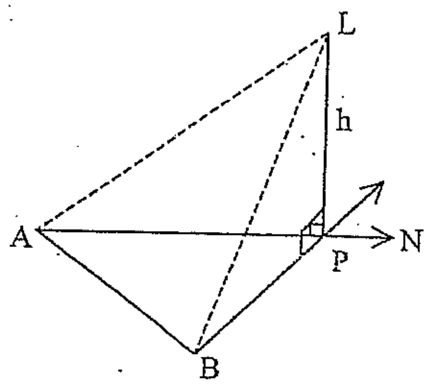
QUESTION 5 MARKS

a) One of the roots of $x^2 - (k+1)x + 2k + 2 = 0$ is twice the other root. Find the value of k 3

b) i) A monic polynomial is odd, has a single root at $x = -1$ and a double root at $x = 3$. Find its equation, if it is of degree 7 (leave in factorised form) 2

ii) Sketch this polynomial 1

c)



PL is a vertical pole, h metres high, standing on horizontal ground. PA is a shadow of the pole when the direction of the sun is due north and when the angle of elevation of the sun is 45°. PB is the shadow cast by the pole when the bearing of the sun is 345°T and its angle of elevation is 31°. The distance AB is 15 metres.

i) Draw a top view sketch and label the information given. 1

ii) Find the height h of the pole to the nearest metre 4

QUESTION 6 MARKS

a) Find the remainder when the polynomial $P\ (x) = x^3 - 3x + 1$ is divided by $x^2 - 4$ 2

b) i) What is the equation of the chord of contact to the parabola $x^2 = 8y$, from the external point $T\ (x_{0},\ y_0)$ 1

ii) If the chord of contact meets the parabola at P and Q, show that the midpoint of PQ is 4

$$M(\,x_{0,}\ \frac{x_{0}{}^{2}}{4} - y_0\,)$$

iii) If the point T lies on the line $x + 2y + 5 = 0$, find the locus of M. 4

Student Name: ____________________ Teacher Name: ____________________

2012 Ext. 1 Yearly Solutions.

1a) $B \cdot (3,1)$ $\cdot P(x,y)$

A $\cdot$ $(-7,5)$

$k : l = -3 : 1$

$x = \frac{-3\times 3 + 1\times -7}{-3+1}$ $y = \frac{-3\times 1 + 1\times 5}{-3+1}$

$= 8$ $= -1$

$(8, -1)$

b) $\frac{x}{x^2-4} \geq 0$

Critical pts

$x = 0, \pm 2$

$-2 < x \leq 0, \quad x > 2$

c) (i) 7

(ii) 3

(iii) $\alpha^3 + \beta^3$

$(\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$(\alpha+\beta)[(\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta]$

$(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$

$7[7^2 - 3\times 3]$

$= 280$

d) $x^3 + 2x^2 + 4x - 5 = 0$

$\alpha+\beta+\gamma = -2, \quad \alpha\beta\gamma = 5, \quad \alpha\beta + \alpha\gamma + \beta\gamma = 4$

$(\alpha-1)(\beta-1)(\gamma-1)$

$(\alpha\beta - (\alpha+\beta) + 1)(\gamma - 1)$

$\alpha\beta\gamma - \alpha\beta - \gamma(\alpha+\beta) + \alpha + \beta + \gamma - 1$

$\alpha\beta\gamma - (\alpha\beta + \gamma\alpha + \gamma\beta) + (\alpha+\beta+\gamma) - 1$

$5 - 4 + -2 - 1 = -2$

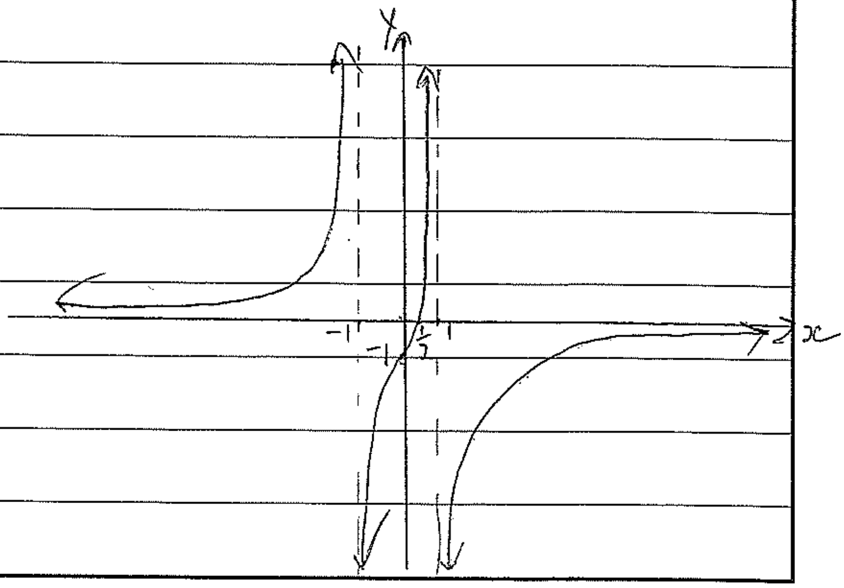
2a) (i) $f(x) = \frac{2x-1}{1-x^2}$

As $x \to \infty$, $f(x) \to 0 \uparrow$

As $x \to -\infty$, $f(x) \to 0 \downarrow$

(ii) $x = \pm 1$ vertical

$y = 0$ horizontal



Student Name: ______________________ Teacher Name: ______________________

b)(i) $A = \sqrt{1^2 + (\sqrt{3})^2} = \underline{2}$

$\alpha = \tan^{-1}\sqrt{3} \quad \therefore \underline{\alpha = \frac{\pi}{3}}$

(ii) $2\cos\left(\theta - \frac{\pi}{3}\right) = 1$

$\cos\left(\theta - \frac{\pi}{3}\right) = \frac{1}{2}$

$\theta - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}$

$\underline{\theta = 0, \frac{2\pi}{3}, 2\pi}$

3a) Use $m=0$ (complementary $\angle$ for $x=2$)

$3x - 2y + 1 = 0$

$2y = 3x + 1$

$y = \frac{3}{2}x + \frac{1}{2}$

$m = \frac{3}{2}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{\frac{3}{2} - 0}{1 + \frac{3}{2} \times 0}\right|$

$\theta = 56°19'$

$\therefore$ Angle is $90 - 56°19'$

$\underline{33°}$

b) $x^2 = 4ay$

$x = 2ap \quad y = ap^2$

$\frac{dx}{dp} = 2a \quad \frac{dy}{dp} = 2ap$

$\frac{dy}{dx} = \cancel{\frac{\frac{dy}{dp}}{\frac{dx}{dp}}} = \frac{\cancel{2ap}}{\cancel{2a}}$

$= p$

$\therefore$ gradient of normal $= \frac{-1}{p}$

Now $y - ap^2 = \frac{-1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$\underline{x + py = 2ap + ap^3}$

c) $P(x) = x^3 - x^2 - 10x - 8$

$P(1) \neq 0, \; P(-1) \neq 0$

$P(2) \neq 0, \; P(-2) = 0 \; \therefore$

$(x+2)$ is a factor

$P(x) = (x+2)(x^2 - 3x - 4)$

by inspection

$\underline{\therefore P(x) = (x+2)(x+1)(x-4)}$

Student Name: ______________________ Teacher Name: ______________________

d) $|2x-1| = 4x+3$

$2x-1 = 4x+3$ $\qquad$ $2x-1 = -(4x+3)$

$-4 = 2x$ $\qquad$ $2x-1 = -4x-3$

$x = -2$ $\qquad$ $6x = -2$

$x = \frac{-1}{3}$

since $4x+3<0$ when $x=-2$

it is no solution

$\therefore x = \frac{-1}{3}$

4 a(i) $\sin(A+B) + \sin(A-B)$

$\sin A\cos B + \cancel{\sin B\cos A} + \sin A\cos B - \cancel{\sin B\cos A}$

$2\sin A\cos B$

(ii) $2\sin 285 \cos 45$

$= \sin 330^\circ + \sin 240$

$= -\sin 30 - \sin 60$

$= \frac{-1}{2} - \frac{\sqrt{3}}{2}$

$= \frac{-1-\sqrt{3}}{2}$

b) $x^2 + (k+1)x + 4$

$\Delta < 0$

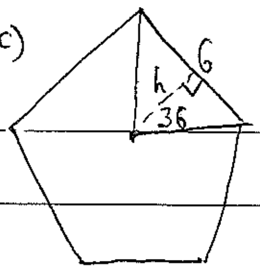
$(k+1)^2 - 4\times1\times4 < 0$

$k^2+2k-15 < 0$

$(k-3)(k+5) < 0$

$-5 < k < 3$

c)



$\tan 36 = \frac{3}{h}$

$h = \frac{3}{\tan 36}$

$= 4.129$

Area $= 5 \times \frac{1}{2} \times 6 \times 4.129$

$= 62\,m^2$

d) $y^2 = -3x+6$

$y^2 = -3(x-2)$

Vertex $(2,0)$

$a = \frac{3}{4}$

$\therefore$ Focus $(1\frac{1}{4}, 0)$

Student Name: ______________________ Teacher Name: ______________________

5 a) $x^2-(k+1)x+2k+2=0$

Let roots be $\alpha, 2\alpha$

$\alpha+2\alpha = k+1$ (1)

$2\alpha^2 = 2k+2$ (2)

From (1), $\alpha = \frac{k+1}{3}$

sub. into (2)

$2\left(\frac{k+1}{3}\right)^2 = 2k+2$

$2k^2+4k+2 = 18k+18$

$2k^2-14k-16=0$

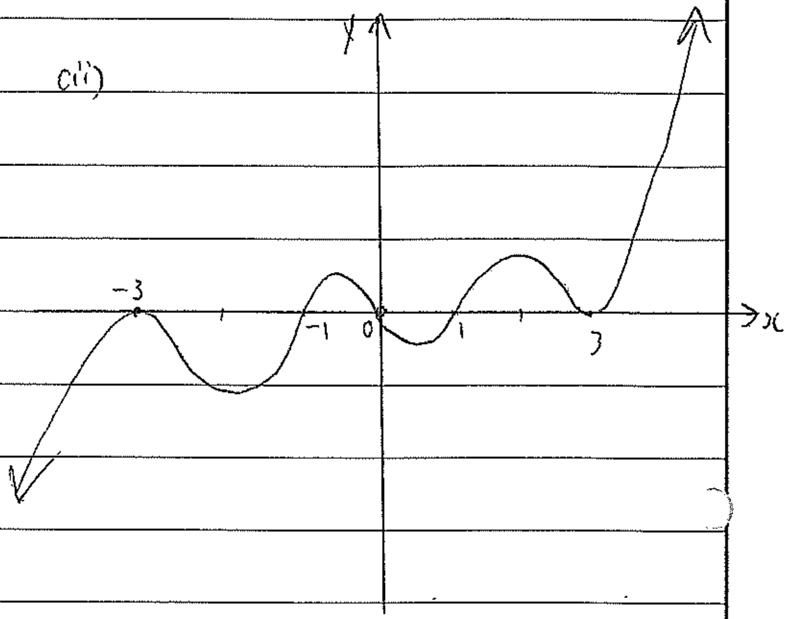
$k^2-7k-8=0$

$(k+1)(k-8)=0$

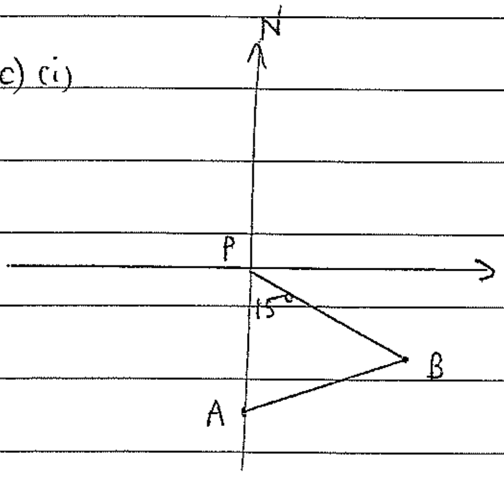
$\underline{k=-1 \text{ or } 8}$

b) (i) $P(x)=(x+1)(x-1)(x-3)^2(x+3)^2x$

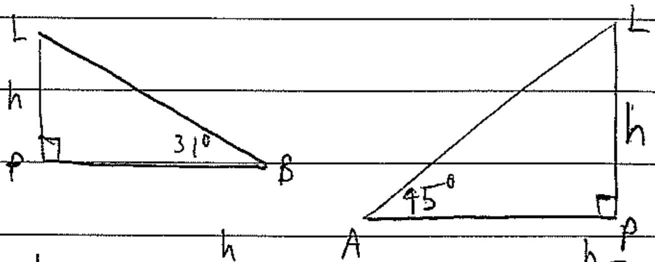
(ii)



c) (i)



(ii)

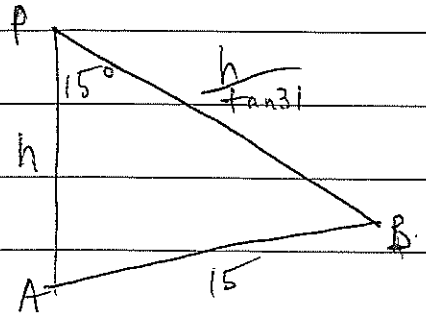


$\tan 31 = \frac{h}{BP}$

$BP = \frac{h}{\tan 31}$

$\tan 45 = \frac{h}{AP}$

$AP = h$



$15^2 = h^2 + \frac{h^2}{\tan^2 31} - 2 \times \frac{h^2}{\tan^2 31} \times \cos 15$

$225 = h^2\left(1 + \frac{1}{\tan^2 31} - \frac{2\cos 15}{\tan 31}\right)$

$h^2 = 405$

$\underline{h = 20\text{ m}}$ to nearest metre

Student Name: ______________________ Teacher Name: ______________________

6 a) $P(x) = x^3 - 3x + 1$

Let remainder be $ax + b$

$\therefore P(2) = 3 = 2a + b$

$P(-2) = -1 = -2a + b$

$\therefore 2 = 2b$

$b = 1 \therefore a = 1$

remainder is $x + 1$

b) (i) $xx_0 = 4(y + y_0)$

(ii) Solve $x^2 = 8y$ and $xx_0 = 4(y + y_0)$ simultaneously

$y = \frac{x^2}{8}$

$\therefore xx_0 = 4\left(\frac{x^2}{8} + y_0\right)$

$2xx_0 = x^2 + 8y_0$

$x^2 - 2x_0 x + 8y_0 = 0$

Now let roots be α and β

$\therefore \alpha + \beta = 2x_0$

$\therefore \frac{\alpha + \beta}{2} = x_0$ is x value of midpoint

Now sub. $x = x_0$ into chord of contact to get y value.

$x_0^2 = 4(y + y_0)$

$\frac{x_0^2}{4} = y + y_0$

$y = \frac{x_0^2}{4} - y_0$

$\therefore M$ is $\left(x_0, \frac{x_0^2}{4} - y_0\right)$

c(iii) T lies on $x + 2y + 5 = 0$

$\therefore x_0 + 2y_0 + 5 = 0$ (1)

Student Name: ______________________ Teacher Name: ______________________

Now for locus of M :

$$x = x_0 \qquad y = \frac{x_0^2}{4} - y_0$$

$$y_0 = \frac{-x_0 - 5}{2}$$

$$\therefore y = \frac{x_0^2}{4} - \left(\frac{-x_0 - 5}{2}\right)$$

$$\therefore y = \frac{x^2}{4} + \frac{x + 5}{2}$$

$$4y = x^2 + 2(x+5)$$

$\underline{4y = x^2 + 2x + 10 \text{ is locus of } M}$