Sydney Technical High School

Name: ....

Maths Class: ....

Year 11

Mathematics Extension 1

Preliminary Course

Assessment 3

September, 2018

Time allowed: 90 minutes

General Instructions:

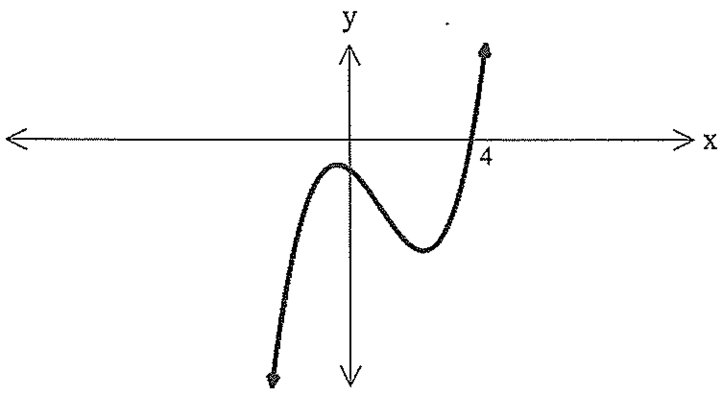
- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided at the rear of this Question Booklet, and may be removed at any time.

Section 1 Multiple Choice
Questions 1-7
7 Marks

Section II Questions 8-13
60 Marks

SECTION I - Multiple Choice (7 marks)

Spend about 10 minutes on this section
Use the multiple choice answer sheet located in the front of your answer booklet.
All questions are worth 1 mark

1	If $f(n) = n(n+1)(n+2)$, then $\frac{f(n)}{f(n+1)} =$ A. $\frac{n}{n+1}$ B. $\frac{n}{n+3}$ C. $\frac{n+1}{n+2}$ D. $\frac{n+2}{n+3}$
2	$\lim_{n \to \infty} \frac{5x^2-2}{3x^2+x} =$ A. -2 B. $-\frac{1}{2}$ C. $\frac{5}{3}$ D. 5
3	 The graph of the polynomial shown above, could be: A. $y = (x-4)(x^2+x+1)$ B. $y = (x+4)(x^2+x+1)$ C. $y = (4-x)(x^2+x+1)$ D. $y = (x-4)^3$

SECTION II

60 Marks

Allow about 80 minutes for this section
START EACH QUESTION AT THE TOP OF A NEW PAGE IN YOUR ANSWER BOOKLET

<u>QUESTION 8</u>: (10 Marks)

Marks

(a) Find, in simplest form, the derivative of: **2**

(i) $y = \frac{x^2-1}{x^2+1}$ (ii) $y = 2x\sqrt{x}$

(b) Find the acute angle between the lines $x + 2y = 5$ and $x - 3y + 3 = 0$ **2**

(c) Find the monic polynomial of degree 3, which is odd, and which, when divided by $x - 1$ has a remainder of -1. **2**

(d) (i) Given that the polynomial Q(x) has a double root at $x = 3$, fully factorize **2**

$$Q(x) = 2x^3 - 7x^2 - 12x + 45$$

(ii) Without the use of calculus, sketch the graph of y = Q(x). **2**
(Use about $\frac{1}{3}$ of a page for your diagram)

QUESTION 10: (10 Marks) Start a New Page

Marks

(a) (i) Show that $\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{\sqrt{x+h}-\sqrt{x}}{h}$ **1**

(ii) By using then definition of the derivative as a limit (ie by First Principles), find the derivative of $y = \sqrt{x}$ **2**

(b) Find the values of θ, where $0^o \leq \theta \leq 360^o$, which solves **4**

$$2\cos^2\theta - \sin\theta = 1$$

(c) The roots of the equation $x^3 - 5x^2 + x + 7 = 0$ are α, β, and γ. **3**

Find the value of $\alpha^2 + \beta^2 + \gamma^2$

QUESTION 12: (10 Marks) Start a New Page

Marks

(a) Solve $\frac{1}{x+2} \leq \frac{2}{x}$ 3

(b) (i) Express $\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha), where\ 0^o < \alpha < 360^o$. 2

(ii) By using the auxiliary angle method, solve the equation $\cos\theta - \sin\theta + 1 = 0$, where 2

$0^o \leq \theta \leq 360^o$

(c) P $(2p, p^2)$ is a point on the parabola $x^2 = 4y$, where the S is the focus.

T is the point which divides the line SP in the ratio 1:2.

(i) Find the co-ordinates of T in terms of p. 1

(ii) As P moves around the parabola, show that T lies on the parabola 2

$$9x^2 = 12y - 8$$

2018 Year 11 Extension 1

Preliminary Final Exam

Solutions

Section 1: 1/ B 2/ C 3/ A 4/ B 5/ C 6/ B 7/ C

Necessary working

Q1 — $\dfrac{f(n)}{f(n+1)} = \dfrac{n(n+1)(n+2)}{(n+1)(n+2)(n+3)}$

$= \dfrac{n}{n+3}$

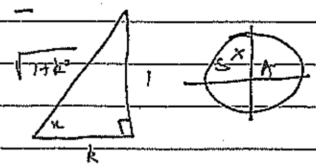
Q5 — $x x_0 = 2a(y + y_0)$

$-x = 4(y - 4)$

$-x = 4y - 16$

$x + 4y - 16 = 0$

Q6/ —



$\therefore \cos x = -\dfrac{k}{\sqrt{1+k^2}}$

Q7/ OQ passes through the centre, (call it P)

P is (1,1)

$OP = \sqrt{2}$ (by Pythagoras) $PQ = 1$ (radius)

$\therefore OQ = \sqrt{2} + 1$

Section 2:

Question 8:

(a)(i) $dy/dx = \dfrac{(x^2+1)2x - (x^2-1)2x}{(x^2+1)^2}$

$= 4x/(x^2+1)^2$

(ii) $\dfrac{d}{dx}\left(2x^{3/2}\right) = 3x^{1/2}$
$= 3\sqrt{x}$

(b) $\tan\theta = \left|\dfrac{-1/2 - 1/3}{1 + (-1/2)(1/3)}\right|$

$= \left|\dfrac{-5/6}{5/6}\right|$

$\therefore \theta = 45°$ or $\pi/4^c$

(c) $P(x) = 1x^3 + bx$

$P(1) = 1 + b = -1$

$b = -2$

$\therefore P(x) = x^3 - 2x$.

(d)(i) $Q(x) = (x-3)^2 A(x)$

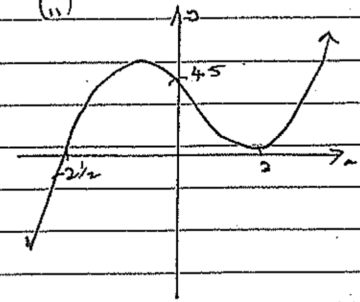
$= (x^2 - 6x + 9)A(x)$

$= (x^2 - 6x + 9)(2x+5)$

by inspection.

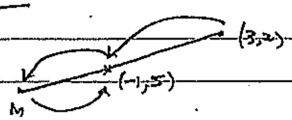
(because $9 \times 5 = 45$)

(ii)



Question 9:

(a)



M is (−5, 8)

(b)(i) LHS $= \dfrac{1 - (1 - 2\sin^2 x)}{2\sin x\cos x}$

$= \dfrac{2\sin^2 x}{2\sin x\cos x}$

$= 2\tan x$

(ii) $\tan 15° = \dfrac{1 - \cos 30°}{\sin 30°}$

$= \dfrac{1 - \sqrt{3}/2}{1/2}$

$= 2 - \sqrt{3}$

(c)(i) $\dfrac{dy}{dx} = x/2a$

At P $m_T = p$

Equation $y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$

(ii) R is $(0, -ap^2)$

(iii) $PS^2 = (ap^2 - a)^2 + 4a^2p^2$

$= (ap^2 + a)^2$

$PS = a(p^2 + 1)$

$RS = a + ap^2$

$= PS$

$\therefore \triangle RPS$ is isosceles $(PS = RS)$

QUESTION 10:

(a) (i) $\frac{1}{\sqrt{x+h}+\sqrt{x}} \times \frac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}}$

$= \frac{\sqrt{x+h}-\sqrt{x}}{h}$

(ii) $\frac{dy}{dx} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$

$= 1/2\sqrt{x}$.

(b) $2(1-\sin^2\theta) - \sin\theta = 1$

$\therefore 2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\therefore \sin\theta = 1/2$ or $\sin\theta = 1$

$\therefore \theta = 30^\circ$ or, 150° or, 270°.

(c) $\alpha+\beta+\gamma = 5$

$\alpha\beta + \alpha\gamma + \beta\gamma = 1$

$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= 25 - 2$

$= 23$

QUESTION 11:

(a) (i) OP: $y = 2mx$

BP $y = mx + 3$

(ii) ~~P is at the intersection~~

$\therefore 2mx = mx + 3$

$\therefore \begin{cases} x = 3/m \\ y = 6 \end{cases}$

(iii) the line $y = 6$.

(b) $\tan\alpha + \tan\beta = 2$

$\tan\alpha\tan\beta = -1$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= 2/2$

$= 1$

$\therefore \alpha+\beta = 45^\circ \ (\pi/4)$

(b) $PS^2 = (x-1)^2 + (y-1)^2$

$PM = (y+3)$

Now $PS^2 = 4 \times PM^2$

$\therefore x^2 - 2x + 1 + y^2 - 2y + 1 = 4y^2 + 24y + 36$

$x^2 - 2x - 3y^2 - 26y - 34 = 0$

QUESTION 12:

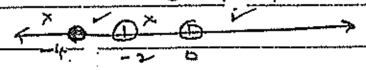
(a) METHOD I

C.V's at $x \neq -2$, $x \neq 0$

To find next C.V. solve.

$2x + 4 = x$

$x = -4$



$-4 \le x < -2$ or $x > 0$

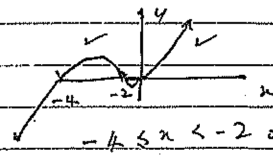
METHOD II

$x^2(x+2) \le 2x(x+2)^2$

$x^3 + 2x^2 \le 2x^3 + 8x^2 + 8x$

$x^3 + 6x^2 + 8x \ge 0$

$x(x+4)(x+2) \ge 0$



$-4 \le x < -2$ or $x \ge 0$

(b) (i) $R = \sqrt{2}$.

$\sqrt{2}\left(\cos\theta \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\sin\theta\right)$

$= \sqrt{2}\cos(\theta+\alpha)$

where $\cos\alpha = 1/\sqrt{2}$

$\therefore$ EXP^N $= \sqrt{2}(\cos(\theta+45^\circ))$

(ii) $\sqrt{2}\cos(\theta+45^\circ) = -1$

$\therefore \theta + 45^\circ = 135^\circ$ or 225°

$\theta = 90^\circ$ or, 180°

(c) (i) T is $\left(2p/3, \frac{p^2+2}{3}\right)$

(ii) $x = 2p/3 \Rightarrow p = 3x/2$

$\therefore y = \frac{9x^2/4 + 2}{3}$

$12y = 9x^2 + 8$

$9x^2 = 12y - 8$

QUESTION 13:

(a) (i) $\frac{dy}{dx} = -4/x^2$

At P $m_T = -4/c^2$

Equation $y - 4/c = -4/c^2(x - c)$

$\therefore c^2y + 4x = 8c$

(ii) R is $(0, 8/c)$ Q is $(2c, 0)$

(iii) Area $\triangle ROQ = 1/2(OR)(OQ)$

$= 1/2(8/c)(2c)$

$= 8u^2$

which is independent of c.

(b) (i) $h/x = \tan 20^\circ \Rightarrow h = x\tan 20^\circ$

$h/2x = \tan\theta \Rightarrow h = 2x\tan\theta$

$\therefore x\tan 20^\circ = 2x\tan\theta$

$\tan\theta = 1/2\tan 20^\circ$

$\theta = 10^\circ 19'$

(ii) $4x^2 = x^2 + 100^2$

$3x^2 = 100^2$

$x = 100/\sqrt{3}$

(iii) $h = \frac{100}{\sqrt{3}}\tan 20^\circ$

$\doteqdot 21$ m

MARKERS COMMENTS – Preliminary Final EXTENSION 1 2018

QUESTION 8:

(a)(ii) Unbelievably, most students did this by the product rule! Convert it to y = $2x^{\frac{3}{2}}$ and it takes one line!

(b) If you can't learn simple formulae you are attempting the wrong level!

(c) An odd function has no even powers and no constants (it goes through the origin). MONIC means the coefficient of x^3 is unity.

(d) (i) There is no need to prove that x-3 is a factor of the polynomial. You were TOLD it was a double root (ie $(x-3)^2$ is a factor)

(ii) You should show – y-intercept, points where it cuts the x-axes and general shape.

QUESTION 9:

(a) You must be able to competently use the dividing ratio formula at Extension 1 level-too many mucked it up!

(b)ii When a question says "Hence", it is guaranteed to be the easiest method using what you have shown in part (i). Many students used the harder tan(A-B) method.

(c)iii Distance SR didn't require the distance formula, simple distance along the Y axis. SP did require the formula which turned out to be a perfect square which many did not see. Needed to finish off with equality statement: PS = SR so triangle must be isosceles.

QUESTION 10:

a)(ii) Students must take care writing the correct limit notation

b) Too many students did not know how to factorise $2\sin^2\Theta+\sin\Theta-1=0$. To factor a quadratic it must equal to zero

QUESTION 11:

a) iii. Once you had y=6 there was no need to go and add an x to it. It was a horizontal line but you did not earn the mark if you just wrote the locus is a line. Needed to give more information.

b) Solving for tanα and tanβ or using the relationship between the roots and coefficients were the 2 most effective methods. Solving separately for α and β and then adding them was often not successful.

c) READ THE QUESTION! Many doubled/halved the wrong distance or added an extra x^2 when finding distance between point and line, which could have been avoided by SKTECHING A DIAGRAM.

QUESTION 12:

a) Many students struggled with this question. If using the method of multiplying both sides by the square of the denominator, you must square both sides by BOTH denominators and simplify. Wrong critical values and incorrect inequality signs. Sketching the graph helps to get the correct values. Be careful when testing.

b)i) $\alpha = 45°$ ONLY. Some students had two solutions.

ii) Many incorrectly solved cos $(\theta + 135) = -1$ or solved the equation using 1 instead of -1 resulting in wrong quadrants and answers. Read the question carefully!

c)i) Order is important in ratio division. Several students divided ratio in order PS instead of SP, leading to wrong answer. Also, a=1 needed to be substituted in.

ii) Carry errors from part i awarded for eliminating p and correctly substituting into y. Marks not awarded for fudging to get final answer! More detail should be shown if substituting x and y in terms of p and saying point lies on the parabola.

QUESTION 13:

a) i. some students used incorrect index law to differentiate and others left x in the dy/dx rather than subbing in the x=c to find the gradient at the point P.

ii. SHOW THAT – really does mean that you should get the answer on the page and therefore check if you don't that you are indeed using the correct gradient.

iii. independent of C means that the area should NOT have a C in it.

b) i. important to show full working and generate two expressions for the ratio of h/x thus setting them equal to each other and evaluating the angle (some incorrect rounding occurred here – be careful)

ii/iii. Mistakes occurred here when students did not remember to square the full side ie (2x) squared when using Pythagoras theorem. This therefore made the question easier and did mean that they were less likely to get mistake carried forward into the final part of the exam.