Advanced Mathematics

December 2010

Time: 50 minutes + 5 minutes reading time

DIRECTIONS

- Full working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only (*not pencils*) to write your solutions.
- No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.
- Write your teacher's name and your name on the cover sheet provided
- At the end of the exam, staple your answers in order behind the cover sheet provided, and your questions on the back
- Approved Maths aids and calculators may be used

1. Find

a) $\int (3x-7)^{15}dx$ — 2

b) $\int x\sqrt{x}\,dx$ — 2

c) $\int_0^5 \frac{7x^4-1}{x^2}dx$ — 3

2. Evaluate

$\sum_{k=2}^{5}(4k+1)$ — 2

3. Archie invests \$1500 at 6.24% per year compound interest, compounding quarterly. Calculate the value of the investment after 5 years. — 2

4. The sum of the first n terms of a certain arithmetic series is given by $S_n = \frac{n(3n+1)}{2}$ — 2

Find T_{17}.

5. Find the number which when added to each of 2, 6 and 13 will give the first 3 terms of a geometric progression. — 3

6. The tenth term of an arithmetic sequence is 29, and the fifteenth term is 44.

a) Find the value of the common difference and the value of T_1. — 3

b) Find the sum of the first 75 terms. — 2

7. For the function $f(x)$, $\int_1^6 f(x)\,dx = 7$ — 2

Hence, calculate $\int_1^6 (f(x)+3)\,dx$

8. The following table lists the values of a function $y = f(x)$ for 3 values of x

x	2.1	2.3	2.5
y	3	3.8	4.2

a) Use Simpson's rule and these values to estimate $\int_{2.1}^{2.5} f(x)\,dx$ to 2 decimal places — 2

b) By using the trapezoidal rule with three function values, find the approximate volume of the solid generated when the area enclosed between the curve $y = f(x)$, the x-axis and the lines $x = 2.1$ and $x = 2.5$ is rotated about the x-axis. Give your answer in terms of π. — 3

9. a) Express $0.1\dot{4}$ as an infinite series — 1

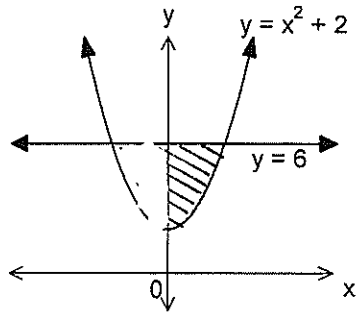
b) Hence express $0.1\dot{4}$ as a fraction with no common factors. — 2

10. To calculate the area of the region bounded by the curve $y = x^2 - 2x$ and the x-axis between $x = 0$ and $x = 3$, Harry used $\int_0^3 x^2 - 2x \; dx$

a) Explain why Harry's method of calculating this area is incorrect. 1

b) Find the required area. 3

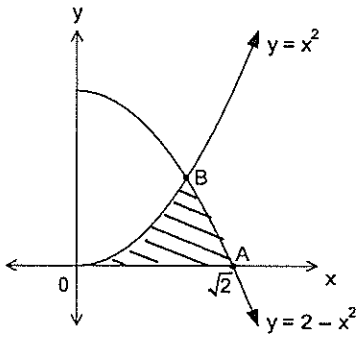
11.



In the diagram, the shaded region is bounded by the parabola $y = x^2 + 2$, the y-axis and the line $y = 6$

Find the volume of the solid formed when the shaded region is rotated about the y-axis 3

12.



The shaded region OAB is bounded by the parabolas $y = x^2$ and $y = 2 - x^2$ and the x-axis between $x = 0$ and $x = \sqrt{2}$

a) Find the coordinates of B 1

b) Calculate the area OAB (2 decimal places) 3

13.

a) Joe invests a fixed amount, P at the beginning of each year into a superannuation fund for 10 years. The compound interest rate is 4%pa, compounded annually. By establishing a series, find the amount P, invested if the value of the superannuation account at the end of the 10^{th} year was \$89900, to the nearest dollar. 4

b) Joe then retires and intends to draw a set amount M from this fund each month. He finds a bank that offers him 4.8%pa on his lump sum compounding monthly and he expects to withdraw M for the next 15 years.

i) Show that at the end of the 2^{nd} month the amount remaining is $A_2 = 89900(1.004)^2 - M(1.004 + 1)$ 1

ii) Hence calculate M to the nearest dollar by first showing that

$$M = \frac{359.6(1.004)^{180}}{1.004^{180} - 1}$$ 3

~ END OF EXAM ~

Solutions

yr 11 2unit Dec 2010 /50.

1. a) $\int (3x-7)^{15}\,dx = \frac{(3x-7)^{16}}{3\times 16} + C$ ①

$= \frac{(3x-7)^{16}}{48} + C$ ①

b) $\int x^{\frac{3}{2}}\,dx = \frac{2x^{\frac{5}{2}}}{5} + C$ ① ①

c) $\int_0^5 7x^2 - x^{-2}\,dx = \left[\frac{7x^3}{3} + \frac{1}{x}\right]_0^5$ ①

$= \left(\frac{7\times 5^3}{3} + \frac{1}{5}\right) - (\frac{7}{3}+1)$ ①

$= 288.53$

[7]

2. $\sum_{K=2}^{5}(4K+1) = 9 + 13 + 17 + 21$ ①

$= 60$ ①

[2]

3. $A_{20} = 1500\left(1 + \frac{0.0624}{4}\right)^{20}$ ①

$= \$2044.30$ ①

[2]

4. $S_n = \frac{n(3n+1)}{2}$

$S_{17} = \frac{17(3\times 17+1)}{2}$ ① $S_{16} = \frac{16(3\times 16+1)}{2}$

$T_{17} = S_{17} - S_{16}$

$= 442 - 392.$

$= 50.$ ①

[2]

5. $2+x$, $6+x$, $13+x$ — G.P.

$\therefore \frac{6+x}{2+x} = \frac{13+x}{6+x}$ ①

$(6+x)^2 = (13+x)(2+x)$

$36 + 12x + x^2 = 26 + 15x + x^2$ ①

$10 = 3x$

$x = \frac{10}{3}$ ①

[3]

6. $T_{10} = a + 9d = 29$ ①

$T_{15} = a + 14d = 44$

a) $T_{15} - T_{10} = 5d = 15$

$d = 3$ ①

sub into T_{10}

$a + 27 = 29$

$a = 2$ ①

$\therefore T_1 = 2,\ d = 3.$

[3]

7. $\int_1^6 f(x)\,dx = 7$

$\int_1^6 (f(x)+3)\,dx = 7 + [3x]_1^6$ ①

$= 7 + (18-3)$

$= 22.$ ①

[2]

6b)

$S_{75} = \frac{75}{2}(2\times 2 + 74\times 3)$ ①

$= 8475$ ①

[2]

8. a) $\int_{2.1}^{2.5} f(x)\,dx = \frac{0.2}{3}\{3 + 4\times 3.8 + 4.2\}$ ①

$= 1.4933\ldots$

$\doteqdot 1.49u^2$ (2dp) ①

b)

y	3	3.8	4.2
y^2	9	14.44	17.64

$V = \pi\int_{2.1}^{2.5} y^2\,dx$ ①

$= \pi\times\frac{0.2}{2}\times\{9 + 2\times 14.44 + 17.64\}$ ①

$= 5.552\pi u^3$ ①

[5]

9. a) $0.1\dot{4} = 0.1 + \frac{4}{100} + \frac{4}{1000} + \frac{4}{10000} + \cdots$

or $= 0.1 + 0.04 + 0.004 + 0.0004 + \cdots$ (1)

b) $\lim S = \frac{a}{1-r}$

$= \frac{\frac{4}{100}}{1-\frac{1}{10}}$

$= \frac{4}{90}$ (1)

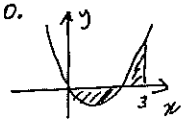
$\therefore 0.1\dot{4} = \frac{1}{10} + \frac{4}{90}$

$= \frac{13}{90}$ (1)

[3]

• some may use other method (still allocate 2 marks)

10.



a) Area under x axis is neg and cancels out area above curve (1) something logical

b) $A = \left| \int_0^2 x^2 - 2x \, dx \right|$ (1) $+ \int_2^3 x^2 - 2x \, dx$

$= \left| \left[\frac{x^3}{3} - x^2 \right]_0^2 \right| + \left[\frac{x^3}{3} - x^2 \right]_2^3$

$= \left| \frac{8}{3} - 4 - 0 \right| + \left(9 - 9 - \left(\frac{8}{3} - 4 \right) \right)$ (1)

$= \frac{4}{3} + \frac{4}{3}$

$= \frac{8}{3} u^2$ (1)

[4]

11. $V = \pi \int_2^6 y - 2 \, dy$ (1)

$= \pi \left[\frac{y^2}{2} - 2y \right]_2^6$ (1)

$= \pi \left(18 - 12 - (2 - 4) \right)$

$= 8\pi u^3$ (1)

or dec answer

25.13...

[3]

12.

a) $y = x^2$ --- ①

$y = 2 - x^2$ --- ② (1)

①=②

$x^2 = 2 - x^2$ $\therefore B = (1, 1)$

$x^2 = 1$

$x = \pm 1$

b) $A = \int_0^1 x^2 \, dx + \int_1^{\sqrt{2}} 2 - x^2 \, dx$ (1)

$= \left[\frac{x^3}{3} \right]_0^1 + \left[2x - \frac{x^3}{3} \right]_1^{\sqrt{2}}$ (1)

$= \frac{1}{3} - 0 + \left(2\sqrt{2} - \frac{2\sqrt{2}}{3} - \left(2 - \frac{1}{3} \right) \right)$

$= \frac{1}{3} + \frac{4\sqrt{2} - 5}{3}$

$= \frac{4\sqrt{2} - 4}{3}$

$= 0.55 u^2$ } (1)

[4]

13.

a) $A_1 = P(1.04)^{10}$ $A_2 = P(1.04)^9$ $A_3 = P(1.04)^8$ . . . (1)

$A_{10} = P(1.04)$

$\therefore$ Total $A = P(\underbrace{1.04 + 1.04^2 + 1.04^3 + \cdots 1.04^{10}}_{\text{G.P}})$ (1)

$a = 1.04$ $r = 1.04$ $n = 10$.

(1) $89900 = P \times \frac{1.04(1.04^{10} - 1)}{1.04 - 1}$

$P = 7199.86...$

$\fallingdotseq \$7200$ (1)

b) $r = 4.8\%$ pa $n = 180$

i) $= 0.004$

$A_1 = 89900(1.004) - M$ (1)

$A_2 = A_1(1.004) - M \rightarrow 89900(1.004)^2 - (M \times 1.004) - M.$

$= 89900(1.004)^2 - M(1.004 + 1)$

ii) $A_3 = 89900(1.004)^3 - M(1.004^2 + 1.004 + 1)$

$\therefore$ (1)

$A_{180} = 89900(1.004)^{180} - M(1 + 1.004 + \cdots$ (1)

but $A_{180} = 0$

$0 = 89900(1.004)^{180} - M\left(\frac{1(1.004^{180} - 1)}{1.004 - 1} \right)$

$M = 89900(1.004)^{180} \times \frac{0.004}{1.004^{180} - 1}$ (1)

$= 359.6(1.004)^{180} / 1.004^{180} - 1 = \$701.59$

[8]