Sydney Technical High School

Name: ....

Maths Class: ....

Year 11
Mathematics Extension 1

Preliminary Course

Assessment 3

September, 2019

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided.

Section 1 Multiple Choice
Questions 1-10
10 Marks

Section II Questions 11-16
50 Marks

Section 1 **Multiple Choice** **(10 marks)**

Use the multiple choice answer sheet located in the front of your answer booklet. All questions are worth one mark.

1. What is the remainder when the polynomial $P(x) = 5x^3 - 17x^2 - x + 11$ is divided by $x - 2$?

   (A) -147

   (B) -95

   (C) -19

   (D) 11

2. A function is represented by the parametric equations:

   $x = 2t + 1$

   $y = t - 2$

   Find the Cartesian equation of the function

   (A) $x - 2y + 3 = 0$

   (B) $x - 2y - 3 = 0$

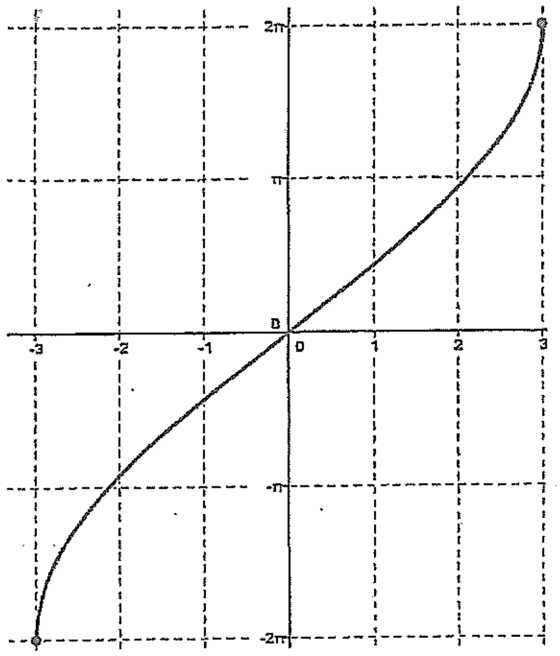
   (C) $x + 2y + 5 = 0$

   (D) $x - 2y - 5 = 0$

3. Given that α and β are both acute angles, evaluate $\sin(\alpha + \beta)$ if $\sin\alpha = \frac{8}{17}$ and $\sin\beta = \frac{4}{5}$

   (A) $\frac{108}{85}$ (B) $\frac{84}{85}$ (C) $\frac{36}{85}$ (D) $\frac{28}{85}$

4. Which function is graphed below?



(A) $2\pi \sin 3x$ (B) $2\pi \sin^{-1} \frac{1}{3}x$ (C) $4 \sin^{-1} 3x$ (D) $4 \sin^{-1} \frac{1}{3}x$

5. The curve $y = 2x^{\frac{1}{3}}$ is reflected in the line $y = x$. What is the equation of the reflected curve?

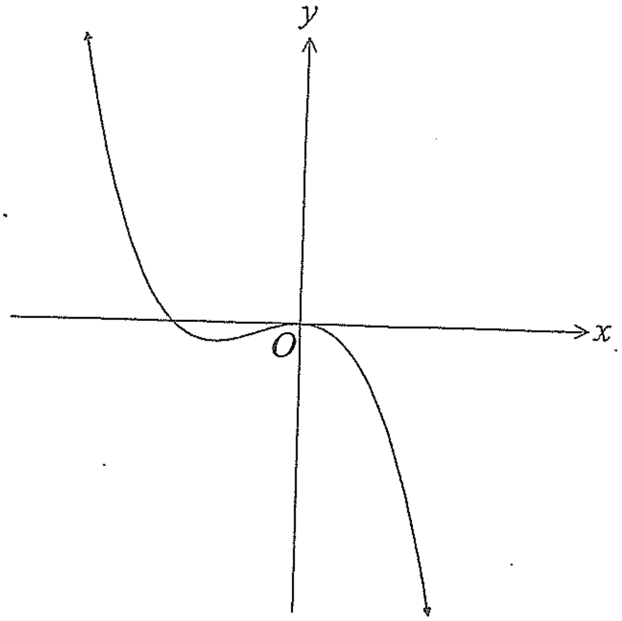
(A) $y = \frac{x^3}{16}$ (B) $y = \frac{x^3}{8}$ (C) $y = \frac{x^3}{4}$ (D) $y = \frac{x^3}{2}$

6. The solution to $\ln(x^3 + 19) = 3\ln(x + 1)$ is:

(A) $x = 2$ (B) $x = -3$

(C) $x = -3$ or $x = 2$ (D) $x = -2$ or $x = 3$

7. What is a possible equation of this function?



(A) $f(x) = -x(x-1)(x+1)$

(B) $f(x) = -x^2(x+1)$

(C) $f(x) = -x^2(x-1)$

(D) $f(x) = x^2(x+1)$

8. If $f(x) = 1 + \frac{2}{x-3}$, which of the following give the equations of the horizontal and vertical asymptotes of $f^{-1}(x)$?

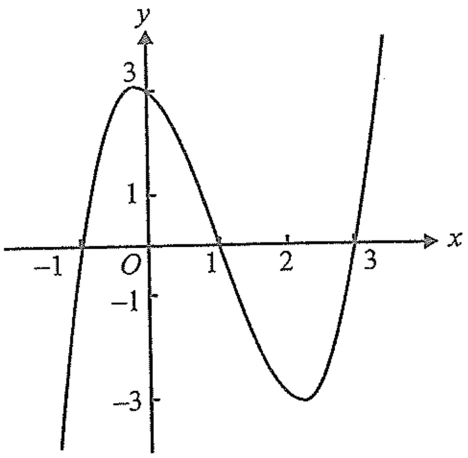
(A) Vertical asymptote is x = 1 and horizontal asymptote is y = 2

(B) Vertical asymptote is x = 1 and horizontal asymptote is y = 3

(C) Vertical asymptote is x = 3 and horizontal asymptote is y = 1

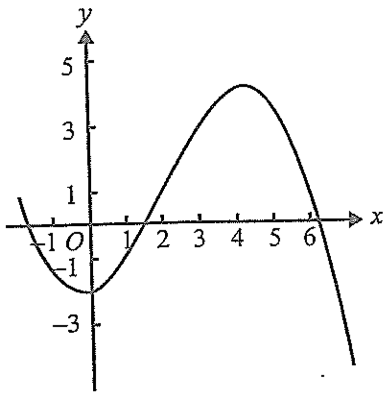
(D) Vertical asymptote is x = 3 and horizontal asymptote is y = 2

9. The graph of a function f, with domain R, is as shown.

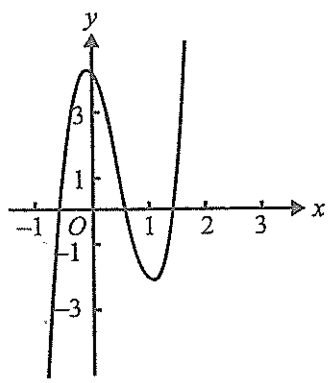


The graph which best represents $1 - f(2x)$ is:

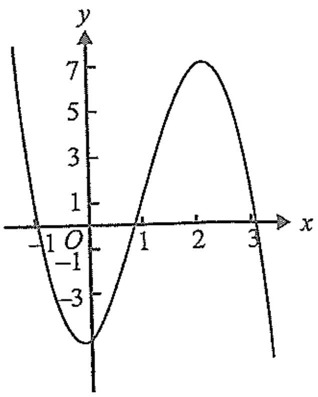
A.



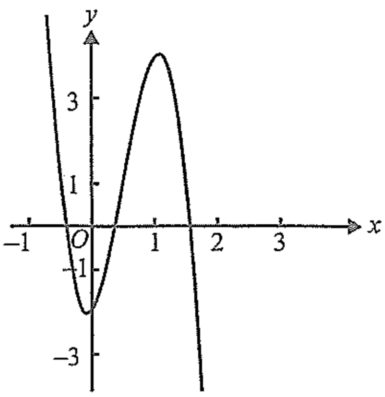
B.



C.



D.



10. The volume of a cube is increasing at a constant rate of 100 cm^3 per second. At what rate is the total surface area of the cube increasing when the side length of the cube is 10cm?

(A) $\frac{5}{6}$ cm^2 per second

(B) $\frac{1}{3}$ cm^2 per second

(C) 40 cm^2 per second

(D) 750 cm^2 per second

Section II (60 marks) Attempt Questions 11-16

Answer each question in your answer booklet

Start each question on a NEW sheet of paper

QUESTION 11 (Start a new page) **(8 Marks)**

(a) Solve $x + 2 \leq \frac{4}{x-1}$ (3)

(b) Solve $\dfrac{6^{3n} \times 9^{n+1}}{8^n} = 1$ (2)

(c) Find the equation of the normal to the curve $y = e^{2x+2}$ at the point where $x = -1$ (3)

End of Question 11

QUESTION 12 (Start a new page) **(10 Marks)**

(a) In the quadratic equation $x^2 + 3x + 2p - 1 = 0$, one root is triple another. Find the value of p. (2)

(b) Xavier is playing a variation of Chess called Makruk on-line. Each game is graded and Xavier begins with a 0.6 probability of winning and a 0.3 probability of losing.

At the end of each game, players received points.

If Xavier wins he receives 5 points. If he loses he receives 2 points and a draw will result in Xavier receiving 3 points.

(i) What is the probability that Xavier's first game ends in a draw? (1)

In the next game, after grading has occurred, Xavier now has a 0.4 probability of winning and a 0.4 probability of losing.

Find the probability that after two games:

(ii) Xavier receives ten points. (1)

(iii) Xavier receives five points or fewer (2)

(c) The number of bacteria in a culture is given by $N = Ae^{kt}$ where t is measured in hours. If 6000 bacteria increase to 9000 after 8 hours, find

(i) k correct to 3 significant figures (2)

(ii) the rate at which the bacteria are increasing after 2 days. (2)

End of Question 12

QUESTION 13 **(Start a new page)** **(8 marks)**

(a) Prove that $\frac{\cos 2\theta+\sin 2\theta-1}{\cos 2\theta-\sin 2\theta+1} = \tan\theta$ (2)

(b) The polynomial $P(x) = x^2 + ax + b$ has a zero at $x = 2$ and when $P(x)$ is divided by $x + 1$, the remainder is 18. Find the values of a and b. (2)

(c) (i) Using at least one-third of a page of your booklet, sketch the graph of $y = |2x - 1|$ (1)

(ii) Hence or otherwise, solve $|2x - 1| \leq |x - 3|$ (2)

(d) By drawing an appropriate right-angled triangle, find the exact value of $\cos\left(\tan^{-1}\left(\frac{1}{3}\right)\right)$ (1)

End of Question 13

QUESTION 14 **(Start a new page)** **(8 marks)**

(a) Solve $1 + \cos 2x = \sin 2x$ for $0 \leq x \leq 2\pi$ (3)

(b) Let $f(x) = \frac{3+e^{2x}}{4}$

(i) Find the range of $f(x)$ (1)

(ii) Find the inverse function $f^{-1}(x)$ (2)

(c) Simplify $\frac{\cos 4x-\cos 6x}{\sin 6x-\sin 4x}$ (2)

End of Question 14

QUESTION 15 (Start a new page) **(8 Marks)**

(a) Use 't' results to prove that

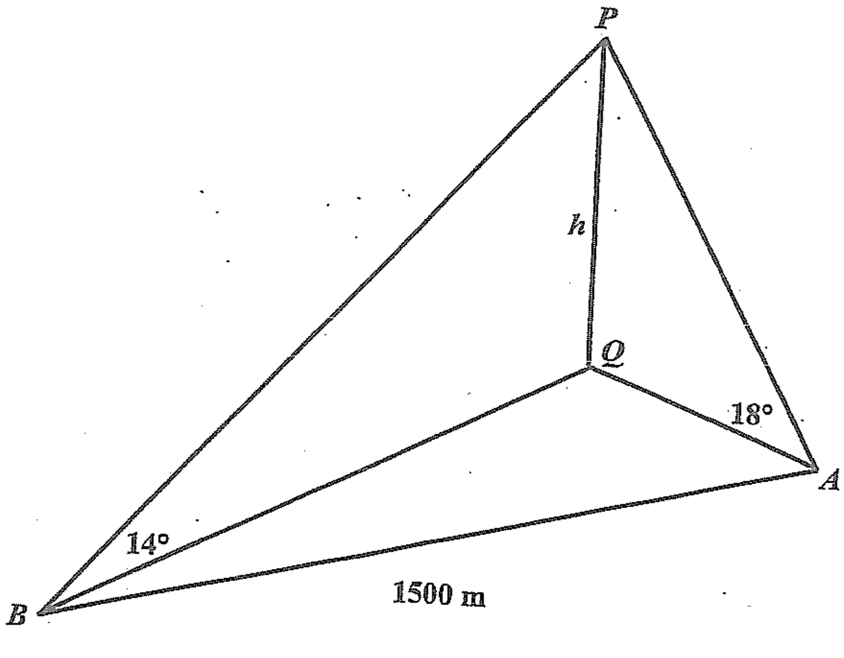
$$\frac{1+\cos x}{\sin x} = \frac{1}{t} \text{ for } t = \tan\frac{x}{2}$$ (2)

(b) The polynomial equation $x^3 - x^2 - 8x + m = 0$ has an integer double root. (3)

(i) Find the value of m

(ii) Hence find the roots of the polynomial equation

(c) The angle of elevation of a tower PQ of height h metres at a point A due east of it is 18°. From another point B, due south of the tower the angle of elevation is 14°. The points A and B are 1500 metres apart on level ground.



(i) Copy the diagram into your answer booklet

(ii) Show that $BQ = h \tan 76°$ (1)

(iii) Find the height h of the tower to the nearest metre (2)

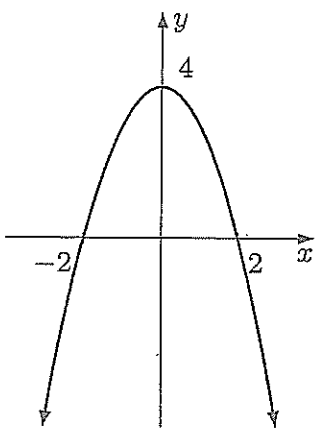
End of Question 15

QUESTION 16 (Start a new page) (8 Marks)

(a) Danuta has an 80% probability of passing her first English assessment and has a 45% probability of passing **both** the first and second assessments. Find the probability that Danuta will pass the second assessment given that she passes the first one. (1)

(b) The roots of the equation $x^3 - 3x^2 + 4x + 2 = 0$ are $\propto, \beta, \gamma$. Find the value of $\propto^{-1} + \beta^{-1} + \gamma^{-1}$ (2)

(c) The diagram below shows the graph of $y = f(x)$ (2)



Sketch the graph of $y = \frac{1}{f(x)}$

(d) Show that the Cartesian equation of (3)

$x = -1 + cos\theta$

$y = 2 - sin\theta$

is a circle and state the centre and radius

END OF PAPER

Section 1

Multiple Choice

1. $P(2) = 5(2)^3 - 17(2)^2 - 2 + 11$

$= -19$ (C)

2. $x = 2t + 1$

$x - 1 = 2t$

$t = \frac{x-1}{2}$

$\therefore y = \frac{x-1}{2} - 2$

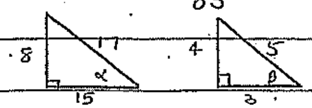
$2y = x - 1 - 4$

$x - 2y - 5 = 0$ (D)

3. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{8}{17} \times \frac{3}{5} + \frac{15}{17} \times \frac{4}{5}$

$= \frac{84}{85}$



(B)

4. For $\sin^{-1} x$ | For given graph

$-1 \le x \le 1$ | $-3 \le x \le 3$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ | $-2\pi \le y \le 2\pi$

$\therefore -1 \le \frac{x}{3} \le 1$

$-\frac{\pi}{2} \le \frac{y}{4} \le \frac{\pi}{2}$

$\frac{y}{4} = \sin^{-1}\frac{x}{3}$

$y = 4\sin^{-1}\frac{x}{3}$ (D)

5. $y = 2x^{1/3}$

$x = 2y^{1/3}$

$\frac{x}{2} = y^{1/3}$

$y = \frac{x^3}{8}$ (B)

6. $\ln(x^3 + 19) = \ln(x+1)^3$

$x^3 + 19 = x^3 + 3x^2 + 3x + 1$

$3x^2 + 3x - 18 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = 2$ $\quad x = -3$ is outside natural domain (A)

7. B

8. $y = 1 + \frac{2}{x-3}$

$x = 1 + \frac{2}{y-3}$

$x - 1 = \frac{2}{y-3}$

$\frac{1}{x-1} = \frac{y-3}{2}$

$\frac{2}{x-1} + 3 = y$ (B)

9. (D)

10 $\frac{dV}{dt} = 100$ Need to find $\frac{dA}{dt}$

$V = x^3$ $A = 6x^2$

$\frac{dV}{dx} = 3x^2$ $\frac{dA}{dx} = 12x$

$\frac{dA}{dt} = \frac{dV}{dt} \times \frac{dx}{dV} \times \frac{dA}{dx}$

$= 100 \times \frac{1}{3x^2} \times 12x$

$= \frac{400}{x}$

When $x = 10$ $\frac{dA}{dt} = \frac{400}{10}$

$= 40 cm^2/sec$ (C)

SECTION II

QUESTION 11

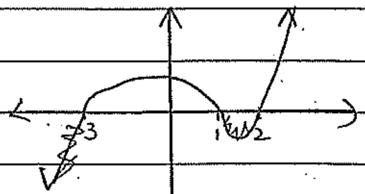
(a) $x + 2 \leq \frac{4}{x-1}$

$(x+2)(x-1)^2 \leq 4(x-1)$

$(x-1)[(x+2)(x-1)-4] \leq 0$

$(x-1)(x^2+x-6) \leq 0$

$(x-1)(x+3)(x-2) \leq 0$



$x \leq -3$ $1 < x \leq 2$ as $x \neq 1$

(b) $\frac{(2 \times 3)^{3n} \times (3^2)^{n+1}}{(2^3)^n} = 1$

$\frac{2^{3n} \times 3^{3n} \times 3^{2n+2}}{2^{3n}} = 3^0$

$3^{5n+2} = 3^0$

$5n = -2$

$n = \frac{-2}{5}$

(c) $y = e^{2x+2}$

$y' = 2e^{2x+2}$

At $x = -1$ $y = e^0$

$y = 1$ $(-1, 1)$

At $x = -1$ $y' = 2$

$\therefore m_T = 2$

$m_N = -\frac{1}{2}$

$\therefore$ Equation of normal

$y - 1 = -\frac{1}{2}(x+1)$

$2y - 2 = -x - 1$

$x + 2y - 1 = 0$

QUESTION 12

(a) Let the roots be α and 3α.

$\alpha + 3\alpha = \frac{-b}{a}$ $3\alpha^2 = \frac{c}{a}$

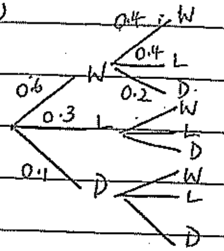
$= -3$ $= 2p - 1$

$4\alpha = -3$ $\therefore \frac{27}{16} = 2p - 1$

$\alpha = -\frac{3}{4}$ $p = \frac{43}{32}$

(b) (i) $P(draw) = 0.1$

(ii)



$P(10pts) = P(WW)$

$= 0.6 \times 0.4$

$= 0.24$

(iii) $P(\leq 5) = P(LL) + P(LD) + P(DL)$

$= 0.3 \times 0.4 + 0.3 \times 0.2 + 0.1 \times 0.4$

$= 0.22$

(c) (i) $N = Ae^{kt}$

When $t = 0$ $N = 6000$

$6000 = Ae^0$

$A = 6000$

$\therefore N = 6000e^{kt}$

When $t = 8$ $N = 9000$

$9000 = 6000e^{8k}$

$1.5 = e^{8k}$

$\log_e 1.5 = \log_e e^{8k}$

$k = \frac{\log_e 1.5}{8}$

$k = 0.0507$

(ii) When $t = 48$ hrs

$N = 6000e^{0.05068 \times 48}$

$= 68344$

$\therefore \frac{dN}{dt} = kN$

$\frac{dN}{dt} = 0.05068 \times 68344$

$= 3464$

QUESTION 13

(a) $LHS = \frac{\cos 2\theta + \sin 2\theta - 1}{\cos 2\theta - \sin 2\theta + 1}$

$= \frac{1 - 2\sin^2\theta + 2\sin\theta\cos\theta - 1}{2\cos^2\theta - 1 - 2\sin\theta\cos\theta + 1}$

$= \frac{2\sin\theta(\cos\theta - \sin\theta)}{2\cos\theta(\cos\theta - \sin\theta)}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$= RHS$

(b) $P(x) = x^2 + ax + b$

$P(2) = 0$ $4 + 2a + b = 0$

$\therefore 2a + b = -4$ --(1)

$P(-1) = 18$ $1 - a + b = 18$

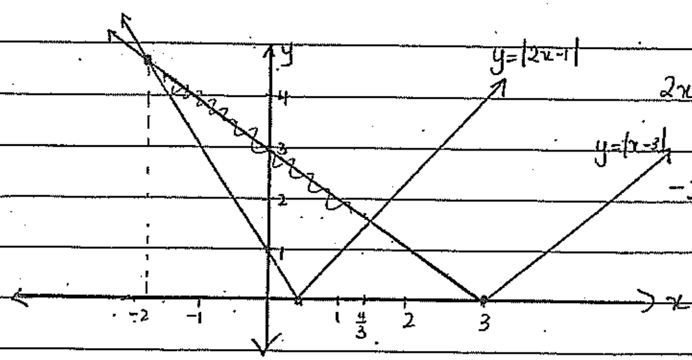
$-a + b = 17$

$b = 17 + a$

Substitute $2a + 17 + a = -4$

$a = -7$ $\therefore b = 10$

(c)



$2x - 1 = -x + 3$

$x = \frac{4}{3}$

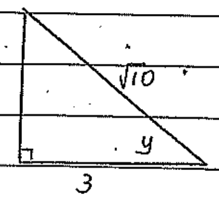
$-2x + 1 = -x + 3$

$-x = 2$

$x = -2$

$\therefore -2 \leq x \leq \frac{4}{3}$

(d)



Let $y = \tan^{-1}\frac{1}{3}$

$\tan y = \frac{1}{3}$

$\therefore \cos y = \frac{3}{\sqrt{10}}$

QUESTION 14

(a) $1 + \cos 2x = \sin 2x$

$1 + \cos^2 x - \sin^2 x = 2\sin x \cos x$

$2\cos^2 x - 2\sin x \cos x = 0$

$2\cos x(\cos x - \sin x) = 0$

$\cos x = 0$ $\cos x = \sin x$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

(b)(i) Since $e^{2x} > 0$ for all x

then $f(x) = \frac{3+0}{4}$

$\therefore$ Range $y > \frac{3}{4}$

(ii) $x = \frac{3 + e^{2y}}{4}$

$e^{2y} = 4x - 3$

$2y = \ln(4x-3)$

$y = \frac{1}{2}\ln(4x-3)$

(c) $= \frac{\cos(5x-x) - \cos(5x+x)}{\sin(5x+x) - \sin(5x-x)}$

$= \frac{2 \sin 5x \sin x}{2 \cos 5x \sin x}$

$= \tan 5x$

QUESTION 15

(a) LHS $= \frac{1 + \cos x}{\sin x}$

$= \frac{1 + \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}}$

$= \frac{1 + t^2 + 1 - t^2}{2t}$

$= \frac{2}{2t}$

$= \frac{1}{t}$ = RHS

(b)(i) $x^3 - x^2 - 8x + m = 0$

$3x^2 - 2x - 8 = 0$

$(3x+4)(x-2) = 0$

$x = 2$ integer double root

$8 - 4 - 16 + m = 0$

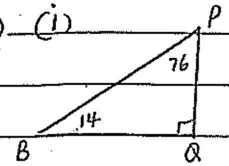
$m = 12$

(ii) $2, 2, \beta$ $\therefore \alpha^2\beta = \frac{c}{a}$

$4\beta = -12$

$\beta = -3$ $\therefore$ 3 roots $2, 2, -3$

(c) (i)



$\tan 76 = \frac{BQ}{h}$

$BQ = h \tan 76$

(ii) Similarly using ΔPQA: $AQ = h \tan 72$

In ΔBQA $BQ^2 + AQ^2 = 1500^2$

$h^2 \tan^2 76 + h^2 \tan^2 72 = 1500^2$

$h^2(\tan^2 76 + \tan^2 72) = 1500^2$

$h^2 = \frac{1500^2}{\tan^2 76 + \tan^2 72}$

$h = 297$ m to the nearest m

QUESTION 16

(a) $P(\text{1st English}) = 80\%$

$P(\text{both}) = 45\%$

$\therefore P(E) = \frac{0.45}{0.8}$

$= 56.25\%$

(b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

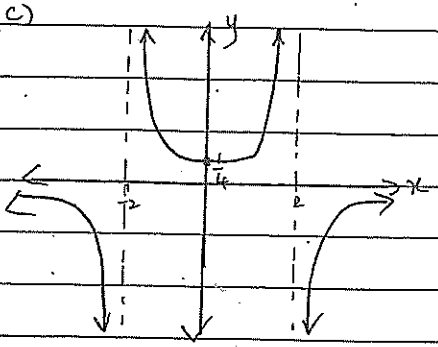
$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{\frac{c}{a}}{-\frac{d}{a}}$

$= \frac{4}{-2}$

$= -2$

(c)



(d) $x = -1 + \cos\theta \qquad \therefore \cos\theta = x + 1$

$y = 2 - \sin\theta \qquad \sin\theta = 2 - y$

$x^2 + y^2 = (\cos\theta - 1)^2 + (2 - \sin\theta)^2$

$x^2 + y^2 = \cos^2\theta - 2\cos\theta + 1 + 4 - 4\sin\theta + \sin^2\theta$

$x^2 + y^2 = -2\cos\theta - 4\sin\theta + 6$

$x^2 + y^2 = -2(x+1) - 4(2-y) + 6$

$x^2 + y^2 = -2x - 2 - 8 + 4y + 6$

$x^2 + 2x + 1 + y^2 - 4y + 4 = -4 + 1 + 4$

$(x+1)^2 + (y-2)^2 = 1$

centre $(-1, 2)$ radius $= 1$

YEAR 11 EXTENSION 1 TEACHER COMMENTS

Question 11:

(a) Teachers teach inequalities in one of two different ways, each with its advantages. Students need to learn ONE method, not confuse two (or more) methods.
But one things is the same: For $x + 2 \leq \frac{4}{x-1}$ You CANNOT multiply by x – 1. (If it is positive, then it is OK, but what if x – 1 is negative? Then thee inequality sign changes.) The same applies to the line $(x+2)(x-1)^2 \leq 4(x-1)$. You cannot DIVIDE by x – 1, for the same reasons.
Simple principle: Inequalities should ring warning bells!!!!

(b) This question was intended to be done by indices (and LOOKS like this!!), not logarithms. People who used a convoluted logarithm method were extremely lucky that the answer was rational, so it was exact. Trouble would arise if it were irrational, say.
Also, learn this ⟶ $6^n = 2^n.3^n$. And no, in $\frac{3^{n+1}}{2^{n+1}}$ you can't cancel the n+1 's.

(c) Well done!

Question 12

a) Students were able to find the value of α yet found difficulties finding the value of p using sums and products of roots.

b) Poorly answered. It would have been best to have drawn a tree diagram to realise that $P(\leq 5) = P(LL) + P(LD) + P(DL)$

ci) Students were able to find the value of k however had difficulties giving the answer to three significant figures.

cii) $\frac{dN}{dt} = k \times N$ needed to be shown with the correct substitution to be awarded full marks.

<u>Question 13:</u>

a) Needed to look at reference sheet to get the substitutions and know which one to use. Also you need to learn HOW TO SET OUT A TRIG PROOF IT WAS DREADFUL!!!!!!

b) Learn the factor and remainder theorem and how to use them as this is a standard type of question and I was surprised how few students knew the process

c) It was a 2 unit graph that half of the EXTENSION ONE KIDS got wrong x – intercept is y=0 and y – intercept is x = 0

d) Use the graph to help solve or at least test the answers

e) Simple and careless errors done here

<u>Question 14</u>:

a) Poorly done. The smartest result for cos2x to use was 2cossquaredx-1 as the ones would then cancel out. From then on needed to get to cosx(cosx –sinx)=0 for first mark. NEVER divide throughout by something with an x in it as you lose a solution. Don't worry you weren't the only one, about ¾ of the cohort did this.

bi) Let e to the x =0 works out y=3/4 but then realise that y can't equal ¾ as e to the 2x only approaches 0.

b) Use the formula sheet, these were on it.

Question 15:

a) The t-results are on the formula sheet and needed to be used to correctly answer the question. Marks were not awarded for simply citing these results. Student should be wary of silly algebraic mistakes such as $\left(1+\frac{1-t^2}{1+t^2}\right)\times\frac{1+t^2}{2t}=1+\frac{1-t^2}{2t}$
Students should take care to clearly set out any proof or show questions.

b) Students who attempted to use the double root as α, α, β were mostly unsuccessful due to algebraic errors, and received few or no marks. The most efficient method was to use multiplicity of roots. Several successful methods were used to find the final root after (or sometimes before), finding the value of m.

c) i) Students needed to show where the angle $76°$ came from. Stating $\tan 76° = \frac{BQ}{h}$ was not sufficient for a show question, as this is such a simple rearrangement of the final answer.
For comparison's sake, if a question asked you to show that $x = 5$, simply writing $x + 1 = 6$, thefore $x = 5$ would not be an acceptable answer.
ii) Students were less likely to make mistakes if they followed the clue in part i): Finding a similar expression for AQ and using Pythagoras' theorem. Many students surprisingly incorrectly used Pythagoras' theorem, or attempted to use the Sine Rule with two different triangles at once. Students should also take care with their calculator work.

Question 16:

a) Successful students recognised that this was a conditional probability questions, and easily arrived at the answer by the correct application of formula, or the use of a tree diagram.

b) This was a fairly conventional sum and product of roots question which was not done particularly well. Successful students recognised that they had to take the common denominator in order to express what they were given $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$ in terms of the sum and product of roots, i.e. $\frac{\gamma\beta+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$, the formulas for which are in the formula sheet.

c) This question was generally well attempted, both extension1 students, at a minimum need to recognise that the zeros of a function become the asymptotes of its reciprocal as we dividing by zero is not possible.

d) There was a short way and a long way to answer this question. The long way involved squaring the given equations for x and y and then completing squares to arrive at the equation of a circle (see solutions). All but 1 students who completed this question successfully went the shorter route of expressing $\sin\theta$ and $\cos\theta$ in terms and x and y and then using the Pythagorean Identity to arrive at the equation of a circle. The marking was **generous,** it is noted that this is a **proof** question and many of the solutions were not explicit enough in their show steps. If you are using the Pythagorean Identity in a proof question you must state this explicitly and not jump straight to the final solution otherwise you are likely to lose marks.