Class Teacher____________________ Name ____________________

SYDNEY TECHNICAL HIGH SCHOOL

MANNERS MAKYTH MAN

MATHEMATICS

Year 11 Preliminary HSC Course

ASSESSMENT TASK 3

September 2013

Time Allowed: 90 minutes

Instructions:

- Write using blue or black pen.
- Approved calculators may be used.
- Attempt all questions.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new side of a page.

Total Marks 73

Section 1 Multiple Choice	Section 2
5 Marks	68 Marks
Answer on sheet	Allow 82 minutes for this section
Allow 8 minutes for this section	

1 What is the acute angle between the lines $2x-y-7=0$ and $3x-5y-2=0$?

(A) 4°24`

(B) 32° 28`

(C) 57°32`

(D) 85°36`

2 If $t=\tan\frac{\theta}{2}$ which of the following expressions is equivalent to $4\sin\theta+3\cos\theta+5$?

(A) $\frac{2(t+2)^2}{1-t^2}$

(B) $\frac{(t+4)^2}{1-t^2}$

(C) $\frac{2(t+2)^2}{1+t^2}$

(D) $\frac{(t+4)^2}{1+t^2}$

3 The interval *DE*, where *D* is $(4,5)$ and *E* is $(19,-5)$, is divided internally in the ratio 2:3 by the point (x,y). What are the values of *x* and *y*?

(A) $(-16,25)$

(B) $(10,1)$

(C) $(12,0)$

(D) $(13,-1)$

4 What is the solution to the inequality? $\frac{3}{x-2}\le 4$

(A) $x<-2$ and $x\ge -\frac{11}{4}$

(B) $x>-2$ or $x\le -\frac{11}{4}$

(C) $x<2$ or $x\ge \frac{11}{4}$

(D) $x>2$ and $x\le \frac{11}{4}$

5 Consider the polynomial $P(x)=3x^3+3x+a$.
If $x-2$ is a factor of $P(x)$, what is the value of *a*?

(A) -30

(B) -18

(C) 18

(D) 30

Question 6 **(11 Marks)** Start a new page **Marks**

a) A tangent to a curve makes an angle of $60°$ to the positive x axis. What is the exact gradient of this tangent. **1**

b) Write $\sec\theta$ in terms of t where $t = \tan\frac{\theta}{2}$ **1**

c) Find a and b if $\frac{\sqrt{7}-5}{-2+3\sqrt{7}} = a + b\sqrt{7}$ **2**

d) By finding an expression for the tangent of the angle between pairs of lines, prove that the triangle formed by the intersection of these 3 lines is an isosceles triangle. You must make your conclusions clear. **3**

$x - \sqrt{3}y + 5 = 0$ and

$x - y + 10 = 0$ and

$\sqrt{3}x - y + 10 = 0$

e) Consider the curve with equation $y = \frac{x-1}{x+2}$.

i. Determine the equation(s) of any asymptotes **2**

ii State the range of the function. **1**

iii State the domain of the function. **1**

End of Question 6

Question 7 (11 Marks) Start a new page

a) If $2x^2 - 9x + 9 \equiv (ax - b)(x - b)$ for all values of x, find the values of a and b 2

b) Derive the equation $\sin 2\theta$ and use this to find the exact value of $\cos 15° \sin 15°$ 2

c) Find the cartesian equation of the curve:

$$x = \sin t \text{ and } y = \sec t$$ 2

d) Find the equation of the tangent line to the curve $y = x^4 + 3x^2 - 1$ at $x = 1$. 2

e) Find the locus of a point P(x,y) which is equidistant from $3x + 4y = 36$ and $4x + 3y = 24$ 3

End of Question 7

Question 8 **(11 Marks)** Start a new page **Marks**

a) Using the auxillary angle method, or otherwise, solve
$6\sin\theta + 8\cos\theta = 4$ for $0° \le \theta \le 360°$ **3**

b) Differentiate and write in simplest factorised form:

i) $(3x^2+2)^{10}$ **2**

ii) $\dfrac{2x^2}{\sqrt{(x^2-4)}}$ **2**

c) By fully factorizing *f(x)* (where possible), sketch the graph of
$f(x) = x^3 - 4x^2 + 8x - 8$ ***identifying all key points*** **4**

End of Question 8

Questin 9 **(10 Marks)** Start a new page Marks

a) Solve for x: $3^{2x} = 6(3)^{x-1} + 3$ 2

b) If α, β, γ are the roots of $3x^3 - 4x^2 + 7x - 11 = 0$

Find:

i) $\alpha + \beta + \gamma$ 1

ii) $\alpha\beta\gamma$ 1

iii) $(\alpha+1)(\beta+1)(\gamma+1)$ 2

c) By equating the coefficients of sin x and cos x, or otherwise, find constants A,B satisfying the identity: 2

$$A\,(2\sin x + \cos x) + B\,(2\cos x - \sin x) \equiv \sin x + 8\cos x$$

d) Prove the trigonometric identity: $\dfrac{\cos 2x}{(\cos x + \sin x)^3} = \dfrac{\cos x - \sin x}{1 + \sin 2x}$. 2

End of Question 9

Question 10 (11 Marks) Start a new page

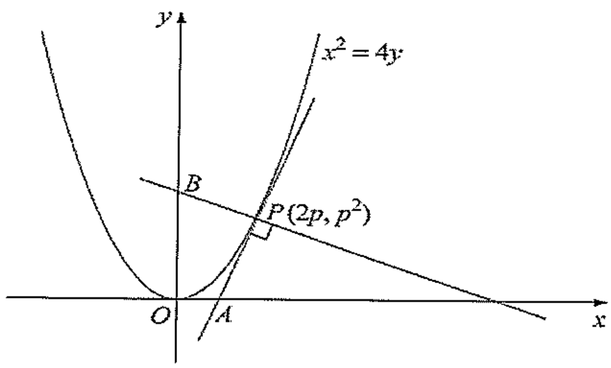
a) Let $P(x) = (x + 1)(x - 3)\, Q(x) + ax + b$ where $Q(x)$ is a polynomial and a and b are real numbers. The polynomial $P(x)$ has a factor of $(x - 3)$. When $P(x)$ is divided by $x + 1$ the remainder is 8.

i. Find the values of a and b **2**

ii. Find the remainder when P(x) is divided by $\left(x^2 - 2x - 3\right)$ **1**

b) From what external point are the tangents to the parabola $x^2 = 8y$ to be drawn so that $4y = 2x + 4$ is the equation of the chord of contact? **2**

c)



The diagram shows the graph of the parabola $x^2 = 4y$. The tangent to the parabola at $P\left(2p, p^2\right)$, $p > 0$, cuts the x axis at A. The normal to the parabola at P cuts the y axis at B.

i) Show that the equation of the tangent AP is $y = px - p^2$ **2**
(Show all working)

ii) Find the coordinates of A **1**

iii) Show that B has coordinates $\left(0,\ p^2 + 2\right)$ **1**

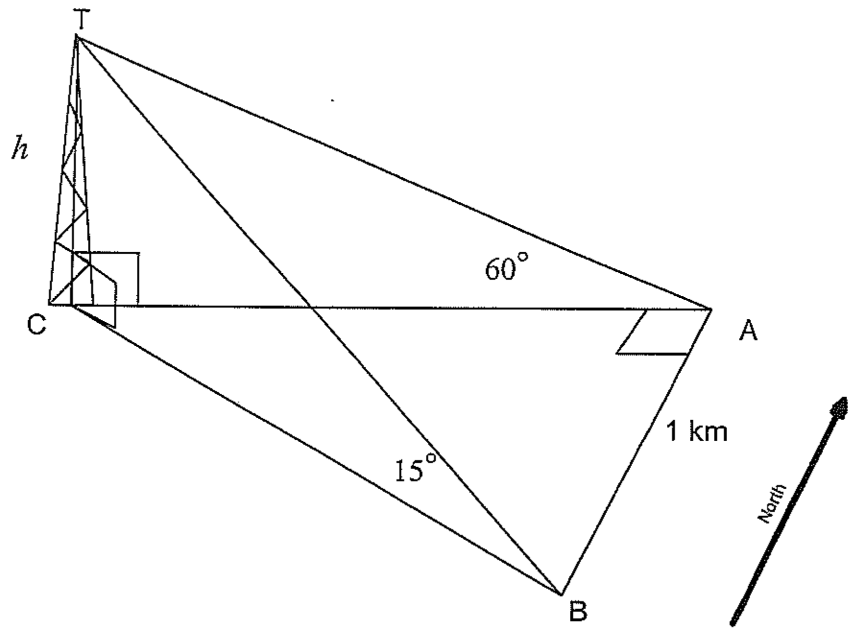
iv) Let C be the midpoint of AB.
Find the Cartesian equation of the locus of C **2**

End of Question 10

Question 11 (12 Marks) Start a new page

a) Find the equation of the normal to the parabola $x^2 = 12y$ at the point where $x = -2$ 2

b) The angle of elevation of the top of a tower (T) from a point A due East of the tower is 60°. From a point B due South of A, the angle of elevation of T is 15°. A and B are at the same elevation as the base of the tower.



The distance $AB = 1$ km

i) Show that $3h^2 \cot^2 15° = 3 + h^2$ 2

ii) Find the value of h to the nearest metre 2

c) Show algebraically that the line $y = x - 4$ is a tangent to the circle $x^2 + y^2 = 8$ and find the coordinates of the point of contact 3

Question 11 continues over the page

d) A piece of wire 24 metres long is cut into two parts, one of which is used to form a square, and the other to form a rectangle whose length is three times its width.

i) If the width of the rectangle is x show that A, the sum of the areas of the rectangle and the square is given by $A=7x^2-24x+36$ 1

ii) Find the vertex of the parabola $A=7x^2-24x+36$ 1

iii) Find the lengths of the two parts if the sum of the areas given in part (i) is a minimum. ***Do not use calculus*** 1

End of Question 11

$\tan 60 = \sqrt{3}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \cos\theta = \frac{1-t^2}{1+t^2}$

$= 1+t^2/1-t^2$

$\frac{\sqrt{7}-5}{3\sqrt{7}-2} \times \frac{3\sqrt{7}+2}{3\sqrt{7}+2} = \frac{21+2\sqrt{7}-15\sqrt{7}-10}{63-4}$

$= \frac{11-13\sqrt{7}}{59}$

$a = 11/59 \quad b = -13/59$

$x - \sqrt{3}y + 5 = 0$ —— (1)

$-\sqrt{3}y = -x + 5$

$y = \frac{1}{\sqrt{3}}x + 5/\sqrt{3}$

$m_1 = \frac{1}{\sqrt{3}}$

$x - y + 10 = 0$

$y = x + 10$

$m_2 = 1$

$\tan\theta = \left|\frac{m_2 - m_1}{1+(m_1)(m_2)}\right|$

$= \frac{1-\frac{1}{\sqrt{3}}}{1+(1/\sqrt{3})(1)}$

$= \left|\frac{1-1/\sqrt{3}}{1+1/\sqrt{3}}\right|$

$\theta = 15^\circ$

(ii) $\sqrt{3}x - y + 10 = 0$

$y = \sqrt{3}x + 10$

$m_1 = \sqrt{3} \quad y = x + 10$

$m_2 = 1$

$\tan\theta = \left|\frac{1-\sqrt{3}}{1+\sqrt{3}}\right|$

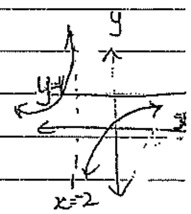
$\theta = 15^\circ$

∴ isosceles triangle

(e) $y = \frac{x-1}{x+2}$

$= \frac{x+2-3}{x+2}$

$= 1 - \frac{3}{x+2}$



(i) HA $y = 1$ VA $x = -2$.

(ii) R: $y < 1 \quad y > 1$

(iii) D: $x < -2 \quad x > -2$

Q7

$2x^2 - 9x + 9 \equiv (ax-b)(x-b)$

$ax^2 - abx - bx + b^2$

$ax^2 - x(ab+b) + b^2$

equating $a = 2$

$ab + b = +9$

$2b + b = +9$

$3b = +9$

$b = +3$

$a = 2 \quad b = +3$

(b) $\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$

$= 2\sin\theta\cos\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$

$\cos 15 \sin 15 = \frac{1}{2}\sin 2\theta$

$= 1/2 \sin 30$

$= (1/2)(1/2)$

$= 1/4$

(c) $x = \sin t \quad y = \sec t$

$x^2 = \sin^2 t \quad y^2 = \frac{1}{\cos^2 t}$

$y^2 = \frac{1}{1-\sin^2 t}$

$y^2 = \frac{1}{1-x^2}$

$y = \frac{1}{\pm\sqrt{1-x^2}}$

(d) $y = x^4 + 3x^2 - 1$

$\frac{dy}{dx} = 4x^3 + 6x$

at $x = 1$, $y = 3$ } $m = 10$

$y - 3 = 10(x-1)$

$y - 3 = 10x - 10$

$y = 10x - 7$

$10x - y - 7 = 0$

(e) $3x + 4y = 36 \quad 4x + 3y = 24$

$\left|\frac{Ax+By+C}{\sqrt{a^2+b^2}}\right| = \left|\frac{Ax_1+By_1+C}{\sqrt{a^2+b^2}}\right|$

$\left|\frac{3x+4y-36}{\sqrt{9+16}}\right| = \left|\frac{4x+3y-24}{5}\right|$

(+)

$5(3x+4y-36) = 5(4x+3y-24)$

$-x - y - 12 = 0$

$x - y + 12 = 0$

(−)

$-3x - 4y + 36 = 4x + 3y - 24$

$-7x - 7y + 60 = 0$

$7x - 7y - 60 = 0$

Q8

(a) $6\sin\theta + 8\cos\theta = 4$

$r = 10 \quad \alpha = 53^\circ 8'$

$10\sin(\theta + 53^\circ 8') = 4$

$\sin(\theta + 53^\circ 8') = 0.4$

$\theta + 53^\circ 8' = 23^\circ 35'$

$\theta = 103^\circ 17', 330^\circ 27'$

OR

$6\left(\frac{2t}{1+t^2}\right) + 8\left(\frac{1-t^2}{1+t^2}\right) = 4$

$12t + 8 - 8t^2 = 4(1+t^2)$

$-12t^2 + 12t + 4 = 0$.

$3t^2 - 3t - 1 = 0$

$t = \frac{3 \pm \sqrt{9-(4)(3)(-1)}}{6}$

$= \frac{3 \pm \sqrt{21}}{6}$

$\tan\frac{\theta}{2} = \frac{3+\sqrt{21}}{6} \qquad \tan\frac{\theta}{2} = \frac{3-\sqrt{21}}{6}$

$\theta/2 = 51^\circ 39' \qquad \theta/2 = 165^\circ 14'$

$\theta = 103^\circ 18' \qquad \theta = 330^\circ 27'$

(b)(i) $y = (3x^2+2)^{10}$

$\frac{dy}{dx} = (10)(3x^2+2)(6x)$

=

(ii) $y = (2x^2)(x^2-4)^{-1/2}$

$u = 2x^2 \qquad v = (x^2-4)^{-1/2}$

$u' = 4x \qquad v' = (-1/2)(x^2-4)^{-3/2}(2x)$

$v' = (-x)(x^2-4)^{-3/2}$

$\frac{dy}{dx} = (2x^2)(-x)(x^2-4)^{-3/2} + (4x)(x^2-4)^{-1/2}$

$= \frac{(2x^2)(-x)}{\sqrt{(x^2-4)^3}} + \frac{4x}{\sqrt{x^2-4}}$

$= \frac{-2x^3 + 4x(x^2-4)}{\sqrt{x^2-4}\,(x^2-4)}$

$= \frac{-2x^3 + 4x^3 - 16x}{\sqrt{x^2-4}\,(x^2-4)}$

$= \frac{2x^3 - 16x}{(x^2-4)(\sqrt{x^2-4})}$

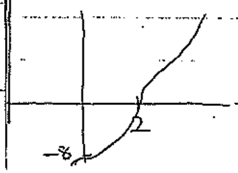
(d) $f(x) = x^3 - 4x^2 + 8x - 8$

$f(2) = 0$

$f(x) = (x-2)(x^2-2x+4)$

$\uparrow \Delta = 4 - (4)(1)(4)$

$\Delta < 0$ no real roots



Q9

(a) $3^{2x} = 6(3)^{-x} + 3$

$3^{2x} = 6(3)(3^{-x}) + 3$

$3^{2x} = (6)\frac{3^x}{3} + 3$

$3^{2x} = (2)(3^x) + 3$

Let $u = 3^x$

$u^2 - 2u - 3 = 0$

$(u-3)(u+1) = 0$

$u = 3 \qquad u = -1$

$3^x = 3 \qquad$ no soln.

$x = 1$

(b) $3x^3 - 4x^2 + 7x - 11 = 0$

(i) $\alpha + \beta + \gamma = -b/a = 4/3$

(ii) $\alpha\beta\gamma = -d/a = +11/3$

(iii) $(\alpha+1)(\beta+1)(\gamma+1)$

$= \gamma\alpha\beta + \gamma\alpha + \gamma\beta + \gamma + \alpha\beta + \alpha + \beta + 1$

$= +11/3 + 7/3 + 4/3 + 1$

$= 22/3 + 1$

$= 7\,1/3 + 1 \quad = 8\,1/3$

(c) $A(2\sin x + \cos x) + B(2\cos x - \sin x)$

$2A\sin x + A\cos x + 2B\cos x - B\sin$ [illegible]

$\sin x(2A - B) + \cos x(A + 2B) = \sin x + 8$ [illegible]

$2A - B = 1$ — ①

$A + 2B = 8$ — ② — ×2

$2A - B = 1$ — ①

$2A + 4B = 16$ — ③

$-5B = -15$

$B = 3$ sub into ①

$2A - 3 = 1$

$2A = 4$

$A = 2$

(d) $\frac{\cos 2x}{(\cos x + \sin x)^3} = \frac{\cos x - \sin x}{1 + \sin 2x}$

LHS

$= \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^3}$

$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2}$

$= \frac{\cos x - \sin x}{\cos^2 x + 2\sin x\cos x + \sin^2 x}$

$= \frac{\cos x - \sin x}{1 + 2\sin x} = RHS$

2010

(a) $P(x) = (x+1)(x-3)Q(x) + ax + b$

) $P(3)$ $3a + b = 0$ — ①

$P(-1)$ − ($-a + b = 8$) — ②

$4a = -8$

$\underline{a = -2}$ into ①

$-6 + b = 0$

$b = 6$

$a = -2 \quad b = 6$

ii) $P(x) = (x+1)(x-3)(Q(x)) + 2x + 6$

remainder $-2x + 6$

) $x^2 = 8y \qquad a = 2$

$4y = 2x + 4 \qquad xx_0 = 2a(y + y_0)$

$2x = 4y - 4$

$= (2)(2)(y-1)$

point $(2, -1)$

) $x^2 = 4y \qquad P(2p, p^2)$

$y = x^2/4$ Eqn tangent:-

$y' = x/2 \qquad y - p^2 = p(x - 2p)$

at $x = 2p \qquad y - p^2 = px - 2p^2$

$m = p \qquad \underline{y = px - p^2}$

(ii) at A $y = 0$

$0 = px - p^2$

$px = p^2$

$x = p$

$(p, 0)$

(iii) Eqn normal:- $m_1 = -1/p$

$y - p^2 = -1/p(x - 2p)$

$yp - p^3 = -x + 2p$

at $x = 0 \quad yp = p^3 + 2p$

$y = p^2 + 2$

B $(0, p^2 + 2)$

(iii) co-ords A

$y = px - p^2 \qquad$ at $y = 0$

$0 = px - p^2$

$x = p \qquad (p, 0)$

mid point $A(p, 0) \quad B(0, p^2 + 2)$

$x = \frac{0 + p}{2} \qquad y = \frac{p^2 + 2}{2}$

$2x = p \qquad y = \frac{(2x)^2 + 2}{2}$

$y = \frac{4x^2 + 2}{2}$

$y = 2x^2 + 1$

2011

(a) $x^2 = 12y$

$y = x^2/12$

$y' = 2x/12$

$y' = x/6$ at $x = -2 \qquad y = 1/3$

$m = -1/3 \qquad m_{norm} = 3$

$y - 1/3 = 3(x + 2)$

$3y - 1 = 9x + 18$

$9x - 3y + 19 = 0$

(b) $\tan 60 = h/AC \qquad \tan 15 = h/BC$

(i) $AC = h/\sqrt{3} \qquad BC = h/\tan 15$

$BC^2 = AB^2 + AC^2$

$\frac{h^2}{\tan^2 15} = 1 + \frac{h^2}{3}$

$3h^2 \cot^2 15 = 3 + h^2$

(ii) $\frac{3h^2}{\tan^2 15} = 3 + h^2$

$3h^2 = 3 + h^2(\tan^2 15)$

$3h^2 = 3\tan^2 15 + h^2 \tan^2 15$

$3h^2 - h^2 \tan^2 15 = 3\tan^2 15$

$h^2(3 - \tan^2 15) = 3\tan^2 15$

$h^2 = \frac{3\tan^2 15}{3 - \tan^2 15}$

$= 271.2$ m

$= 271$ m.

(c)

$y = x - 4 \qquad x^2 + y^2 = 8$

$x^2 + (x-4)^2 = 8$

$x^2 + x^2 - 8x + 16 - 8 = 0$

$x^2 - 4x + 4 = 0$

$\Delta = b^2 - 4ac$

$= 16 - (4)(1)(4)$

$= 0 \quad \therefore$ tangent

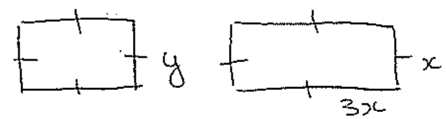
$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2 \quad y = -2$

$(2, -2)$

2011



$4y + 8x = 24$

$4y = 24 - 8x$

$y = 6 - 2x$

$A = y^2 + 3x^2$

$= (6 - 2x) + 3x^2$

$= 36 - 24x + 4x^2 + 3x^2$

$= 7x^2 - 24x + 36$

(ii) $x = \frac{-b}{2a}$

$= 24/14$

$= 12/7$

(iii) square:- $2^4/_7 \times 2^4/_7$

rectangle:- $1^5/_7 \times 5^1/_7$