STUDENT'S NAME: ____________________

TEACHER'S NAME: ____________________

BAULKHAM HILLS HIGH SCHOOL

MATHEMATICS ADVANCED ASSESSMENT

December 2008

Time allowed – Seventy minutes

DIRECTIONS TO CANDIDATES:

- Start each question on a new page.
- Show all relevant working.
- Use black or blue pen.
- **<u>NO</u>** liquid paper is to be used.
- Approved Maths aids and calculators may be used.

QUESTION 1 [9 marks]

(a) Find the primitive function of:

(i.) $3x^5 + 1$ **2**

(ii.) $\dfrac{12x^4 - 7x}{x^3}$ **2**

(iii.) $(x^2 - 6)^2$ **2**

(b) Find the focus, focal length and the equation of the directrix for $x^2 = \dfrac{1}{2}y$. **3**

6

QUESTION 2 [8 marks] **(Start on a new page)**

(a) If α and β are the roots of the equation $x^2 + 7x + 12 = 0$, find:

(i.) $\alpha + \beta$ **1**

(ii.) $\alpha\beta$ **1**

(iii.) $\alpha^2 + \beta^2$ **2**

(b) Find the equation of the curve when the curve passes through (1, 6) and $\dfrac{dy}{dx} = 3x$. **2**

QUESTION 3 [7 marks] **(Start on a new page)**

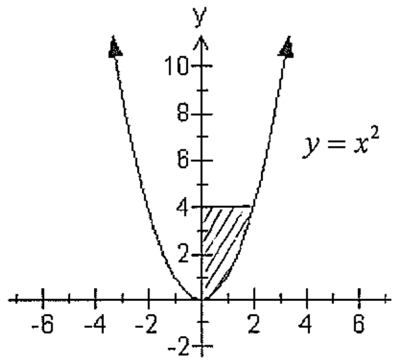
(a) Find the vertex, focal length, focus and directrix for $x^2 + 4x + 12y - 8 = 0$. **4**

(b) Find the exact volume when $y = x + 5$ is rotated about the x-axis from $x = 1$ to $x = 2$. **3**

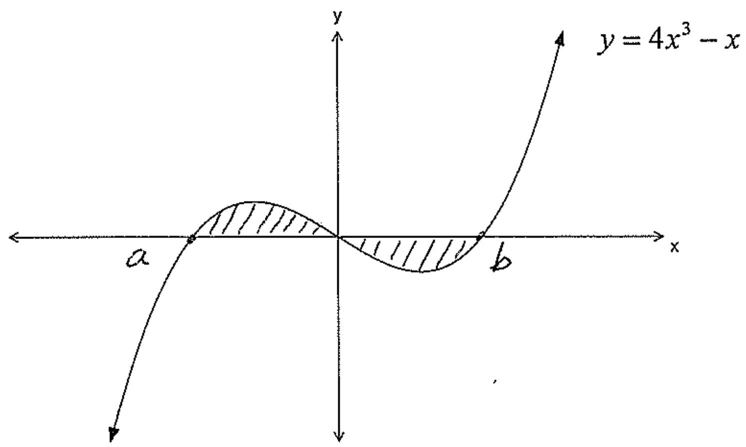
QUESTION 4 [9 marks] **(Start on a new page)**

(a) Find the shaded region:

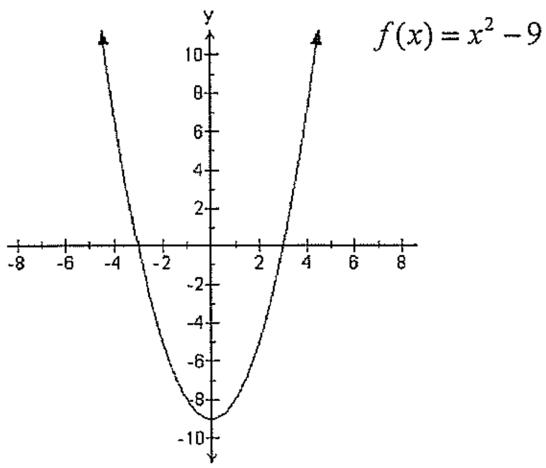
(i.) **3**



(ii.) Find the coordinates of a and b and hence find the shaded region. **4**



(b) Evaluate $\int_{-3}^{3} f(x)dx$ for $f(x)$ as shown below. **2**

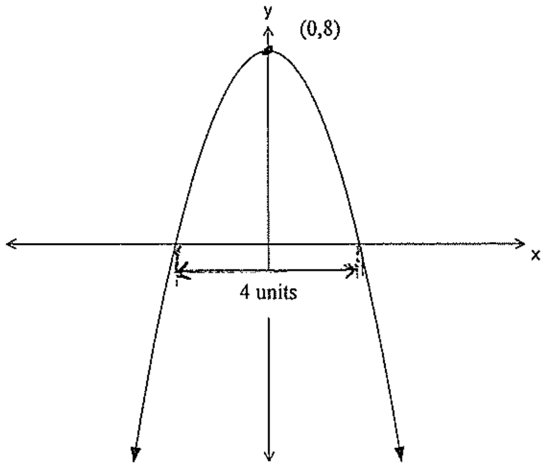


QUESTION 5 [7 marks] **(Start on a new page)**

(a) **(i.)** For $x^2+(k+3)x+4=0$ find the discriminant in terms of k. **1**

(ii.) Hence find the values of k for which $x^2+(k+3)x+4=0$ is positive definite. **2**

(b) The parabola in the diagram below is symmetrical about the vertical axis.



(i.) Find the equation of the parabola. **2**

(ii.) Hence calculate the area bounded by the parabola and the x-axis. **2**

QUESTION 6 [6 marks] **(Start on a new page)**

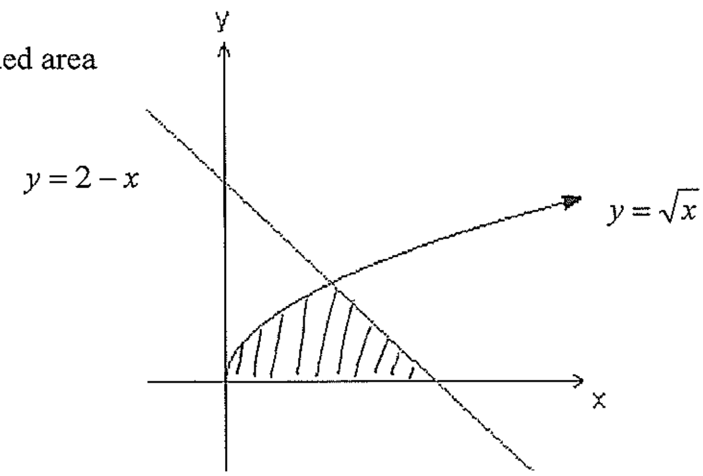
(a) Solve $x^6-28x^3+27=0$. **3**

(b) Find the locus of $P(x,y)$ which moves so it is equidistant from $y=-5$ and from (-1, 3). Describe the locus stating all important features. **3**

QUESTION 7 [8 marks] **(Start on a new page)**

(a) Find the locus of $P(x,y)$ which moves so it is equidistant from $3x+4y=20$ and $4x+3y=12$. **4**

(b) Find the shaded area **4**



QUESTION 8 [8 marks] **(Start on a new page)**

(a) If $y = (x+3)\sqrt{2x-3}$

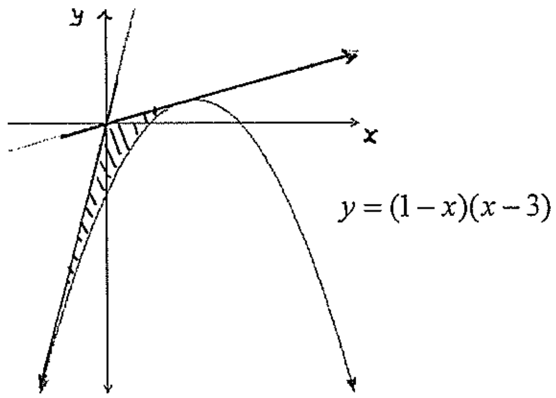
(i.) Show that $\dfrac{dy}{dx} = \dfrac{3x}{\sqrt{2x-3}}$ **2**

(ii.) Hence find $\displaystyle\int \frac{x}{\sqrt{2x-3}}dx$ **2**

(b) Find the volume of the solid generated by rotating $y = x^2 - 2$ about the y-axis between $y = -2$ and $y = 0$, using the Simpson's rule with 3 function values. **4**

QUESTION 9 [6 marks] **(Start on a new page)**

The tangents to the curve $y = (1-x)(x-3)$ that pass through the origin are drawn below.



(i.) Show that the x coordinates of contact between the curve and the tangents are $x = \pm\sqrt{3}$. (Let the equations of the tangents be $y = mx$). **3**

(ii.) Hence find the area enclosed by the curve and the tangents to the curve. **3**

END OF EXAMINATION

Total 66

Answers Yr. 11 2 unit Dec. Ass. task 2008

Question 1

9

a) i) $\int 3x^5+1\,dx = \frac{x^6}{2} + x + C$ (1) (1)

ii) $\int \frac{12x^4-7x}{x^3}\,dx = \int 12x - 7x^{-2}\,dx$ (1)

$= 6x^2 - \frac{7x^{-1}}{-1} + C$

(1) $= 6x^2 + \frac{7}{x} + C$

iii) $\int (x^2-6)^2\,dx = \int x^4 - 12x^2 + 36\,dx$ (1)

$= \frac{x^5}{5} - 4x^3 + 36x + C$ (1)

b) $x^2 = \frac{1}{2}y$ $\qquad \frac{1}{2} = 4a$

vertex $(0,0)$ $\qquad a = \frac{1}{8}$ (1)

focus $(0, \frac{1}{8})$ (1)

directrix $y = -\frac{1}{8}$ (1)

Question 2

6

a) $x^2 + 7x + 12 = 0$

i) $\alpha + \beta = -7$ (1)

ii) $\alpha \cdot \beta = 12$ (1)

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ (1)

$= (-7)^2 - 2 \times 12 = 25$ (1)

b) $y = \int 3x\,dx$
$= \frac{3x^2}{2} + C$ (1)

$6 = 3 \times \frac{1}{2} + C \therefore C = \frac{9}{2}$ (1)

$\therefore y = \frac{3x^2}{2} + \frac{9}{2}$

Question 3

7

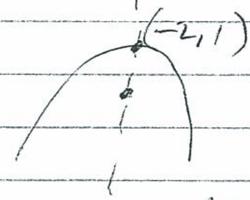
a) $x^2 + 4x + 12y - 8 = 0$

$x^2 + 4x + 4 = -12y + 8 + 4$

$(x+2)^2 = -12(y-1)$ (1)

$\therefore$ vertex $(-2,1)$ $\qquad a = 3$ (1)

focus $S(-2,-2)$ (1)



directrix $y = 4$ (1)

b) $V = \pi \int_1^2 y^2\,dx$ (1) $\qquad y = x+5$

$y^2 = (x+5)^2$

$= \pi \left[\frac{(x+5)^3}{3}\right]_1^2$

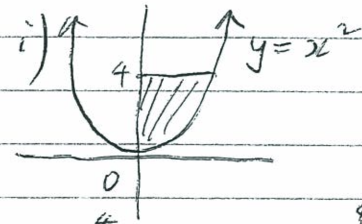
(1)

$= \frac{\pi}{3}\left[7^3 - 6^3\right] = \frac{127}{3}\pi$

Question 4

9

a) i)



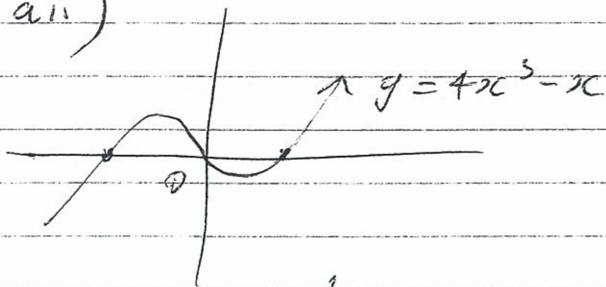
$x = \sqrt{y}$

$A = \int_0^4 x\,dy = \int_0^4 \sqrt{y}\,dy$ (1)

$= \left[\frac{2y^{\frac{3}{2}}}{3}\right]_0^4 = \frac{2}{3} \cdot 4^{\frac{3}{2}} = \frac{16}{3}$

(1) (1)

4 a ii)



$0 = 4x^3 - x$

$0 = x(4x^2 - 1)$

$x = 0 \quad x = \pm\frac{1}{2}$

$\therefore a = -\frac{1}{2} \qquad b = \frac{1}{2}$ (1)

$\text{Area} = \int_{-\frac{1}{2}}^{0} 4x^3 - x\,dx + \left|\int_{0}^{\frac{1}{2}} 4x^3 - x\,dx\right|$

or $2 \times \int_{-\frac{1}{2}}^{0} 4x^3 - x\,dx$ (1)

$= 2\left[x^4 - \frac{x^2}{2}\right]_{-\frac{1}{2}}^{0}$ (1)

$= 2 \times \left[0 - \left(\frac{1}{16} - \frac{1}{8}\right)\right] = \frac{1}{8}$ (1)

b) $\int_{-3}^{3} f(x)\,dx = \int_{-3}^{3} x^2 - 9\,dx$ (1)

$= \left[\frac{x^3}{3} - 9x\right]_{-3}^{3} = \left[-18 - (18)\right]$

$= -36$ (1)

$\boxed{\text{Question 5}}$ (7)

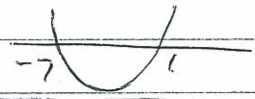
a) i) $\Delta = (k+3)^2 - 16$ (1)

ii) $\Delta < 0$

$(k+3)^2 - 16 < 0$ (1)

$(k+3+4)(k+3-4) < 0$

$(k+7)(k-1) < 0$



$\boxed{-7 < k < 1}$ (1)

b) i) $y = a(x-2)(x+2)$ (1)

$8 = a(\overset{=0}{x^2} - 4) \therefore a = -2$ (1)

$\therefore y = -2(x^2 - 4) \overset{\text{or}}{=} 8 - 2x^2$

ii) $A = \int_{-2}^{2} 8 - 2x^2\,dx = \left[8x - \frac{2x^3}{3}\right]_{-2}^{2}$ (1)

$= 2 \times \left[16 - \frac{16}{3} - 0\right] = 64/3$ (1)

$\boxed{\text{Question 6}}$ (6)

a) $x^6 - 28x^3 + 27 = 0$

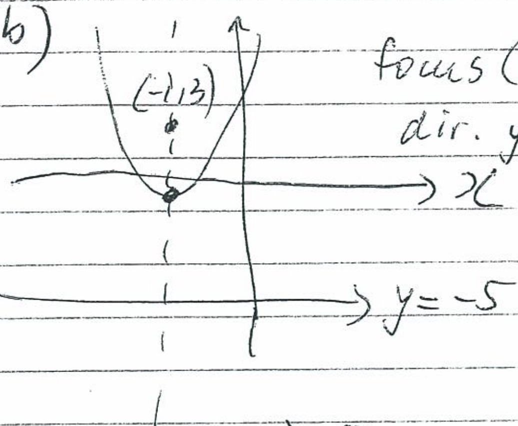
$m^2 - 28m + 27 = 0$ (1) $\boxed{m = x^3}$

$(m - 27)(m - 1) = 0$

$m = 27 \qquad m = 1$ (1)

$\therefore x = 3 \qquad x = 0$

Q. 6b)



focus $(-1,3)$

dir. $y=-5$

vertex $(-1,-1)$ ①

$a = 4$, concave up ①

$\therefore (x+1)^2 = 16(y+1)$ ①

Question 7 (8)

a) $\dfrac{|3x+4y-20|}{\sqrt{25}} = \dfrac{|4x+3y-12|}{\sqrt{25}}$ ①

$|3x+4y-20| = |4x+3y-12|$ ①

$3x+4y-20 = 4x+3y-12$

$0 = x - y + 8$ ①

OR $3x+4y-20 = -(4x+3y-12)$

$7x+7y-32 = 0$ ①

b) pt. of int.

$2 - x = \sqrt{x}$

$4 - 4x + x^2 = x$

$x^2 - 5x + 4 = 0$

$x = 1$, $x = 4$ ①

only ; not possible

$\therefore x = 1$

$2 - x = 0 \quad \therefore x = 2$ ①

$\therefore A = \displaystyle\int_0^1 \sqrt{x}\,dx + \int_1^2 2 - x\,dx$ ①

$= \left[\dfrac{2x^{\frac{3}{2}}}{3}\right]_0^1 + \dfrac{1}{2} \times 1 \times 1 =$

$\dfrac{2}{3} + \dfrac{1}{2} = \boxed{7/6}$ ①

Question 8 (8)

a) i) $\dfrac{dy}{dx} = 1 \times \sqrt{2x-3} + 2(x+3)\cdot\frac{1}{2}(2x-3)$ ① prod. rule

$\therefore \dfrac{dy}{dx} = \dfrac{\sqrt{2x-3}}{1} + \dfrac{x+3}{\sqrt{2x-3}} = \dfrac{2x-3+x+3}{\sqrt{2x-3}}$ ① simp.

$= \dfrac{3x}{\sqrt{2x-3}} \quad \therefore$ shown

ii) $\dfrac{dy}{dx} = \dfrac{3x}{\sqrt{2x-3}} \therefore y = \displaystyle\int \frac{3x}{\sqrt{2x-3}}\,dx$ ①

$\dfrac{y}{3} = \displaystyle\int \frac{x}{\sqrt{2x-3}}\,dx$

$\therefore \displaystyle\int \frac{x}{\sqrt{2x-3}}\,dx = \frac{1}{3}(x+3)\sqrt{2x-3} + c$ ①

b) $V = \pi \displaystyle\int_{-2}^{0} x^2\,dy$ ① $\quad y = x^2 - 2$

$y + 2 = x^2$

y	-2	-1	0
x^2	0	1	2

①

$\therefore V \doteqdot \pi\left[\frac{1}{3}(0 + 4\times 1 + 2)\right]$ ①

$= \dfrac{\pi}{3} \times 6 = 2\pi$ ①

Question 9

6

i) $y=(1-x)(x-3)=-x^2+4x-3$

$\underline{y=mx}$

$-x^2+4x-3=mx$

$-x^2+4x-mx-3=0$ (1)

$-x^2+x(4-m)-3=0$

$\Delta=(4-m)^2-4\times-1\times-3=0$

$(4-m)^2-12=0$ 1pt. of intersect. of each tangent

$(4-m-\sqrt{12})(4-m+\sqrt{12})=0$

(1)

$m=4-\sqrt{12}$ or $m=4+\sqrt{12}$

$\therefore -x^2+x(4-m)-3=0$

$-x^2+x(\cancel{4}-\cancel{4}\pm\sqrt{12})-3=0$

$-x^2\pm\sqrt{12}x-3=0$ (1)

$x=\dfrac{\mp\sqrt{12}\pm\sqrt{12-12}}{2}=\mp\dfrac{\sqrt{12}}{2}=\underline{\underline{\pm\sqrt{3}}}$

ii) $A_1=\int_0^{\sqrt{3}}(4+\sqrt{12})x\,dx-\int_1^{\sqrt{3}}-x^2+4x-3\,dx$

$=\left[2x^2\pm\sqrt{3}x\right]_0^{\sqrt{3}}-\left[-\dfrac{x^3}{3}+2x^2-3x\right]_1^{\sqrt{3}}$

$A_2=\left|\int_{-\sqrt{3}}^{1}-x^2+4x-3\,dx\right|-\int_{-\sqrt{3}}^{0}(4-\sqrt{12})x\,dx$

$=\left[-\dfrac{x^3}{3}+2x^2-3x\right]_{-\sqrt{3}}^{1}-\left[\dfrac{4x^2}{2}-\sqrt{3}x\right]_{-\sqrt{3}}$

$\boxed{\text{Total area}=2\sqrt{3}}$

OR

$A_1=\int_0^{\sqrt{3}}(4-\sqrt{12})x-(-x^2+4x-3)\,dx$

$=\left[(2-\sqrt{3})x^2+\dfrac{x^3}{3}-2x^2+3x\right]_0^{\sqrt{3}}$

$=\underline{\underline{\sqrt{3}}}$

$A_2=\int_{-\sqrt{3}}^{0}(4+\sqrt{12})x-(-3+4x-x^2)\,dx$

$=\left[(2+\sqrt{3})x^2+3x-2x^2+\dfrac{x^3}{3}\right]_{-\sqrt{3}}^{0}$

$=\sqrt{3}$

$\text{Total}=A_1+A_2=\underline{\underline{2\sqrt{3}}}$