SYDNEY TECHNICAL HIGH SCHOOL

Year 11

# PRELIMINARY HIGHER SCHOOL CERTIFICATE

## ASSESSMENT TASK 3

## SEPTEMBER 2016

# Mathematics Extension 1

General Instructions

- Working time – 90 minutes
- Write using black or blue pen
- Approved calculators may be used
- In questions 6 to 11, show relevant mathematical reasoning and/or calculations
- Start each question in section 2 on a new page
- Full marks may not be awarded for careless or badly arranged work

Total marks – 71

**Section 1 – 5 marks**

Attempt Questions 1 – 5

Allow about 8 minutes for this section.

**Section 2 – 66 marks**

Attempt Questions 6 – 11

Allow about 82 minutes for this section.

Name: ______________________

Teacher: ______________________

## Section 1 5 marks

Allow about 8 minutes for this section. Use the multiple choice answer sheet in your answer booklet for questions 1 – 5. Do not remove the multiple choice answer sheet from your answer booklet.

1. Which of the following is an expression for:

$$\frac{d}{dx}\left(x\sqrt{x^2+2}\right)?$$

A) $\frac{x}{\sqrt{x^2+2}}$

B) $1+\frac{x}{\sqrt{x^2+2}}$

C) $\frac{2x^2+2}{\sqrt{x^2+2}}$

D) $\frac{2x^2+x+4}{\sqrt{x^2+2}}$

2. A (-7, 10) and B (1, -2) are two points. What is the value of $k$ such that the point P($k$, 1) divides the interval AB internally in the ration of 3 : 1?

A) -5

B) -3

C) -1

D) 5

3. A curve has parametric equations $x = at$ and $y = \frac{1}{2}(a + at^2)$. What is the Cartesian equation of the curve?

A) $x^2 = 4ay$

B) $x^2 = 2a$

C) $x^2 = a(2y - a)$

D) $x^2 = 2y - a$

4. The graph of the even polynomial passes through the point (1, 2). What is the remainder when $P(x)$ is divided by $(x + 1)$?

A) -2

B) -1

C) 1

D) 2

5. Which of the following is an expression for:

$$\frac{sin4x}{sinx}$$

A) $\frac{1}{2}\cos 2x\ cosx$

B) $\cos 2x\ \cos x$

C) $2\cos 2x\ \cos x$

D) $4\cos 2x\ \cos x$

## Section 2 66 marks

Attempt questions 6 – 11.
Allow about 82 minutes for this section.

Answer each question in your answer booklet. Start each question on a new page.

### Question 6 **11 Marks**

a) Make $x$ the subject in the formula 2

$$y = \frac{x-3}{2-x}$$

b) Consider the polynomial $P(x) = x^3 + ax + b$ where $a$ and $b$ are real numbers. $(x-2)$ is a factor of $P(x)$ and when $P(x)$ is divided by $(x+1)$ the remainder is 6.

(i) Show that $2a + b = -8$ and $b - a = 7$ 2

(ii) Find the values of $a$ and $b$. 1

c) A parabola has equation

$$y^2 - 2y - 8x + 17 = 0$$

(i) Find the coordinates of its vertex. 1

(ii) Sketch the parabola showing its $x$ intercept 1

(iii) On your sketch, display the focus and directrix. 2

d) The equation $x^3 + 2x^2 + 3x + 1 = 0$ has roots $\alpha, \beta$ and $\gamma$. Find the value of $\alpha^2 + \beta^2 + \gamma^2$ 2

**Question 7** (Start a new page) **11 Marks**

a) Find the values of $a$ and $b$ if: 2

$$x^2 + 10x - 3 = ax(x+1) + b(x-1)^2$$

b) Given the polynomial $P(x) = -x^3 - ax$ is odd and has $(x+2)$ as a factor:

Find $a$ 1

(i) Sketch the graph of $y = P(x)$ showing the intercepts on the $x$ axis. 1

(ii) Solve the inequality $P(x) \leq 0$ 1

c) (i) Use the substitution $t = \tan\frac{x}{2}$ to show that the equation $3\,sinx - \cos x = 1$ is equivalent to the equation $t = \frac{1}{3}$ 2

(ii) Solve the equation $3sinx - cosx = 1$ over the domain $0° \leq \theta \leq 360°$ giving the answers correct to the nearest minute. 2

d) Solve the inequality $\frac{1}{x} - \frac{1}{x-2} > 0$ 2

**Question 8** (Start a new page) **11 Marks**

a) Solve the equation $\sqrt{2}\cos(x + 60°) - 1 = 0$ for $0° \leq x \leq 360°$ 3

b) P(2ap, ap²) and Q(2aq, aq²) are two points on the parabola $x^2 = 4ay$ with parameter values p = 2 and q = -1. Find the angle made correct to the nearest degree between the tangents at these points. 2

c) If $secx - tanx = k$ for some real number $k$, show that $secx + tanx = \frac{1}{k}$ 2

d) Show that $3x - 4y + 10 = 0$ is a tangent to the circle $x^2 + y^2 = 4$ 2

e) (i) Show that $\sin(A+B) + \sin(A-B) = 2sinAcosB$. 1

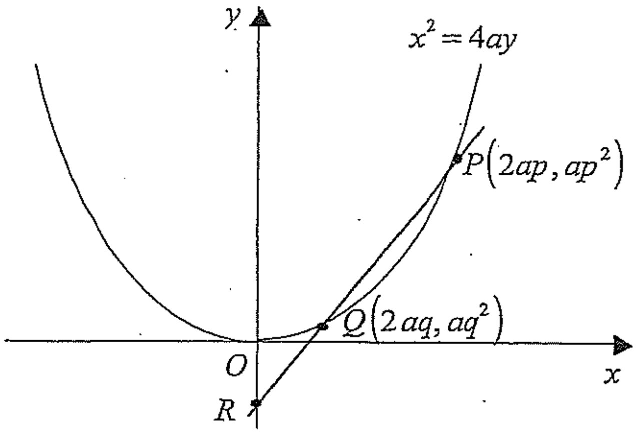
(ii) Hence or otherwise evaluate $2sin45cos15$ and leave your answer in exact form. 1

## Question 9 (Start a new page) 11 Marks

a) (i) Sketch $y = |x-2|$ 1

(ii) Hence or otherwise solve $|x-2| = 2-x$ 1

b)




P(2ap, ap²) and Q(2aq, aq²) are two points on the parabola $x^2 = 4ay$.

(i) Show that the chord PQ has equation $(p+q)x - 2y = 2apq$. 2

(ii) If P and Q move on the parabola such that $pq = 1$, where $p \neq 0$ and $q \neq 0$, show that the chord PQ (produced) always passes through a fixed pt. R on the $y$ axis. 1

c) Find all values of $k$ which will make the expression: 2

$(k+1)x^2 - 2(k-1)x + (k-5)$ a perfect square

d) The polynomial equation $x^3 + bx^2 + cx + d = 0$ has roots $\alpha, \alpha^2$ and $\alpha^3$ for some real number $\alpha \neq 0$

(i) Find in terms of c and d the value of $\frac{1}{\alpha} + \frac{1}{\alpha^2} + \frac{1}{\alpha^3}$. 2

(ii) Show that $b^3 d - c^3 = 0$ 2

Question 10 (Start a new page) 11 Marks

a) For what value(s) of $x$ is the function $y = \frac{x+1}{x-1}$ not differentiable and give a brief reason why not. 1

b) For $f(x) = \frac{x^2}{x^2-4}$

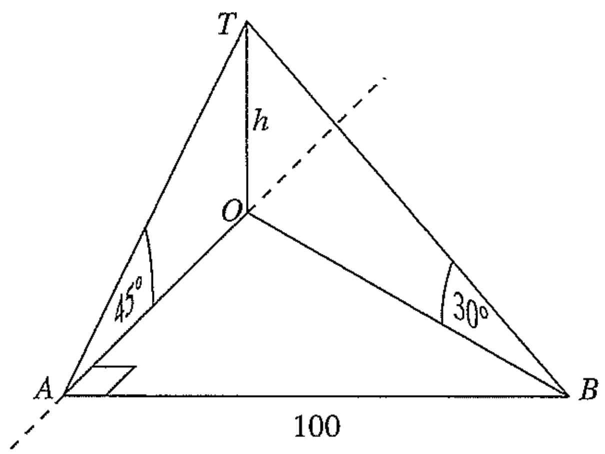
(i) Find $\lim\limits_{x \to \infty} \frac{x^2}{x^2-4}$ 1

(ii) Show that it is an even function 1

(iii) State the domain 1

(iv) Without using calculus, sketch the curve showing all important features. 2

c)




A surveyor stands at a point $A$, which is due south of a tower $OT$ of height $h$ m. The angle of elevation of the top of the tower from $A$ is 45°. The surveyor then walks 100 m due east to a point $B$, from where she measures the angle of elevation of the top of the tower to be 30°.

(i) Express the length of $OB$ in terms of $h$. 1

(ii) Show that $h = 50\sqrt{2}$ 2

(iii) Calculate the bearing of $B$ from the base of the tower, correct to the nearest degree. 2

**Question 11** (Start a new page) **11 Marks**

a) Given $(x-3)$ is a factor of the polynomial $f(x) = 2x^3 - 7x^2 - 7x + 30$, find all solutions for $f(x) = 0$. 2

b) Find the coordinates of the point on the curve $y = x^2 + 3x - 1$ where the tangent is parallel to the line $y = 5x + 6$. 2

c) $A(2ap, ap^2)$ and $B(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Find the coordinates of the midpoint M of the chord joining $A(2ap, ap^2)$ and $B(2aq, aq^2)$ in simplified and factorised form. 1

(ii) Given the equation of the chord is $y = \frac{1}{2}(p+q)x - apq$ and it passes through the focus of the parabola, show that $pq = -1$. 1

(iii) Hence show by substitution that M lies on the parabola $x^2 = 2ay - 2a^2$ 3

(iv) Find the vertex and focus of the parabola given in part (iii). 2

END OF TEST

4:03 – 4:30 27 4:33 – 4:39 6 5:22 – 6. 38 = 71 min

Student Name: Teacher Name:

# 2016 Year 11 Ext. 1 Final Prelim. Solutions

## Section 1

1. C 2. C 3. C 4. D 5. D

## Section 2

6. a) $y = \frac{x-3}{2-x}$

$2y - xy = x - 3$

$2y + 3 = x + xy$

$2y + 3 = x(1+y)$

$x = \frac{2y+3}{y+1}$

b) (i) $P(2) = 8 + 2a + b = 0$

$P(-1) = -1 - a + b = 6$

$\therefore 2a + b = -8$

$b - a = 7$

(ii) $3a = -15$

$a = -5 \quad \therefore b = 2$

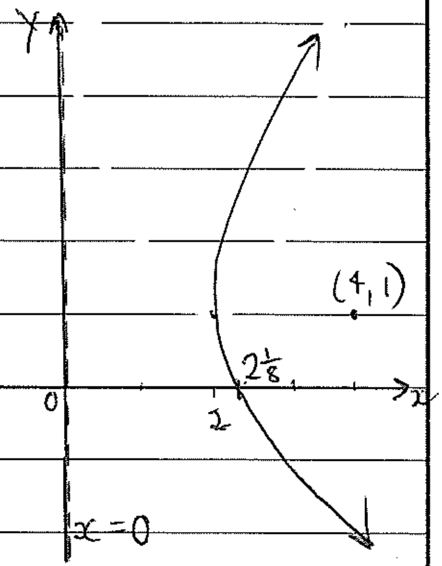
c) (i) $y^2 - 2y = 8x - 17$

$y^2 - 2y + 1 = 8x - 16$

$(y-1)^2 = 8(x-2)$

Vertex $(2, 1)$

(ii)




d) $x^3 + 2x^2 + 3x + 1 = 0$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= \left(-\frac{2}{1}\right)^2 - 2 \times \frac{3}{1}$

$= -2$

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7. $x^2 + 10x - 3 \equiv ax(x+1) + b(x-1)^2$

$\equiv ax^2 + ax + bx^2 - 2bx + b$

$= (a+b)x^2 + (a-2b)x + b$

$\therefore$ $\underline{b = -3}$, $a + b = 1$ $a - 2b = 10$

$a - 3 = 1$ $a - 2 \times -3 = 10$

$\underline{a = 4}$ $a = 4$

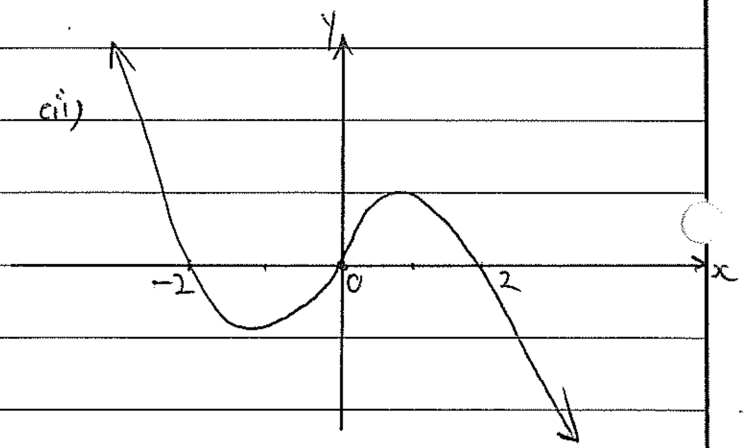
b)(i) $P(x) = -x^3 - ax$

$P(-2) = -(-2)^3 - a \times -2 = 0$

$= 8 + 2a = 0$

$\underline{a = -4}$

(ii)




(iii) $\underline{-2 \le x \le 0,\ x \ge 2}$

c) (i) $3\sin x - \cos x = 1$

$3 \times \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2} = 1$ where $t = \tan\frac{x}{2}$

$6t - 1 + t^2 = 1 + t^2$

$6t = 2$

$\underline{t = \frac{1}{3}}$

(ii) $\tan\frac{x}{2} = \frac{1}{3}$

$\frac{x}{2} = 18°26'$

$\underline{x = 36°52'}$

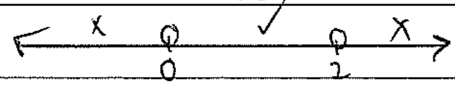
d) $\frac{1}{x} - \frac{1}{x-2} > 0$

$x \ne 0$ $x \ne 2$

$\frac{1}{x} - \frac{1}{x-2} = 0$

$x - 2 - x = 0$

No solution




$\underline{0 < x < 2}$

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8. a) $\sqrt{2}\cos(x+60) = 1$

$\cos(x+60) = \frac{1}{\sqrt{2}}$

$x + 60 = 45, 315^\circ, 405^\circ$

$x = 255^\circ, 345^\circ$

b) Gradients of tangents are 2, -1

$\tan\theta = \left|\frac{2 - -1}{1 + -2}\right|$

$= \left|\frac{3}{-1}\right|$

$= 3$

$\theta = 72^\circ$

c) $\sec x - \tan x = k$

$\sec^2 x - \tan^2 x = k(\sec x + \tan x)$

Now since $\tan^2 x + 1 = \sec^2 x$

$1 = k(\sec x + \tan x)$

$\therefore \sec x + \tan x = \frac{1}{k}$

d) If distance (0,0) to $3x - 4y + 10 = 0$ equals 2, line must be a tangent.

$d = \left|\frac{3\times 0 + -4\times 0 + 10}{\sqrt{3^2+(-4)^2}}\right|$

$= \left|\frac{10}{5}\right|$

$d = 2$ ✓

e)(i) $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$\sin A\cos B + \sin B\cos A + \sin A\cos B - \sin B\cos A = LHS$

$2\sin A\cos B = LHS = RHS$

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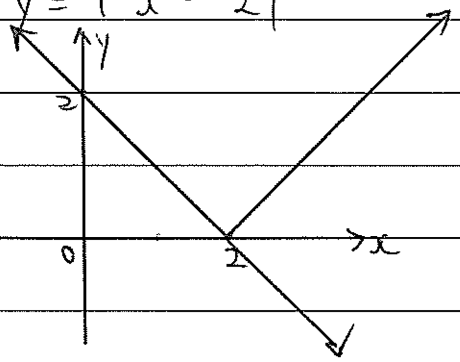
(ii) $2\sin 45 \cos 15 = \sin(45+15) + \sin(45-15)$

$= \sin 60 + \sin 30$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{1+\sqrt{3}}{2}$

9a. $y = |x-2|$




(ii) $|x-2| = 2-x$

for $x \le 2$

b)(i) $P(2ap, ap^2)$ $Q(2aq, aq^2)$

$m_{PQ} = \frac{aq^2 - ap^2}{2aq - 2ap}$

$= \frac{a(q-p)(q+p)}{2a(q-p)}$

$m_{PQ} = \frac{p+q}{2}$

$y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$2y - 2ap^2 = (p+q)x - 2ap(p+q)$

$2y - 2ap^2 = (p+q)x - 2ap^2 - 2apq$

$\therefore (p+q)x - 2y = 2apq$

(ii) If $pq = 1$

$\therefore (p+q)x - 2y = 2a$

when $x = 0$

$y = -a$

$\therefore (0, -a)$ fixed

c) $(k+1)x^2 - 2(k-1)x + (k-5)$

is a perfect square if

$\Delta = 0$

$4(k-1)^2 - 4(k+1)(k-5) = 0$

$4k^2 - 8k + 4 - 4(k^2 - 5k + k - 5) = 0$

$4k^2 - 8k + 4 - 4k^2 + 20k - 4k + 20 = 0$

$8k + 24 = 0$

$k = -3$

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d)(i) $\frac{1}{\alpha} + \frac{1}{\alpha^2} + \frac{1}{\alpha^3}$

$\frac{\alpha^2}{\alpha^3} + \frac{\alpha}{\alpha^3} + \frac{1}{\alpha^3}$

$= \frac{1 + \alpha + \alpha^2}{\alpha^3}$

$= \frac{\alpha^5 + \alpha^4 + \alpha^3}{\alpha^6}$

$= \frac{c}{-d}$

(ii) $\alpha^3 + \alpha^4 + \alpha^5 = c$

$\alpha^2(\alpha + \alpha^2 + \alpha^3) = c$

$\alpha^2 \times -b = c$

Cube both sides

$\alpha^6 \times -b^3 = c^3$

$-d \times -b^3 = c^3$

$b^3 d - c^3 = 0$

10. a) Not differentiable at $x = 1$ because curve doesn't exist at that point.

b)(i) $\lim_{x \to \infty} \frac{x^2}{x^2 - 4} = 1$

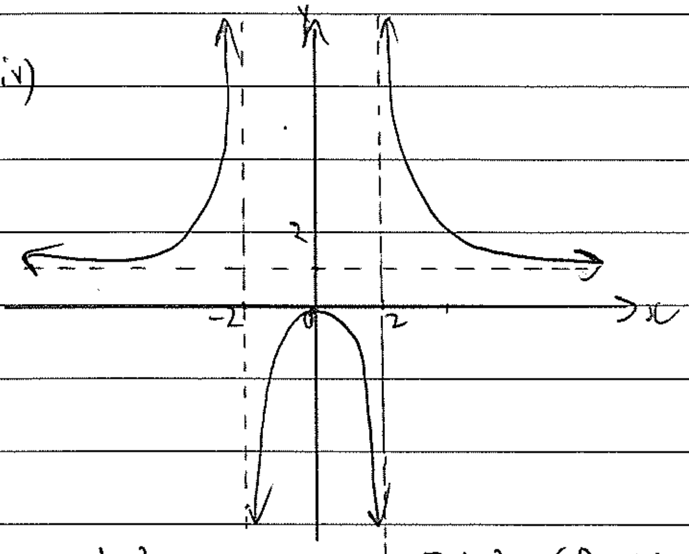
(ii) $f(-x) = f(x)$ if even

$\frac{(-x)^2}{(-x)^2 - 4} = \frac{x^2}{x^2 - 4}$

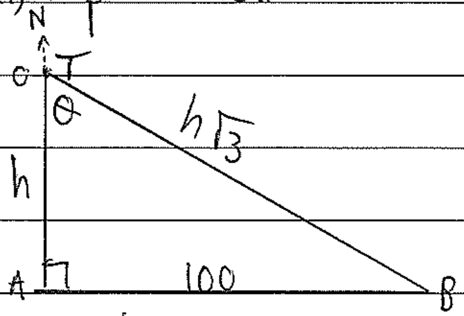
$\frac{x^2}{x^2 - 4} = \frac{x^2}{x^2 - 4}$

(iii) Domain all real $x$ except $x = \pm 2$

(iv)




c)(i) Top View:




$OA = h$ as $\Delta AOT$ is isosceles

$\tan 30 = \frac{h}{OB}$

$\frac{1}{\sqrt{3}} = \frac{h}{OB}$

$OB = h\sqrt{3}$

c.(ii) $h^2 + 10000 = 3h^2$ (Pythag.)

$h^2 = 5000$

$h = 50\sqrt{2}$

(iii) $\tan\theta = \frac{100}{50\sqrt{2}} = \sqrt{2}$

$\theta = 54°44'$ or

$\theta = 55°$

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11a) $f(x) = 2x^3 - 7x^2 - 7x + 30 = (x-3)(2x^2 - x - 10)$

By inspection $= (x-3)(2x-5)(x+2)$

$\therefore$ Solutions are $3, -2, 2\frac{1}{2}$

b) Let $\frac{dy}{dx} = 5$

$\frac{dy}{dx} = 2x+3 = 5$

$2x = 2$

$\therefore$ at $x = 1$

$(1, 3)$

c) (i) M is $\left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right)$

$M\left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$

(ii) $y = \frac{1}{2}(p+q)x - apq$

Sub. in $(0, a)$

$a = 0 - apq$

$1 = -pq$

$\therefore pq = -1$

(iii) $x^2 = 2ay - 2a^2$

Sub. in M

$a^2(p+q)^2 = 2a \times \frac{a(p^2+q^2)}{2} - 2a^2$

$a^2(p+q)^2 = a^2(p^2+q^2) - 2a^2$

$= a^2(p^2+q^2-2)$

$= a^2(p^2+q^2+2pq)$ from part (ii)

$= a^2(p+q)^2$

$\therefore$ M lies on this parabola.

(iv) $x^2 = 2ay - 2a^2$

$x^2 = 2a(y-a)$

Vertex is $(0, a)$

Focus is $\left(0, \frac{3a}{2}\right)$