STUDENT NUMBER: ______________________

TEACHER NAME: ______________________

BAULKHAM HILLS HIGH SCHOOL

MATHEMATICS ADVANCED ASSESSMENT

December 2009

Time allowed – Fifty minutes
Plus 5 minutes reading time

DIRECTIONS TO CANDIDATES:

- Show all relevant working.
- Use black or blue pen.
- <u>**NO**</u> liquid paper is to be used.
- Approved Maths aids and calculators may be used.

QUESTION 1 [6 marks]

Find the primitive function of:

(i.) $2x^2 + 5x - 1$ **2**

(ii.) $\dfrac{x^2+2}{x^2}$ **2**

(iii.) $(2x+5)^{10}$ **2**

QUESTION 2 [3 marks]

Redraw and complete the table below for the function $y = x\sqrt{4-x}$ correct to 4 decimal places. **1**

x	0	1	2	3	4
y	0			3	0

Hence estimate $\int_0^4 x\sqrt{4-x}\,dx$ using Simpson's rule with 5 function values correct to 3 decimal places. **2**

QUESTION 3 [4 marks]

Find the vertex, focal length, focus and directrix of $x^2 - 6x + y + 18 = 0$. . **4**

QUESTION 4 [5 marks]

(a) Find the area bounded by $y = x^2 - 6x + 8$, the x $axis$ and the lines $x = 0$ and $x = 4$. **3**

(b) If $\int_0^k 4 - 2x\,dx = 4$, find k. **2**

QUESTION 5 [4 marks]

If $y = 2x\sqrt{x+1}$

(i) Show that $\dfrac{dy}{dx} = \dfrac{3x+2}{\sqrt{x+1}}$ **2**

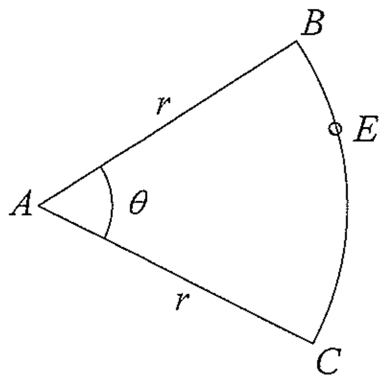
(ii) Hence evaluate $\int_3^8 \dfrac{3x+2}{\sqrt{x+1}}\,dx$. **2**

QUESTION 6 [4 marks]

Sketch $y = 1 + cos2x$ for $0 \leq x \leq 2\pi$. **4**
State the period and amplitude.

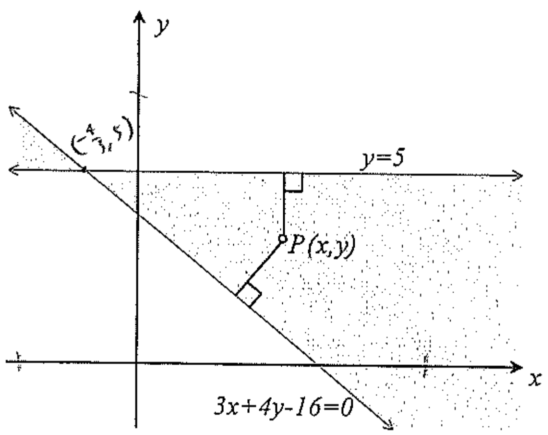
QUESTION 7 [4 marks]

In the figure AB and AC are radii of a circle with centre A. $AB = AC = r$ metres.
E lies on arc BC. **4**



If the perimeter of the figure $ABEC$ is 16 metres, find the area Y (in m^2) of the sector $ABEC$. Answer in simplest form in terms of r.

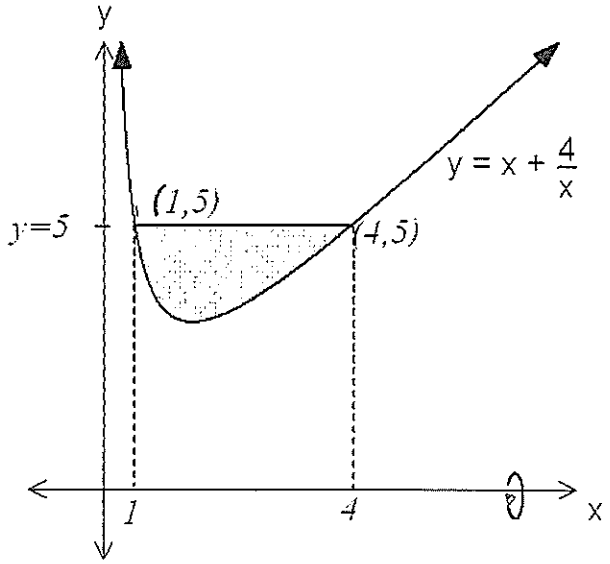
QUESTION 8 [4 marks]



The point $P(x, y)$ is equidistant from the lines $y = 5$ and $3x + 4y - 16 = 0$ and lies in the shaded region of the diagram. Find the equation of the locus of P. **4**

QUESTION 9 [4 marks] **(Start on a new page)**

Given $= x + \frac{4}{x}$. **4**



The shaded region is rotated around the $x - axis$. Find the volume of the solid of revolution in terms of π.

END OF EXAMINATION

TOTAL

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1) a) $\int 2x^2+5x-1\,dx$

$= 2\frac{x^3}{3} + 5\frac{x^2}{2} - x + C$ (1) (1)

ii) $\int \frac{x^2+2}{x^2}\,dx = \int 1+2x^{-2}\,dx$ (1)

$= x + \frac{2x^{-1}}{-1} + C$ (1) OR $= x - \frac{2}{x} + C$

iii) $\int (2x+5)^{10}\,dx = \frac{(2x+5)^{11}}{11\times 2} + C$ (1) (1)

2)

x	0	1	2	3	4
y	0	$\sqrt{3}$ 1.7321	$2\sqrt{2}$ 2.8284	3	0

(1) $h=1$

$y = x\sqrt{4-x}$

$A \doteqdot \frac{1}{3}\left(0+0+4\left(\sqrt{3}+3\right)+2\times 2\sqrt{2}\right)$ (1)

$= \frac{1}{3}\times 24.5852 = \underline{8.195}$ (1)

3) $x^2-6x+y+18=0$

$x^2-6x = -y-18$

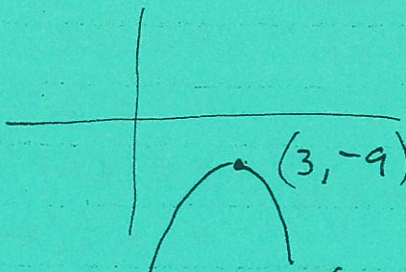
$x^2-6x+3^2 = -y-18+9$

$(x-3)^2 = -(y+9)$

vertex $(3,-9)$ (1)

focal length $= \boxed{a = 1/4}$ (1)

$4a=1$



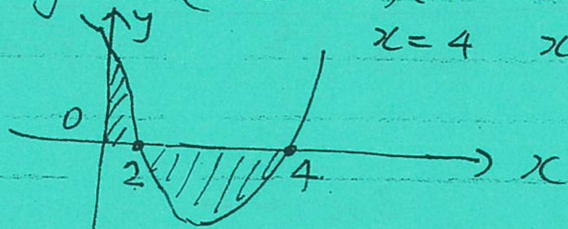
Focus $S\left(3,-9\frac{1}{4}\right)$ (1)

directrix $y = -8\frac{3}{4}$ (1)

4) $y = x^2-6x+8$

$y = (x-4)(x-2)$

(a) $x=4 \quad x=2$



$A = \int_0^2 x^2-6x+8\,dx + \left|\int_2^4 x^2-6x+8\,dx\right|$ (1)

$= \left[\frac{x^3}{3}-\frac{6x^2}{2}+8x\right]_0^2 + \left|\left[\frac{x^3}{3}-3x^2+8x\right]_2^4\right|$ (1)

$= \frac{8}{3}-3\times 4+16-0+\left|\frac{4^3}{3}-3\times 16+32 - \left(\frac{8}{3}-3\times 4+16\right)\right|$

$= \frac{20}{3}+\left|-\frac{4}{3}\right| = \underline{\underline{8}}$ (1)

b) $\int_0^k 4-2x\,dx = 4$

$\left[4x-x^2\right]_0^k = 4$ (1)

$4k-k^2 = 4$ $\therefore$ $k=2$ (1)

$0 = k^2-4k+4$

(5)

(i) $y = 2x\sqrt{x+1}$

$u = 2x$

$u' = 2$

$v = (x+1)^{\frac{1}{2}}$

$v' = \frac{1}{2}(x+1)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = u'v + v'u$$

$$= 2(x+1)^{\frac{1}{2}} + \frac{1}{2}(x+1)^{-\frac{1}{2}} \cdot 2x \quad (1)$$

$$= \frac{2(x+1)^{\frac{1}{2}}}{1} + \frac{x}{(x+1)^{\frac{1}{2}}} \quad (1)$$

$$= \frac{2(x+1) + x}{(x+1)^{\frac{1}{2}}} = \frac{3x+2}{\sqrt{x+1}}$$

(ii) $$\frac{dy}{dx} = \frac{3x+2}{\sqrt{x+1}}$$

$$y = \int \frac{3x+2}{\sqrt{x+1}}\, dx$$

$$\therefore \int_3^8 \frac{3x+2}{\sqrt{x+1}}\, dx = \Big[2x\sqrt{x+1}\Big]_3^8 \quad (1)$$

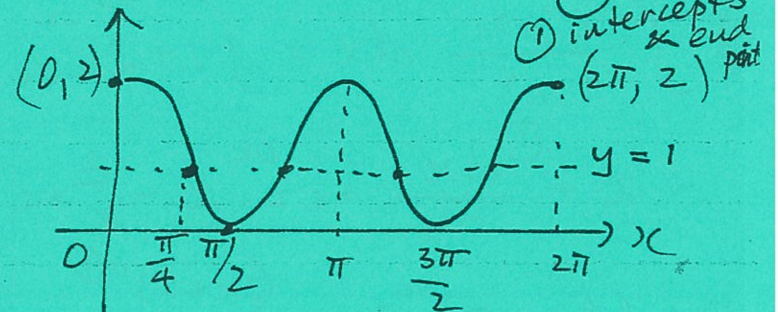
$$= \Big[16\sqrt{9} - 6\sqrt{4}\Big] = 36 \quad (1)$$

(6) $y = 1 + \cos 2x$

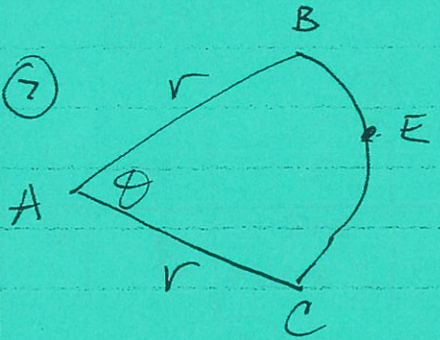
period $T = \frac{2\pi}{2} = \pi$ (1)

amplitude $= 1$ (1)

x	0	$\pi/2$	$\frac{3\pi}{2}$	π	2π
y	2	0	0	2	2



(7)



$P = 16$

$P = 2r + r\theta = 16$ (1)

$r\theta = 16 - 2r$

$$\theta = \frac{16-2r}{r}$$

$$\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times \frac{16-2r}{r} \quad (1)\ (1)$$

$$\therefore A = \frac{1}{2}r\,\frac{2(8-r)}{1} = r(8-r)$$

or $\boxed{A = 8r - r^2}$ (1)

(8) $P(x,y)$

$y = 5$ $\quad\quad$ $3x + 4y - 16 = 0$

$y - 5 = 0$

$$\therefore \frac{|y-5|}{\sqrt{0^2+1^2}} \overset{(1)}{=} \frac{|3x+4y-16|}{\sqrt{3^2+4^2}}$$

$$5|y-5| \overset{(1)}{=} |3x+4y-16|$$

(i) $5(y-5) = 3x + 4y - 16$

$0 = 3x + 4y - \sqrt{5}\,y - 16 + 5\sqrt{5}$

$0 = 3x - y + 9 \quad \therefore \boxed{y = 3x+9}$

OR

(ii) $-5(y-5) = 3x + 4y - 16$

$-5y + 25 = 3x + 4y - 16$

$-9y = 3x - 41$

$$\boxed{y = -\frac{3}{9}x - \frac{41}{9}}$$

locus of P is

(1) i) $y = 3x + 9$

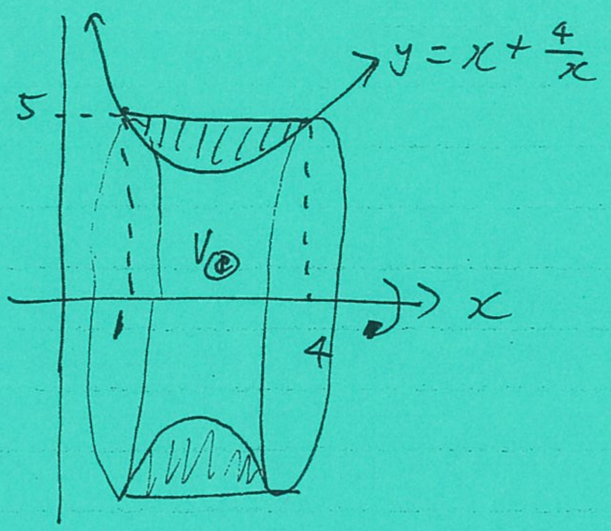
or ii) $y = -\frac{1}{3}x - \frac{41}{9}$

if it has to lie in shaded region

$\therefore$ gradient must be negative

(1) $\boxed{\therefore y = -\frac{1}{3}x - \frac{41}{9} \quad \text{or } 0 = 3x + 9y - 41}$

(9) $y = x + \frac{4}{x}$



$V = V_{cylinder} - V_{©}$

$$= \pi r^2 \times h - \pi\int_1^4 \left(x + \frac{4}{x}\right)^2 dx$$

$V_{©}$ = volume under $y = x + \frac{4}{x}$

$$\therefore V_{©} \overset{(1)}{=} \pi\int_1^4 \left(x + \frac{4}{x}\right)^2 dx$$

$$= \pi\int_1^4 x^2 + 8 + \frac{16}{x^2}\,dx \quad \rightarrow 16x^{-2}$$

$$= \pi\left[\frac{x^3}{3} + 8x + \frac{16x^{-1}}{-1}\right]_1^4$$

$$= \pi\left[\frac{4^3}{3} + 32 - \frac{16}{4} - \left(\frac{1}{3} + 8 - 16\right)\right] \quad (1)$$

$$\boxed{V_{(1)} = 57\pi}$$

$r = 5 \quad h = 3$

$V_{cylinder} = \pi \times 5^2 \times 3 = 75\pi$ (1)

$\therefore V = 75\pi - 57\pi = \boxed{18\pi}$ (1)